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A

SYSTEM

OF

MECHANICS AND HYDROSTATICS,

BEING THE

SUBSTANCE OF LECTURES UPON THOSE BRANCHES OF NATURAL PHILOSOPHY,

By the Rev. T. PARKINSON, M.A. F.R.S.
FELLOW AND TUTOR OF CHRIST'S COLLEGE, CAMBRIDGE.

CAMBRIDGE,

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MDCCLXXXIX.

A

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OF

MECHANICS AND HYDROSTATICS,



BEING THE

SUBSTANCE OF LECTURES UPON THOSE SUBJECTS,

IN NATURAL PHILOSOPHY,

By the Rev. T. PARKINSON, M.A. F.R.S.

FELLOW AND TUTOR OF CHRIST'S COLLEGE, CAMBRIDGE.

CAMBRIDGE:

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ERRATA.

MECHANICS.

Page 24. Art. 48. *for* $LA : L\,a$ *read* $L\,m : LA$.
26. Art. 57. *for* $A : a :: B : b$ *read* $A : a :: b : B$, and *for* $b : B :: a : A$ *read* $b : B :: A : a$.
42. *dele* Art. 89.
128. line 12 from the bottom, *for* FD *read* FP.
147. line 1st *for* EF *read* DC.
152. Art. 384. *for* sin. $\angle BCV$ *read* sin. $\angle BVC$.
157. *for* Plate XIV. *read* XV.
162. Art. 406. in this analogy $A : B :: BP : AB$ *for* AB *read* AP.
211. Art. 530. *for* S *read* F.
247. Art. 639. *read* $\angle MDR = \frac{1}{4} \angle RCD$ and $DF = 4\,NR$.

HYDROSTATICS.

105. Art. 181. *for* $lm \times ln$ *read* $Dm \times Dp$, in line 10 *dele* at
108. Art. 195. line at the bottom *read* which is diminished by heat and encreased by cold.

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PREFACE.

THE want of ſyſtematic treatiſes on mechanics and hydroſtatics hath long been conſidered as equally troubleſome to the tutor, and diſcouraging to his pupils, and firſt induced a deſire to facilitate the attainment of theſe branches of ſcience by a collection, and methodical arrangement, of their ſcattered parts. The preſent coincides, very nearly, with the propoſitions, or heads of lectures, upon this ſubject, adopted by the generality of tutors in the univerſity, and is recommended by an obvious connexion and regular order of dependancy. In all natural ſcience, the analytic method of reaſoning neceſſarily precedes the ſynthetic, and the converſe of this order would terminate in uncertainty and chimera. The exiſtence and delineation of thoſe properties of matter, which are conceived to generate the phenomena of preſſure and motion, are therefore conciſely and generally premiſed in the firſt ſix chapters, and ſupply data, derived from experience and uncontroverted facts, for ſynthetic demonſtrations. The general properties of motion are deſcribed in the chapter upon ſolidity, becauſe the cleareſt conceptions of motion are derived from impulſe, which ariſes from ſolidity; and the laws of motion are introduced in the chapter upon the inertia of matter, becauſe they are inſeparably connected

with

with it. The compoſition and reſolution of forces are immediately ſubſequent; becauſe every propoſition upon this ſubject is deducible from the ſecond law of motion, and may be eſteemed a corollary to it. The connection of the other parts of the treatiſe is, I truſt, equally natural and obvious. To have diveſted this elementary branch of philoſophy of that dry, uninviting aſpect, ſo frequently and feelingly lamented by the young pupil, was certainly moſt deſirable; but the ſame complaint is applicable to the elements of all ſcience, and could not, I believe, be removed without an entire change of the formality of preciſe definitions and propoſitions into a more undefined and leſs ſcientific form of compoſition, nor conſequently without ſacrificing real advantage to leſs material amuſement. The following performance, therefore, claims little more than the inferior merit of facilitating the progreſs of the ſtudent by a ſelection, from the works of others, which may ſuperſede the neceſſity of applying to a multitude of books, and an arrangement coinciding with his lectures. The whole is written in the ſame language for the ſake of uniformity; demonſtrations are dilated or contracted as was deemed expedient; and ſometimes, though as ſeldom as poſſible, new proofs are given. It is no eaſy taſk to compoſe a ſyſtem equally accommodated to every deſcription of underſtandings, and I dare not hope to have accompliſhed it: for the benefit of thoſe, who may be diſſatisfied with the diſcuſſion of any ſubject here, accurate references to the page or chapter of the beſt writers where it is treated, are always inſerted. To be of ſervice to the ignorant and uninformed was the chief motive for undertaking this work, and the ſole object of attention in the execution of it: this laſt conſideration may ſerve to obviate ſome objections, which naturally will occur to more enlightened readers already converſant with the ſubject. Leſt I appear to preſume upon the experience that may be ſuppoſed to attend my ſituation, I beg leave to explain my-

myſelf. When ſeveral demonſtrations of the ſame propoſition are exhibited, let it not be inferred that one was deemed inſufficient to eſtabliſh its truth and required an auxiliary: when any important ſubject is treated with prolixity, and extended beyond the limits uſually preſcribed to an elementary treatiſe, let it not be attributed to negligence or inadvertency: they are both the effects of deſign, and I am warranted, by my own experience at leaſt, to aſſert their utility. It muſt be remembered, that clear and adequate ideas upon a new ſubject are not communicated by a tranſient impreſſion, and that the capacity and comprehenſion of an uninformed mind are only expanded and improved by repeated exertions and aſſiduity. Science implies ſomething more than a mere ability to go through a demonſtration; and a clear comprehenſive knowledge of a queſtion, which may be competent for the ſolution of problems and diſſipation of doubts and objections, is perhaps only to be attained by long reflection, and ſtudiouſly contemplating it on every ſide; and theſe are certainly much aſſiſted by different demonſtrations, which place the ſubject in different points of view, and by tracing its affinity with other truths ſimilar to, or deducible from, it. A ſingle demonſtration may eaſily be too long, and too anxious a deſire to be perſpicuous may occaſion perplexity and confuſion; but it is not eaſy to give too many demonſtrations of a fundamental propoſition, or too many problems and corollaries reſulting from it. This ſpecies of prolixity appears to me to be extremely uſeful, as it may, at leaſt, teach a young ſtudent the art of thinking, an art which can only be acquired, and is found indeed to be no eaſy acquiſition. For theſe reaſons, the reader will obſerve ſome leading ſubjects extended beyond the bounds abſolutely neceſſary, and ſome parts will apparently be ſuperfluous: I ſhall hold him juſtified in thinking ſo, when he ſhall be ſo informed on the ſubject as to pronounce them uſeleſs to him.

I can-

PREFACE.

I cannot conclude this preface without expreſſing my ſincere thanks to Mr. Fiſher and Mr. Vince, of Caius College; to the firſt for his general theorem of the wedge; to the latter for his friendly reviſion of the whole work. Had I conſulted theſe friends more frequently, many errors and imperfections, for I believe there are many, might poſſibly have been avoided. My thanks are alſo juſtly due to thoſe gentlemen who have kindly honoured this undertaking with their approbation and ſupport, and I willingly embrace this public opportunity of acknowledging my obligation.

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INTRO-

INTRODUCTION

TO THE STUDY OF

NATURAL PHILOSOPHY.

1. NATURAL phenomena, in the wideſt acceptation of the terms, denote any effects in the material part of the creation addreſſed to one or more of the ſenſes; and natural philoſophy is the hiſtory of theſe phenomena, and an inveſtigation of the cauſes employed in their production. A phenomenon may itſelf be a natural cauſe productive of numberleſs effects, and each of theſe may alſo be a cauſe of others, &c.; for, of that infinite variety of events obſervable in the material world, none are induced per ſaltum, but effect is dependent upon effect in contiguous ſucceſſion. As matter is totally inactive, and incapable of communicating motion to itſelf, all its motions, and powers of producing a change of motion, in the various operations of nature, are derivative: but the inſtruments immediately directing the movements of the ſeveral parts of the ſyſtem elude the inquiſition of human ability, and whether any inexplicable effect be owing to the Creator's immediate fiat, or ſome ſecondary material power, cannot be known; for the action of a pure ſpirit upon matter cannot be comprehended: but many ſubordinate inſtruments in the government of nature are conſpicuous, matter being impreſſed by its great Creator with ſeveral attributes, which appear, and are conceived by ſome philoſophers, to reſide in it, miniſterial to the continuance of exiſtence and preſervation. The rules by which theſe attributes are directed in their operations, are called natural principles or laws, becauſe in ſimilar circumſtances, they are invariably the ſame; and the attributes themſelves are generally called powers or forces,

from the ſimilitude of their effects to thoſe produced by animal exertions: ſuch are gravity, coheſion, elaſticity, magnetiſm, electricity. The uniform and regular action and utility of ſome of theſe natural powers are very obſervable; but the laſt powers ſeem to be ſtill in a ſtate of analyſis, and the laws, by which their influence is directed, very imperfectly aſcertained. The attraction of coheſion, or of that power by whoſe influence the minute particles of matter tend to each other at ſmall diſtances, is the cement which prevents the diſperſion of the component parts of matter, and, as far as we are competent to decide, adminiſters to the growth of bodies in the animal, vegetable, and foſſil kingdoms. The figures and motions of the great bodies compoſing the ſolar ſyſtem are preſerved by an uninterrupted exertion of the attraction of gravity. Nor is the oppoſite quality of elaſticity or repulſion leſs regular or important. The particles of air are endued with this repulſive power which is eſſential to the preſervation of all bodies contiguous to it, and a diminution of it would proportionably diminiſh its ſalutary influence, rendering it noxious to animal and vegetable life; and many hard bodies are obviouſly poſſeſſed of this quality probably to preſerve their ſpecific nature by protecting their conſtituent particles from the effects of attrition, &c. The operations of other attributes of matter, however deſultory and accidental apparently, are, upon the fulleſt information, diſcovered to be reſtrained within preſcribed limits, whoſe obſervance is productive of harmony, and violation of diſorder. Plants and animals are always produced from their proper ſeed; the reflection and refraction of light are effected according to inviolate laws; matter, by its vis inertiæ, reſiſts the action of any material power, and when in motion moves, according to determinate rules, &c. To deſcribe the various phenomena exhibited in the production and change of motion in the material world, whether explicable or not, promiſcuouſly, is the buſineſs of the natural hiſtorian; and to ſelect thoſe which are explicable, and inveſtigate the ſecondary powers, or qualities, conceived to be reſident in matter, by whoſe inſtrumentality they appear to be effected, is the province and proper occupation of the natural philoſopher.

2. The

2. The hiſtorical part of philoſophy, or deſcription of natural phenomena, is immediately tranſcribed from the works of nature; and, from the inadequacy of our ideas of matter, an inveſtigation of the ſecondary material powers producing them muſt be derived from the ſame ſource. Some phenomena being obſerved to be invariably coexiſtent or ſucceſſive, a connexion is underſtood, or preſumed, to obtain between them; and, the object of philoſophy being an evolution of this connexion, the works of nature where only it is diſcoverable, muſt be attentively explored, the relation noted, and extended, by a cautious ſelection of all the circumſtances of ſimilitude in other phenomena, to a common principle or law of nature. The following proceſs is therefore adopted by the beſt philoſophers, and is the only certain method of philoſophizing.

Firſt, By repeated and accurate obſervations upon matter, or upon ſome general ſimilar phenomena in the material world, the exiſtence and qualities of any individual cauſe are demonſtrated from the phenomena evidently and invariably attached to it; the cauſe of this cauſe, and the cauſe of this laſt, &c. and the effects ſucceſſively more ſubordinate and particular reſulting from the phenomena, conſidered as mechanical cauſes, are then inveſtigated by again exploring the works of nature, and this proceſs is repeated to the limit of human ability. How far the chain of theſe ſecondary material powers, either progreſſive or regreſſive, from thoſe that are more general to their ſubordinate effects, and vice verſa, is extended, is undiſcoverable; but the philoſopher's reſearches are ſoon limited by the occurrence of bodies inconceivably minute, or removed to immeaſurable diſtances. An examination of the few powers that are known is, however, a rational and not unprofitable employment; for they exhibit proofs of unbounded power, conſummate wiſdom, and paternal benevolence in the great Creator; and by their connexion with many uſeful arts, are made ſubſervient to the wants and infirmities of his creatures. *Secondly*, The intenſity at a given diſtance, and law of variation at different diſtances, of any material power being aſcertained by the menſuration of its effects, it is aſſumed as a principle, and from its influence, all ſimilar phenomena are demonſtrated to reſult by

mathematical or other ſcientific methods of reaſoning. For many effects are, upon examination, found to bear ſtrong marks of ſimilitude, which to a negligent obſerver exhibit no likeneſs; and, theſe being diveſted of all adventitious particularities, and ranged under the ſame cauſe, a few principles are diſcovered to pervade the whole ſyſtem of matter, producing innumerable phenomena. The firſt of theſe is called the analytic, and the ſecond the ſynthetic method of philoſophizing.

3. Examples illuſtrating the analytic method of reaſoning.

EXAMP. I. The ſpherical figure of the earth proved analytically.

PHENOMENA.

PHENOM. I. The altitude of either pole of the equator is always equal to the latitude of the obſerver.

PHENOM. II. In an eclipſe of the moon a ſection of the earth's ſhadow is always terminated by a circular arc.

Many other phenomena might be adduced in proof of the globular figure of the earth, but it is eſtabliſhed by theſe, becauſe no other afford a ſolution of them. This figure was impreſſed at the original formation of the earth, and is preſerved by an unremitted exertion of the attraction of gravity; and here the analyſis terminates, the cauſe of gravity being undiſcovered.

4. EXAMP. II. The diurnal revolution of the earth proved analytically.

PHENOMENA.

PHENOM. I. The ſun and moſt of the planets revolve round their axes, and a ſimilar motion of the earth is analogous to theſe.

PHENOM. II. The ſun, planets and fixed ſtars appear to revolve uniformly in circles, whoſe planes are perpendicular to, and centers in, a line paſſing through the center of the earth, and perform one revolution in twenty-four hours nearly.

PHE-

PHENOM. III. The gravity of bodies is leaſt under the equator, and increaſes as they recede from it.

If this motion of the heavenly bodies were real, they would be retained in their orbits by forces tending to their correſponding centers in the axis of the world (Newt. ſect. 2. prop. 2.), which are imaginary points in which no attractive powers reſide; and ſome of theſe motions could not be effected without the influence of forces infinitely greater than any mechanical powers yet diſcovered. They are therefore apparent only, and reſult from the rotation of the earth round an axis; and this is confirmed by phenom. 2. which is eaſily explicable by, and inexplicable without, ſuch rotation.

5. EXAMP. III. The properties of light inveſtigated analytically.

PHENOMENA.

PHENOM. I. If the ſun's rays be admitted, through a ſmall aperture, into a dark room, and refracted through a glaſs priſm, an oblong image conſiſting of ſeveral colours will be formed upon a ſheet of paper placed at a proper diſtance.

PHENOM. II. If another priſm be placed behind the former, ſo that their axes be perpendicular, the image will be refracted on one ſide, of the ſame length and breadth as before.

PHENOM. III. Any one colour refracted through any number of priſms, always exhibits a circular image of that colour.

If the ſun's rays were equally refrangible, its image would be circular, and, if differently refrangible, oblong; and conſequently (phenom. 1ſt and 2d) light is compoſed of heterogeneous parts differently refrangible. If the ſun's rays ſplitted or dilated by refraction, the image would be circular and increaſed by every refraction, therefore (phenom. 2d) they do not ſplit or dilate; and (phenom. 3d) the properties of homogeneous rays are innate and unalterable by refraction.

6. The

6. The concluſions thus derived by analyſis are analogical, as they reſult from a compariſon between preſent and paſt phenomena, and will be juſt only when theſe compariſons are repeatedly made, and the ſimilitude of the phenomena perfectly eſtabliſhed. In example the 1ſt. the altitude of the pole being found equal to the latitude of the obſerver, and a ſection of the earth's ſhadow to be a circular arc, and theſe phenomena being invariably the ſame in innumerable trials, an aſſurance that they reſult from ſome fixed cauſes and will never vary, ariſes and commands our aſſent.

Philoſophy therefore, when real and not fantaſtical, has for its baſis uniform and uncontroverted experience; and hence appears the neceſſity of adhering to the firſt rule of philoſophizing.

RULE I.

No more cauſes of natural events ought to be admitted than are real, and ſufficient to explain the phenomena.

7. Of thoſe bodies which are neither concealed by their minuteneſs, nor remote diſtance, few can undergo an analytical diſcuſſion; and philoſophy would be very limited, were it confined to thoſe only which have actually been the objects of experiments; but it is rendered univerſal by obſerving the following rules.

RULE II.

Effects of the ſame kind are to be aſcribed to the ſame cauſe.

The ſame cauſe occaſions the deſcent of bodies in different places of the earth, reſpiration in different animals, the ſenſation of heat from a culinary fire and the ſun, ſolutions of bodies in different menſtruums, and of water in air, &c.

RULE III.

8. *Thoſe qualities of matter which do not vary, and are found in all bodies that admit of experimental examination, ought to be conſidered as qualities of all bodies in general.*

Thus

Thus the attraction of gravity is diſcovered, by experiments and aſtronomical obſervations, to pervade all matter with which we are acquainted: all bodies near the earth's ſurface, and the moon, gravitate towards the earth; the waters of the ſea gravitate towards the moon; the ſun, planets, and comets, gravitate towards each other mutually; and of this principle no bodies acceſſible to experiments can be diveſted, and therefore, from this rule, it is inferred to be an univerſal attribute of matter.

9. As much falſe philoſophy hath been diſſeminated by fanciful hypotheſes and mere metaphyſical conſiderations, unſupported by the reality of facts, the only ſources of natural ſcience, the utility of the following rule is apparent.

RULE IV.

In experimental philoſophy, propoſitions collected from phenomena by induction are to be deemed, notwithſtanding contrary hypotheſes, either accurately true or very nearly ſo; until other phenomena occur by which they may be rendered either more accurate, or liable to exception.

10. The exiſtence, quantity at a given diſtance, and law of variation at different diſtances, of any natural power being demonſtrated by the analytic method of reaſoning, the only way by which they can be known; they are aſſumed as eſtabliſhed general principles, and their adequacy to the production of other phenomena, ſimilar to thoſe uſed in the analyſis, is proved ſynthetically.

Examples illuſtrating the ſynthetic method of reaſoning.

EXAMP. I. The influence of the attraction of gravity, its quantity and law of variation, being aſcertained analytically, it is aſſumed as an eſtabliſhed general principle, and demonſtrated geometrically to be an adequate cauſe of the motions of pendulums, projectiles, preceſſion of the equinoxes, irregularities of the moon's motion, riſing and ebbing of the ſea, and innumerable other phenomena.

EXAMP.

EXAMP. II. The weight of the air being aſcertained by experiments, it is ſuppoſed to be allowed, and affords an eaſy ſolution of the phenomena of pumps, ſyringes, barometers, &c.

EXAMP. III. The equality of the angles of incidence and reflection of light, of the conſtant ratio obtaining between the ſines of incidence and refraction, and the unequal refrangibility of different colours, are diſcovered by experiments; and, being ſuppoſed to be univerſally true, they afford eaſy explanations of the figure and magnitude of images formed by reflection and refraction, of the rainbow, &c.

EXAMP. IV. The ſpherical figure of the earth, its diurnal motion, and obliquity of the ecliptic are diſcovered by repeated obſervation, and, being aſſumed as known principles, they afford an eaſy ſolution of the doctrine of the ſphere, of the art of dialling, of day and night and their inequalities, of heat and cold, and many other phenomena.

11. Of theſe two methods of reaſoning, the latter may ſtrictly be denominated ſcience; for the exiſtence of any natural power, its magnitude and variation being preſuppoſed, it is demonſtrated geometrically to be competent to the production of various effects, and, were this power changed or annihilated, the truth of this reaſoning would remain unaltered. Whether this ſcience be chimerical, or accord with actual exiſtence, depends upon the truth of the preſuppoſed data, or analytic concluſions, which admit of various degrees of conviction from bare preſumption to certainty. The exiſtence of ſome undiſcovered and ſtill unknown quality of matter, collected only from a few experiments, amounts only to a preſumption; but the degree of probability increaſes with their number, and, by repeated and uniformly concurring trials, eſtabliſhes at length an evidence as unqueſtioned as the exiſtence of matter or the ſenſes. That the qualities of matter will remain unchanged and continue always to produce the ſame effects, depends upon the will of that divine Agent, who ordained that the

various

various events in the material world ſhould be effected by the intervention of ſimple and general material cauſes, acting with invariable conſtancy and uſefulneſs, and only ſuffered to tranſgreſs their preſcribed bounds according to the dictates of infinite wiſdom, power and goodneſs. Preſuming therefore upon the identity of the ſenſes and the uniformity of natural operations, the ſtudy of philoſophy may be deemed ſcientific; for, being founded upon the baſis of uniform and uncontroverted experience, and geometric demonſtration, its certainty will continue with them, and conſequently can only be affected or ſubverted by a change or ſubverſion of the conſtitution of nature.

12. Of all magnitudes, numbers are the moſt ſimple, and their minute differences moſt eaſily diſcriminated, and therefore more ſuſceptible of clear ideas than any other magnitudes; and the relations ſubſiſting between powers or forces, and the changes or velocities produced by their exertions, are ſaid to be known, when reduced to numeral expreſſions, though the mode of producing theſe effects be unknown, and perhaps undiſcoverable. The next, in degree of ſimplicity and clearneſs of comprehenſion, are lines, ſurfaces and ſolids, being permanent and eaſily compared by juxta-poſition. The conception of other magnitudes is more remote and difficult. Forces, velocities, and times, having no permanent repreſentatives like numbers and geometric magnitudes, and diſappearing when not actually ſubſiſting, are beſt underſtood from their effects; and this ſtudy is therefore much ſimplified by finding magnitudes of eaſier conception, and more eaſily meaſurable and permanent, as numbers, lines and ſurfaces, whoſe relation is the ſame with that of times and velocities, and natural powers. Philoſophy may conſequently be reduced to an inveſtigation of the relation ſubſiſting between lines, ſurfaces, &c. which vary as, and may be denominated the repreſentatives of, natural powers and their effects; and a knowledge of ratios, or of the rules by which theſe relations are increaſed and diminiſhed, is a neceſſary lemma to this ſtudy and here premiſed.

RATIOS.

13. DEF. *WHATEVER is capable of increaſe or decreaſe, is called magnitude or quantity, as numbers, lines, velocities, forces, &c.*

14. DEF. *Ratio is the mutual relation of two magnitudes of the ſame kind in reſpect of their greatneſs or ſmallneſs; the firſt is called the antecedent and the ſecond the conſequent of the ratio.*

15. Ratio is therefore a compariſon, which always implies a ſimilitude, and is limited by the definition to the relative greatneſs or ſmallneſs of the quantities compared. The magnitude of the ratio of equal quantities is equal to nothing; for the exiſtence of ratios reſults from the inequality of the quantities compared, though not meaſured by it; and it is evident, that the magnitude of a ratio may increaſe or decreaſe through every ſtage of aſſignable quantity.

EXAMPLES.

FIG. I. EXAMP. I. If the diſtance of E from B be equal to nothing, the diameter QB, and chord QE coincide, and their ratio is a ratio of equality, and its magnitude equal to nothing; but as the arc BE increaſes from nothing to a ſemicircle, the magnitude of this ratio increaſes from nothing through every ſtage of aſſignable magnitude, and, when E and Q coincide, is unaſſignably great.

EXAMP. II. The ratio of the tangent TA to the ſine SN of an arc SA, or of the ſecant CT to the radius CS, is a ratio of equality, and its magnitude equal to nothing, when the arc vaniſhes, and S and A coincide. But as the arc SA increaſes from nothing to a quadrant, the magnitude of this ratio increaſes from nothing through

through every ſtage of aſſignable magnitude, and, when S and Q coincide, is unaſſignably great. If the magnitude of this ratio (when e. g. $SA = 30°$) be expounded by any finite line L, and twice this ratio by twice this line, &c. when S and A coincide, L is equal to nothing; and as S recedes from A, L increaſes and becomes infinitely great when S arrives at Q.

Ratios therefore are magnitudes, and like all other magnitudes the object of ratios, and capable of addition, ſubtraction, multiplication and diviſion. They are poſitive or negative, according to their different effects in addition. The ſum of two poſitive, or of two negative ratios, will conſtitute a poſitive, or negative ratio, greater than either of them; and, according as a poſitive ratio is greater or leſs than a negative one, their ſum will be poſitive or negative; and, if they be equal, their ſum is nothing. The magnitudes of the ratio of $L : M$ and of $M : L$, or of 3 : 2 and of 2 : 3, are clearly equal, but of different denominations; and any third ratio, of the ſame denomination with the ratio of $L : M$, will be equally encreaſed by the addition of the ratio of $L : M$ and ſubtraction of the ratio of $M : L$. If the ratio obtaining between L and M, of which L is the greateſt, be called poſitive, and repreſented by any line or number a, twice this ratio will be repreſented by twice a, and n times this ratio by $n \times a$. The ratio alſo of $M : L$ will be repreſented by $-a$, and $n \times$ the ratio of $M : L$ by $n \times -a$; and a line or number, repreſenting any third affirmative ratio, will be equally encreaſed by the addition of $n \times a$ and ſubtraction of $n \times -a$. As magnitudes are only meaſurable by magnitudes, ſui generis, a line by a line, a ſurface by a ſurface, and a ratio by a ratio: to aſcertain the quantity of any ratio, ſome more ſimple ratio may be uſed as a criterion. Thus logarithms are a ſeries of numbers expreſſing the relation which ſubſiſts between any given ratio, conſidered as a criterion, and all other ratios with which it is compared.

16. DEF. *In a ſeries of magnitudes of the ſame kind, either increaſing or decreaſing, the ratio of the extremes is ſaid to be compounded of the ratios of the intermediate terms.*

In the ſeries of magnitudes A, B, C, D, &c. the ratio of the firſt to the laſt, or of $A : D$, is ſaid to be compounded of the ratios of $A : B$, $B : C$, $C : D$; and any ratio, as that of $P\mathcal{Q} : PR$, is ſaid to
FIG. II. be reſolvable into, and equal to, the ratios of $P\mathcal{Q} : Pa$, $Pa : Pb$, $Pb : Pc \ldots . PL : PR$; for the ratio of $P\mathcal{Q} : PR$ is a real magnitude, and like other magnitudes diviſible into its component parts, which are the ratios of $P\mathcal{Q} : Pa$, $Pa : Pb$, $Pb : Pc$, &c.: therefore the ſum of any number of continued ratios, where the antecedent of any ratio is the conſequent of the preceding, is equal to the ratio of the firſt and laſt terms.

FIG. II. 17. Cor. When any ratio, as that of $P\mathcal{Q} : PR$, is to be divided into any other ratios by an arbitrary inſertion of other quantities Pa, Pb, &c.; theſe are not neceſſarily intermediate, or contained between $P\mathcal{Q}$ and PR, the ratio of $P\mathcal{Q} : PR$ being equal to the ratios of $P\mathcal{Q} : Pv$ and of $Pv : PR$. For the ratio of $P\mathcal{Q} : Pv$ is equal to the ratios of $P\mathcal{Q} : PR$ and of $PR : Pv$ (16), and conſequently the ratio of $P\mathcal{Q} : PR$ is equal to the ratio of $P\mathcal{Q} : Pv$ diminiſhed by the ratio of $PR : Pv$, or added to the ratio of $Pv : PR$ (15).

ADDITION OF RATIOS.

18. PROP. *To add the ratios of* A : B, C : D, E : F, *&c. together.*

When the two ratios of $A : B$ and $C : D$ are to be added, let A
FIG. III. and C, B and D be reſpectively the ſides of two rectangular parallelograms X and Z; and (Euclid. VI. 23.) the ratios of $A : B$ and $C : D$, when compounded, are equal to the ratio of $X : Z$, or of $A \times C : B \times D$, theſe quantities being reſpectively equal to X and Z. When the ratios of $A : B$, $C : D$, $E : F$ are to be added, the ſum of the two firſt is equal to the ratio of $AC : BD$ by the proceſs above; and, by making $A \times C$ and E, $B \times D$ and F, reſpectively the ſides of two rectangles, it appears, by the ſame proceſs, that the ſum of the ratios of $AC : BD$ and of $E : F$ is equal to

the

the ratio of $ACE : BDF$. Whatever be the number of ratios to be added the proceſs is ſimilar, and gives the following

RULE.

Multiply the antecedents together for a new antecedent, and the conſequents for a new conſequent.

19. EXAMP. I. The ſum of the ratios of $1:2$, $3:4$, $5:6$, $7:8$ is equal to the ratio of $1\times3\times5\times7 : 2\times4\times6\times8$, or of $105:384$. This alſo appears from the following analogies and article (16); for (38)

$$1 : 2 :: 105 : 210$$
$$3 : 4 :: 210 : 280$$
$$5 : 6 :: 280 : 336$$
$$7 : 8 :: 336 : 384.$$

The ſum of the ratios of $1:2$, $3:4$, $5:6$, $7:8$, is equal to the ſum of the ratios of $105:210$, $210:280$, $280:336$, $336:384$, or (16) to the ratio of $105:384$, as above.

20. EXAMP. II. When two bodies move with uniform motions, philoſophical writers ſay, that the ſpaces deſcribed S and s are to each other in a ratio compounded of the velocities and times; the meaning of which is, that the ratio of the ſpaces is equal to the ſum of the ratio of the velocities when changed, and the ratio of the times when changed, or that the ratio of $S : s$ is equal to the two ratios of $V : v$ and of $T : t$, or to the ratio of $V\times T : v\times t$, ſuppoſing v and t to repreſent any correſponding values of the velocity and time.

21. Cor. 1. If there be any number of quantities A, B, C, D, &c. of which $A : B :: R : r$ and $B : C :: S : s$ and $C : D :: T : t$, A will be to D as $RST : rst$; for $A : D$ in a ratio compounded of the ratios of $A : B$, $B : C$, $C : D$ (16), or their equals $R : r$, $S : s$, $T : t$, or as $RST : rst$ (18).

SCHO-

SCHOLIUM.

22. When the terms of any ratios are forces, times, velocities, &c. lines or numbers are ſuppoſed to be taken whoſe ratios are the ſame with them, and rectangular parallelograms whoſe ſides are theſe lines, or the products of theſe numbers actually multiplied together, are always implied in the multiplication of ſuch magnitudes. The ſum of the ratios of $L : M$ (denoting forces), of $N : P$ (denoting velocities), of $\mathcal{Q} : R$ (denoting ſpaces), and of $S : \mathcal{T}$ (denoting times), is equal to the ratio of $L \times N \times \mathcal{Q} \times S : M \times P \times R \times \mathcal{T}$, which ſignify two products of numbers whoſe factors are as $L : M$, $N : P$, $\mathcal{Q} : R$, $S : \mathcal{T}$.

SUBTRACTION OF RATIOS.

23. PROP. *To ſubtract the ratio of* C : D *from that of* A : B.

Let the ratio of $A : B$ be equal to the ratios of $C : D$ and $x : y$; and (18) $A : B :: xC : yD$, and $x : y :: \frac{A}{C} : \frac{B}{D}$ (Euc. v. 4.) $:: AD : BC$ (Euc. v. 10.); from hence we have the following

RULES.

RULE I. *Divide the antecedent of the ſubtrahend by the antecedent of the ratio to be ſubtracted for a new antecedent, and the conſequent by the conſequent for a new conſequent.*

Or, II. *Invert the terms of the ratio to be ſubtracted, and proceed as in addition.*

24. EXAMP. I. The ratio of 6 : 5 ſubtracted from the ratio of 3 : 2 is equal to the ratio of $\frac{3}{6} : \frac{2}{5}$ or of 15 : 12, which is thus confirmed. The ratio of 3 : 2 is equal to the ratio of 6 : 4, which is equal to the ratio of 6 : 5 and of 5 : 4 (16); and if the ratio of

6 : 5

6 : 5 be taken away, the remainder is the ratio of 5 : 4 or of 15 : 12 (Euc. v. 15.).

25. EXAMP. II. The ratio of 2 : 3 diminiſhed by the ratio of 4 : 5 is equal to the ratio of $\frac{2}{4} : \frac{3}{5}$ or of 10 : 12, which is thus confirmed. The ratio of 2 : 3 is equal to the ratio of 4 : 6 (Euc. v. 15.), that is, of 4 : 5 and 5 : 6 (16); and, if the ratio of 4 : 5 be taken away, the remainder is the ratio of 5 : 6 or of 10 : 12 (Euc. v. 15.).

MULTIPLICATION OF RATIOS.

26. PROP. *To multiply the ratio of* A : B *by any number* m.

The ratio of $A : B$ added to itſelf is equal to the ratio of $A^2 : B^2$, and this added to the ratio of $A : B$ is equal to the ratio of $A^3 : B^3$, and this repeated m times will clearly give the ratio of $A^m : B^m$ (18), and we have this rule.

RULE.

Involve the terms of the ratio to a dimenſion equal to the multiplier.

27. EXAMP. I. Four times the ratio of $A : B$ is equal to the ratio of $A^4 : B^4$, which is confirmed by the following proceſs; for (38)

$$A : B :: A^4 : A^3B$$
$$A : B :: A^3B : A^2B^2$$
$$A : B :: A^2B^2 : A^2B^2$$
$$A : B :: AB^3 : B^4.$$

Therefore four times the ratio of $A : B$ is equal to the ratios of $A^4 : A^3B$, $A^3B : A^2B^2$, $A^2B^2 : AB^3$, $AB^3 : B^4$, or the ratio of $A^4 : B^4$ (16).

28. EXAMP.

28. EXAMP. II. Five times the ratio of 2:3 is equal to the ratio of $2^5 : 3^5$ or 32 : 243, which alſo appears from the proceſs above and (16). For (38)

$$\begin{aligned} 2 : 3 &:: 32 : 48 \\ 2 : 3 &:: 48 : 72 \\ 2 : 3 &:: 72 : 108 \\ 2 : 3 &:: 108 : 162 \\ 2 : 3 &:: 162 : 243. \end{aligned}$$

Therefore the ratio of 2 : 3 multiplied into five is equal to the ratios of 32 : 48, 48 : 72, 72 : 108, 108 : 162, 162 : 243; or to the ratio of 32 : 243 (16).

DIVISION OF RATIOS.

29. PROP. *To divide the ratio of* A : B *by any number* m.

Let $A = x^m$ and $B = y^m$; and the ratio of $A : B$ is equal to the ratio of $x^m : y^m$ or to m times the ratio of $x : y$ (18), which is therefore $\frac{1}{m}$ part of the ratio of $A : B$, and equal to the ratio of $A^{\frac{1}{m}} : B^{\frac{1}{m}}$; becauſe $x = A^{\frac{1}{m}}$ and $y = B^{\frac{1}{m}}$. Whence we have the following

RULE.

Extract that root of the terms of the ratio which is expreſſed by the diviſor.

30. EXAMP. The ratio of 32 : 162 divided by four is equal to the ratio of $\overline{32}^{\frac{1}{4}} : \overline{162}^{\frac{1}{4}}$ or of 2 : 3. And this is confirmed by the proceſs uſed in example (28), where the ratio of 32 : 162 is equal to the ratio of $\overline{2 : 3 \times 4}$, and conſequently the ratio of 2 : 3 is $\frac{1}{4}$th of the ratio of 32 : 162.

31. PROP. *When the difference of two magnitudes is very ſmall compared with the magnitudes themſelves, their ratio is multiplied or divided*

divided by any number m, *by increaſing or diminiſhing their difference* m *times.*

DEM. Let the two magnitudes be A and $A \pm y$, whoſe difference y is very ſmall compared with A; and m times the ratio of $A : A \pm y$ is equal to the ratio of $A^m : \overline{A \pm y}^m$, or of $A^m : A^m \pm mA^{m-1}y + m \cdot \frac{m-1}{2} A^{m-2}y^2$, &c. or (dividing the antecedent and conſequent by A^{m-1}) to the ratio of $A : A \pm my$; becauſe the terms involving $\frac{y^2}{A}$, $\frac{y^3}{A^2}$, &c. are evaneſcent compared with the two firſt, and may be neglected.

By a ſimilar proceſs $\frac{1}{m}$th part of the ratio of $A : A \pm y$ appears to be equal to the ratio of $A : A \pm \frac{y}{m}$. Q. E. D.

32. EXAMP. Twice the ratio of 11 : 10 is equal to the ratio of $\overline{11}^2 : \overline{10}^2$ or of 121 : 100; and, according to this propoſition, it is the ratio of 11 : 9, or of 121 : 99.

Twice the ratio of 101 : 100 is equal to the ratio of $\overline{101}^2 : \overline{100}^2$ or of 10201 : 10000; and according to this propoſition it is equal to the ratio of 101 : 99 or of 10201 : 9999, which is nearly equal to the former.

33. EXAMP. II. A half of the ratio of 101 : 100 is equal to the ratio of $\overline{101}^{\frac{1}{2}} : \overline{100}^{\frac{1}{2}}$, or of 10.049 : 10 nearly; and by this propoſition it is equal to the ratio of $100\frac{1}{2} : 100$, or of 201 : 200, or of 10.05 : 10.

METHODS OF COMPARING RATIOS.

34. DEF. *Proportion is an equality of ratios. When the ratios of* A : B *and* C : D *are equal, they are ſaid to be proportional, and uſually written thus* A : B :: C : D, *or* A *is to* B *as* C *to* D.

35. Cor. If A, B, C, D be proportional, and $A = \frac{1}{4}B$, or $\frac{m}{n} \times B$, C will be equal to $\frac{1}{4}D$, or to $\frac{m}{n} \times D$; or if these magnitudes be incommensurate, and A be greater, or less, than any part or parts of B, C will be greater, or less, than the same part or parts of D. If A be contained between $\frac{m}{n} \times B$ and $\frac{m+1}{n} \times B$, (m and n being any numbers whatever,) C will also be contained between $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$. This is evident from the definitions of ratios and proportion; for if A were contained between $\frac{18}{17} \times B$ and $\frac{19}{17} \times B$, or $\frac{m}{n} \times B$ and $\frac{m+1}{n} \times B$, and C were not contained between $\frac{18}{17} \times D$ and $\frac{19}{17} \times D$, or $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$, A's magnitude compared with B's, would not be equal to C's magnitude compared with D's, or the ratios of $A : B$ and $C : D$ would not be equal, and they would not be proportional.

36. PROP. *If* A *cannot be greater than, equal to, or less than, any part or parts of* B, *but at the same time* C *is greater than, equal to, or less than, the same part or parts of* D, *they will be proportional, or* A : B :: C : D.

DEM. Let $A : B :: E : D$; and if $A = \frac{m}{n} \times B$, $E = \frac{m}{n} \times D$ (35), and, from the hypothesis, $C = \frac{m}{n} \times D$: therefore $E = C$ and $A : B :: C : D$. Let these magnitudes be incommensurate, and if A be contained between $\frac{m}{n} \times B$ and $\frac{m+1}{n} \times B$, E will be contained between $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$ (35), and, from the supposition,

C will be contained between $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$. The difference therefore between C and E is between $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$, and consequently is not greater than $\frac{D}{n}$; and because this is true whatever be the magnitnde of the number n, which may be unassignably great and $\frac{D}{n} = 0$, C will be equal to E, and $A : B :: C : D$. Q. E. D.

37. Cor. 1. If four magnitudes A, B, C, D be proportional, the products of the extreme, and middle, terms are equal, or $A \times D = B \times C$. For let $A = \frac{m}{n} \times B$, and therefore $C = \frac{m}{n} \times D$ (35): consequently $A \times D = \frac{m}{n} \times B \times D$, and $B \times C = B \times \frac{m}{n} \times D = A \times D$. If they be incommensurate, let A be greater than $\frac{m}{n} \times B$, and less than $\frac{m+1}{n} \times B$, and C will be contained between $\frac{m}{n} \times D$ and $\frac{m+1}{n} \times D$ (35); therefore $A \times D$ is contained between $D \times \frac{m}{n} \times B$ and $D \times \frac{m+1}{n} \times B$, and $C \times B$ is contained between $B \times \frac{m}{n} \times D$, and $B \times \frac{m+1}{n} \times D$. The difference of these products is therefore not greater than $\frac{DB}{n}$, which, because n may be taken unassignably great, $= 0$; and consequently $A \times D = B \times C$.

38. Cor. 2. If two products be equal, their factors are proportional. Let $A \times D = B \times C$; and if $A = \frac{m}{n} \times B$, then $C = \frac{A \times D}{B}$ $= ($by

$=$ (by ſubſtituting A's value) $\frac{m}{n} \times D$; or if A be leſs than $\frac{\overline{m+1}}{n} \times B$, but greater than $\frac{m}{n} \times B$; then C, being equal to $\frac{A \times D}{B}$, will be leſs than $\frac{\overline{m+1}}{n} \times D$, and greater than $\frac{m}{n} \times B$; conſequently (36) $A : B :: C : D$. In the ſame manner it may be proved, that $A : C :: B : D$, if they be ſimilar; or that $A : BC :: 1 : D$, and $B : A :: D : C$, or $B : A \times D :: 1 : C$, or $A : B :: C : \frac{BC}{A}$. Any equation is therefore reſolvable into a proportion by ſo arranging the terms, that the rectangle of the extreme terms may be one ſide of the equation, and that of the mean terms the other.

39. Cor. 3. Ratios, which are equal to any ratio, are equal to each other. Let $A : B :: C : D$ and $C : D :: E : F$; and if A be equal to $\frac{m}{n} \times B$, or contained between $\frac{m}{n} \times B$ and $\frac{\overline{m+1}}{n} \times B$, C will be equal to $\frac{m}{n} \times D$, or contained between $\frac{m}{n} \times D$ and $\frac{\overline{m+1}}{n} \times D$ (35), and alſo E will be equal to $\frac{m}{n} \times F$, or contained between $\frac{m}{n} \times F$ and $\frac{\overline{m+1}}{n} \times F$ (35); therefore A and E are either equal to, or contained between, the ſame parts of B and F reſpectively, and $A : B :: E : F$ (36).

40. Cor. 4. If $A : B :: C : D$ they will be proportional inverſely, or $B : A :: D : C$. Let A be equal to $\frac{m}{n} \times B$ or contained between $\frac{m}{n} \times B$ and $\frac{\overline{m+1}}{n} \times B$, and C will be equal to $\frac{m}{n} \times D$, or contained between $\frac{m}{n} \times D$ and $\frac{\overline{m+1}}{n} \times D$ (35); therefore $\frac{n}{m} \times A$ is equal to

or

or greater than, and $\frac{n}{m+1} \times A$ is leſs, than B, or B is contained between $\frac{n}{m+1} \times A$ and $\frac{n}{m} \times A$, or equal to $\frac{n}{m} \times A$; and for the ſame reaſon D is contained between $\frac{n}{m+1} \times D$ and $\frac{n}{m} \times D$, or equal to $\frac{n}{m} \times C$; therefore (36) $B : A :: D : C$.

41. Cor. 5. Magnitudes are proportional to their equimultiples or equal parts. Let $A = \frac{m}{n} \times B$, and $2A$ will be equal to $\frac{m}{n} \times 2B$, and $\frac{1}{2}A = \frac{m}{n} \times \frac{1}{2}B$; therefore (36) $A : B :: 2A : 2B :: \frac{1}{2}A : \frac{1}{2}B$; or let A be contained between $\frac{m}{n} \times B$ and $\frac{\overline{m+1}}{n} \times B$, and $2A$ will be contained between $\frac{m}{n} \times 2B$ and $\frac{\overline{m+1}}{n} \times 2B$; and $\frac{1}{2}A$ between $\frac{m}{n} \times \frac{1}{2}B$ and $\frac{\overline{m+1}}{n} \times \frac{1}{2}B$; therefore (36) $A : B :: 2A : 2B :: \frac{1}{2}A : \frac{1}{2}B$, &c.

42. Cor. 6. If $A : B :: C : D$, $A^m : B^m :: C^m : D^m$ and $A^{\frac{1}{m}} : B^{\frac{1}{m}} :: C^{\frac{1}{m}} : D^{\frac{1}{m}}$. For m times, or an mth part of, the ratio of $A : B$ is equal to m times, or an mth part of, that of $C : D$, becauſe theſe ratios are equal to each other; therefore (34) $A^m : B^m :: C^m : D^m$; and $A^{\frac{1}{m}} : B^{\frac{1}{m}} :: C^{\frac{1}{m}} : D^{\frac{1}{m}}$.

43. Cor. 7. If $A : B :: C : D$; then $A+B : B :: C+D : D$. Let A be equal to $\frac{m}{n} \times B$, or contained between $\frac{m}{n} \times B$ and $\frac{\overline{m+1}}{n} \times B$, and (35) C will be equal to $\frac{m}{n} \times D$, or contained between $\frac{m}{n} \times D$ and

and $\overline{\frac{m+1}{n}} \times D$; therefore adding B to both, $A+B$ will be equal to $\overline{\frac{m+n}{n}} \times B$, or greater than $\overline{\frac{m+n}{n}} \times B$ and leſs than $\frac{m+1+n}{n} \times B$; and $C+D$ will be equal to $\frac{m+n}{n} \times D$, or greater than this, and leſs than $\overline{\frac{m+1+n}{n}} \times D$; and conſequently (36) $A+B : B :: C+D : D$. In the ſame manner it may be proved, that $A-B : B :: C-D : D$; and $A : B :: A \pm C : B \pm D$.

44. DEF. *That ratio is ſaid to be greater or leſs than another, whoſe antecedent has a greater proportion to its conſequent than the antecedent of the other to its conſequent.*

Two ratios, whoſe antecedents, or conſequents, are the ſame, are eaſily compared: for it is evident that the ratio of 6 : 3 is greater, and the ratio of 4 : 3 leſs, than the ratio of 5 : 3; and the negative ratio of 3 : 6 is greater, and of 3 : 4 leſs, than the ratio of 3 : 5.

45. *The firſt method of comparing unequal ratios is to reduce them to other ratios equal to them with a common antecedent, or conſequent.*

EXAMP. To compare the ratios of $A : B$ and of $C : D$. Find a ratio equal to either of them, whoſe antecedent or conſequent is the antecedent or conſequent of the other; thus, $A : B :: C : \frac{BC}{A}$ (38), and conſequently the ratio of $A : B$ being equal to that of $C : \frac{B \times C}{A}$, is greater or leſs than the ratio of $C : D$, according as $\frac{BC}{A}$ is leſs or greater than D.

Let

Let the ratios of 3 : 5 and of 6 : 9 be compared; and 3 : 5 :: 6 : $\frac{30}{3}$ (10), therefore the ratio of 3 : 5, being the ſame with that of 6 : 10, is a greater negative ratio than that of 6 : 9.

46. *The ſecond method of comparing ratios, is to divide the antecedents by their reſpective conſequents, and that ratio will be the greateſt, the quotient of whoſe terms is the greateſt.*

EXAMP. The ratio of $A : B$, being equal to that of $C : \frac{BC}{A}$ (38), is greater than, equal to, or leſs than the ratio of $C : D$, according as $\frac{B \times C}{A}$ is leſs than, equal to, or greater than D, or $\frac{C}{A}$ leſs than, equal to, or greater than $\frac{D}{B}$, or $\frac{A}{C}$ greater than, equal to, or leſs than $\frac{B}{D}$. This method is expeditious and true, but let it be remembered that $\frac{A}{B}$ is a number as every quotient is, and does not meaſure the ratio of $A : B$, nor is the relative magnitude or ratio of the ratios of $A : B$ and $C : D$ equal to the ratio of $\frac{A}{B} : \frac{C}{D}$.

47. *The third method of comparing ratios. If* A *be contained between* $\frac{m}{n} \times$ B *and* $\frac{m+1}{n} \times$ B, *and* C *be contained between* $\frac{m}{n} \times$ D *and* $\frac{m+1}{n} \times$ D, *the ratios of* A : B *and* C : D *are equal: if therefore* A *be greater than* $\frac{m}{n} \times$ B *and* C *be not greater than* $\frac{m}{n} \times$ D, *or if* nA *be greater than* mB, *but* nC *not greater than* mD, *the ratio of* A : B *is greater than that of* C : D.

EXAMPS.

EXAMP. Let the ratios to be compared be 7 : 5 and 4 : 3. Multiply the firſt and third by 3, and the ſecond and fourth by 4, and the reſulting numbers are 21, 20, 12, 12; and the firſt multiple being greater than the ſecond, but the third not greater than the fourth, 7 has to 5 a greater ratio than 4 : 3.

48. DEF. *Any magnitude* A *is ſaid to be, or vary, directly as another* B, *when* A *is to any new value of* A *as* B *to a correſponding new value of* B, *that is, when* A : a :: B : b, a *and* b *being correſponding new values of* A *and* B.

FIG. IV. EXAMP. I. If the line *mn* move parallel to itſelf, and its extremities *m* and *n* be always in the lines *LP*, *LQ*; *Lm* varies as *mn*. For *LA* : *La* (a new value of *Lm*) :: *mn* : *AB* (a correſponding new value of *mn*).

EXAMP. II. The area *Lmn* varies directly as the ſquare of *Lm* or *mn*; for *Lmn* : *LAB* (a new value of *Lmn*) :: $mn^2 : AB^2$ (the ſquare of a new value of *mn*).

49. DEF. *Any magnitude* A *is ſaid to be, or vary, inverſely as another* B, *when* A *is to a new value of* A, *as a cotemporary new value of* B *is to* B, *or when* A : a :: b : B.

EXAMP. I. In the lever the power is inverſely as the perpendicular let fall from the center of motion upon its direction; for let *F* and *f* be two powers in equilibrio, *P* and *p* the perpendiculars let fall from the center of motion upon their directions; and *F* : *f* (a new value of the power) :: *p* (the perpendicular let fall upon *f*'s direction) : *P*.

EXAMP.

EXAMP. II. If a given ſpace be described in different times T and t, with unequal uniform velocities V and v, the velocities and times are inverſely as each other; for $V : v$ (a new value of the velocity) $:: t$ (a new value of the time correſponding to v) $: T$.

EXAMP. III. If two variable right lines AB, AC form the equal rectangles AD, EF, they are inverſely as each other; for (Euc. VI. 14.) $AB : CF$ (a new value of AB) $:: CE$ (a new value of AC) $: AC$. FIG. V.

SCHOLIUM.

50. When any quantity A varies directly, or inverſely as any other B, an analogy is always implied, and A and B are not neceſſarily homogeneous.

51. Cor. 1. If A be directly as B, and B directly as C, A will be directly as C; for (48) $A : a :: B : b$ and $B : b :: C : c$; therefore (39) $A : a :: C : c$, and A is directly as C (48).

52. Cor. 2. If A be as B and B as C; A will be as $B \pm C$, or as $\sqrt{BC}$; for $A : a :: B : b :: C : c$, and (43) $B : b :: B \pm C : b \pm c$; therefore (39) $A : a :: B \pm C : b \pm c$, and A is as $B \pm C$ (48): and becauſe the ratios of $B : b$ and $C : c$ are equal, $B^2 : b^2 :: BC : bc$ and $B : b :: \sqrt{BC} : \sqrt{bc}$. B therefore, and conſequently A (51), is as $\sqrt{BC}$.

53. Cor. 3. A varies as $m \times A$, or as $\frac{A}{m}$; for $A : a :: m \times A : m \times a$, or as $\frac{A}{m} : \frac{a}{m}$ (41); therefore (48) A is as $m \times A$, or as $\frac{A}{m}$.

54. Cor. 4. If A be as B, A^2 will be as B^2, and A^m as B^m, and $A^{\frac{1}{m}}$ as $B^{\frac{1}{m}}$; for ſince $A : a :: B : b$ (48) $A^m : a^m :: B^m : b^m$, or $A^{\frac{1}{m}} : a^{\frac{1}{m}} :: B^{\frac{1}{m}} : b^{\frac{1}{m}}$ (42); therefore A^m is as B^m and $A^{\frac{1}{m}}$ as $B^{\frac{1}{m}}$ (48).

55. Cor. 5. If S be as $V \times T$, $\frac{S}{T}$ will be as V, $\frac{S}{V}$ as T, and $\frac{S}{V \times T}$ a given quantity: for (48) $S : s :: VT : vt$ and $\frac{S}{V} : \frac{s}{v} :: T : t$ and $\frac{S}{T} : \frac{s}{t} :: V : v$, and $\frac{V}{VT} : \frac{s}{vt} :: 1 : 1$ (41); therefore $\frac{S}{V}$ is as T, $\frac{S}{T}$ as V, and $\frac{S}{VT}$ does not vary (48).

56. Cor. 6. If A be as B, and C as D, $A \times C$ will be as $B \times D$; for $A : a :: B : b$ and $C : c :: D : d$; therefore the ratios of $A : a$ and $C : c$ are equal to the ratios of $B : b$ and $D : d$, and the ſums of theſe ratios are equal, or the ratio of $AC : ac$ is equal to that of $BD : bd$ (18), and $AC : ac :: BD : bd$ (34), and AC is as BD (48).

57. Cor. 7. If A be inverſely as B, then B will be inverſely as A; for $A : a :: B : b$ (49) and $b : B :: a : A$; and if B be to b as $4 : 5$, $A : a :: 5 : 4$.

58. Cor. 8. If A be directly as B, and B inverſely as C, A will be inverſely as C; for $A : a :: B : b$ (48) and $B : b :: c : C$; therefore $A : a :: c : C$ (39), and A is inverſely as C (49).

59. Cor. 9. If A be inverſely as B, and B inverſely as C; A will be directly as C: for $A : a :: b : B$ and $b : B :: C : c$, therefore $A : a :: C : c$ and A is as C.

60. Cor. 10. If $A \times B$ be always the same, A is inverſely as B; for $A \times B : a \times b :: 1 : 1$ and $A : a :: b : B$; or otherwiſe, ſince $A \times B = a \times b$, $A : a :: b : B$ (38).

61. Cor. 11. If A be inverſely as B, it will be as $\frac{1}{B}$ and $v : v$; for $A : a :: b : B$ (49) $:: \frac{1}{B} : \frac{1}{b}$ (41), therefore A is as $\frac{1}{B}$ (48). If A be as $\frac{1}{B}$, $A : a :: \frac{1}{B} : \frac{1}{b}$ (48) $:: b : B$ (41); and A is inverſely as B (49).

62. Cor. 12. If A be as $\frac{L}{M}$, A will be inverſely as $\frac{M}{L}$; for $A : a :: \frac{L}{M} : \frac{l}{m}$ (48) $:: \frac{m}{l} : \frac{M}{L}$ (41); and A is inverſely as $\frac{M}{L}$ (49).

63. DEF. *If any quantity* A *be dependent upon ſeveral others* P, Q, R, S, T, *all independent of each other, ſo that none of the quantities* P, Q, R *can vary, but* A *varies in the ſame ratio, nor either* S *or* T *can vary, but* A *varies in a contrary ratio;* A *is ſaid to be as* P *and* Q *and* R *directly, and* S *and* T *inverſely.*

Thus, the fraction $\frac{LMN}{XZ}$ varies as L and M and N directly and X and Z inverſely; becauſe none of the factors in the numerator can vary, but the value of the fraction varies in the ſame ratio; and none of the factors in the denominator can vary, but the value of the fraction varies in a contrary ratio. If any one of the factors in the numerator be multiplied by 2, 3, &c. the value of the fraction is 2, 3, &c. times its former value; and if any of the factors in the denominator be multiplied by 2, 3, &c. the value of the fraction becomes $\frac{1}{2}$, $\frac{1}{3}$, &c. of its former value.

64. Cor. 1. If any magnitude A vary as P and Q and R directly, and as X and Z inverſely, theſe magnitudes being independent of each other, and any one may vary without affecting the reſt, A will vary as $\frac{P \times Q \times R}{X \times Z}$. For let any new cotemporary values of P, Q, R, X, Z, be reſpectively equal to p, q, r, x, z, and let A, when changed in the ratios of $P : p$, $Q : q$, $R : r$, $\frac{1}{X} : \frac{1}{x}$, and $\frac{1}{Z} : \frac{1}{z}$, become a; that is,

Let $P : p :: A : B =$ A's value after the change of P.
$Q : q :: B : C =$ A's value after the change of P, Q.
$R : r :: C : D =$ A's value after the change of P, Q, R.
$\frac{1}{X} : \frac{1}{x} :: D : E =$ A's value after the change of P, Q, R, X.
$\frac{1}{Z} : \frac{1}{z} :: E : a =$ A's value after the change of P, Q, R, X, Z.

The ſum of the ratios of $A : B$, $B : C$, $C : D$, $D : E$, $E : a$, or the ratio of $A : a$ (16) is equal to the ſum of the ratios of $P : p$, $Q : q$, $R : r$, $\frac{1}{X} : \frac{1}{x}$, $\frac{1}{Z} : \frac{1}{z}$, or to the ratios of $\frac{PQR}{XZ} : \frac{pqr}{xz}$ (18); and A varies as $\frac{PQR}{XZ}$ (48).

65. Cor. 2. If any of the quantities P, Q, R, X, Z be given or remain invariable, they are to be rejected, and A will vary as the reſt. Let P be given or $P = p$, and, becauſe $P : p :: A : B$, $A = B$ (38), and the ratios of $B : C$, $C : D$, $D : E$, $E : a$, or the ratio of $B = A : a$, is equal to the ratio of $\frac{QR}{XZ} : \frac{qr}{xz}$ (18), and A is as $\frac{QR}{XZ}$ (48). Let Q be alſo given or $Q = q$, and $A (= B = C) : a :: \frac{R}{XZ} : \frac{r}{xz}$, or A is as $\frac{R}{XZ}$ (48). Hence may be underſtood what is meant by philoſophical writers, when they ſay the velocity (V) varies

varies as the ſpace (S) directly, and the time (T) inverſely, or as $\frac{S}{T}$; and, if T be given, V is as S, and, if S be given, V varies as $\frac{1}{T}$.

66. EXAMP. I. The number of feet (S) deſcribed in any time (T), with an uniform velocity (V), encreaſes or decreaſes directly with V and T, both of which are independent of each other, and therefore varies as $V \times T$. If V be doubled or encreaſed in the ratio of 2 : 1, and T be encreaſed in the ratio of 3 : 1, S will be encreaſed in the ratios of 2 : 1 and 3 : 1, or of 6 : 1.

67. EXAMP. II. The quantity of matter (Q) in different bodies varies as the magnitude (M) multiplied into the denſity (D). For if the magnitude vary in the ratio of $M : m$, and the denſity or cloſeneſs of the conſtituent parts, in the ratio of $D : d$, the quantity of matter will be changed according to both theſe ratios, or $Q : q :: M : m$ and $D : d :: M \times D : m \times d$, or Q varies as $M \times D$ (48).

68. EXAMP. III. The velocity (V) of a body moving uniformly in the peripheries of different circles, varies as the radius (R) directly, and periodic time (P) inverſely. For, if P remain conſtant whilſt the ſpace deſcribed or periphery varies, the velocity will encreaſe or decreaſe directly as the ſpace, or as R, becauſe the peripheries of circles are as their diameters; and if the periodic time be encreaſed in the ratio of 2 or 3 : 1, or univerſally in the ratio of $P : p$, the velocity will be changed in the ratio of 1 : 2 or 3, or of $p : P$. If therefore the radius be changed in the ratio of $R : r$, and periodic time in the ratio of $P : p$, and V become v, $V : v$ as $R : r$, and $\frac{1}{P} : \frac{1}{p}$, or as $\frac{R}{P} : \frac{r}{p}$, and the velocity is as $\frac{R}{P}$ (48).

69. Ex-

69. EXAMP. IV. If V be the velocity communicated, by the action of a force F, to a quantity of matter represented by $\mathcal{Q}$, and these quantities be supposed to vary, V will be as $\frac{F}{\mathcal{Q}}$; for if the force become $2F$, $3F$, &c. the velocity will become $2V$, $3V$, &c. and if a quantity of matter become $2\mathcal{Q}$, $3\mathcal{Q}$, &c. the velocity communicated will be $\frac{V}{2}$, $\frac{v}{3}$, &c. and V therefore is as $\frac{F}{\mathcal{Q}}$ (64).

A SYSTEM

A

SYSTEM

OF

NATURAL PHILOSOPHY.

MECHANICS.

MECHANICAL philoſophy acquired the name from its utility in the conſtruction of machines; but it is now, in a more general ſenſe, underſtood to comprehend two branches of ſcience, cultivated at different periods of time, denominated Statics, or the ſcience of the equilibrium and relation of powers, and Dynamics, or the ſcience of actual motion. The firſt profeſſes to deſcribe the conſtruction, properties, and mechanical advantages of machines, and their various combinations, calculated to ſuſtain the preſſure of heavy bodies and facilitate their motion; and to inveſtigate the equilibrium of powers acting upon them, or their relative magnitudes, when by oppoſite exertions they deſtroy each other's effect or remain quieſcent. This branch comprehends that part called the mechanical powers, and is ſometimes called practical mechanics. The object of dynamics is the nature, geneſis, and change of actual motion, or an inveſtigation of the direction,

rection, quantity, and law of variation, of a force or power capable of generating any motion or change of motion; and vice versâ. This comprehends the laws by which all motions are regulated, the motions reſulting from colliſions, the theory of oſcillations, projectiles, and centripetal forces, and is ſometimes called rational mechanics. The principles of ſtatics were calculated and eſtabliſhed by Archimedes, and have, ſince that period, been almoſt exhauſted by the labours of ſucceeding writers. Galileo demonſtrated the laws of deſcent of heavy bodies, and from him originated the ſcience of dynamics, which has ſince been proſecuted to an amazing extent in Euler's Mechanica, and Newton's Principia. As the objects of mechanical philoſophy are the equilibrium and motions of bodies; a deſcription of the different qualities, or mechanical affections of matter, producing preſſure, motion, and other phenomena, ought to be premiſed. Mechanics therefore might with great propriety be divided into two parts; of which the firſt would contain the properties of matter, or the exiſtence, intenſity at a given diſtance, and laws of variation at different diſtances, of thoſe general principles which are the apparent origin of motion, and the continuation of motion in the material world, inveſtigated by analyſis: and the ſecond would be an application of theſe principles, containing ſynthetic demonſtrations of their effects upon machines, commonly called the mechanical powers, when in equilibrio, and their relation to actual motions, rectilineal and curvilineal, uniform and variable. This is the moſt obvious diviſion of the ſubject, and every deviation from it muſt be attributed to the deſign of this ſelection, which is ſolely the utility of the academic ſtudent.

CHAP.

MATTER, AND ITS PROPERTIES.

CHAP. I.

OF MATTER.

70. MATTER is the ſubſtance employed in the formation of that part of the creation, whoſe exiſtence is evidenced by the teſtimony of the ſenſes; and the moſt characteriſtic and prominent marks of it ſeem to be extenſion and ſolidity. Indeed ſolidity is the moſt diſcriminating mark of matter, as it diſtinguiſhes it from every thing elſe; but taſtes, ſmells, ſounds equally indicate the exiſtence, though whatever is ſolid and extended is the common and moſt general deſcription, of matter. Many other qualities invariably adhere to all matter with which we are acquainted, whether hard, ſoft, or fluid; and from our ignorance of its internal conſtitution, or that latent principle, by whoſe influence its qualities are connected, and from which they neceſſarily derive their origin, theſe, and many others inacceſſible to the ſenſes, may be ingredients in its eſſence. All our knowledge of matter, as far as it is related to the preſent ſubject, is either geometrical, or philoſophical: the firſt conſiders matter as being of ſome magnitude, or circumſcribing ſpace and having ſome figure, then called body, and is uſually denominated ſtereometry, or the menſuration of magnitudes of three dimenſions, length, breadth, and thickneſs; and the ſecond comprehends all the properties of matter addreſſed to the ſenſes, which may be ſtiled phyſical or philoſophical, becauſe all the phenomena of nature are conceived to reſult imme-

diately from them; as extenſion, ſolidity, inertia, and thoſe apparently more active qualities, gravity, magnetiſm, electricity, coheſion, elaſticity. Theſe laſt are called mechanical affections of matter, or mechanical cauſes, becauſe they are conceived to reſide in matter, and all mechanical phenomena, or changes of motion obſervable in the material world, appear to be directed by their neceſſary and uniform agency. The philoſophical properties of matter are diſtinguiſhable into two kinds, general, and ſpecific: the firſt are ſuch as univerſally adhere to every ſpecies of matter, and of which no art hath been able to diveſt them, as *extenſion, ſolidity, mobility, quieſcibility, inertia, figure, attractions and repulſions*, and probably many more which are too ſubtle for the obſervation of ſenſe, or have yet eluded the inquiſition of the philoſopher; and the ſecond are ſuch as are attached only to particular ſpecies of matter, as opacity, tranſparency, hardneſs, fluidity, colour, magnetiſm, elaſticity, &c. Another diſcrimination of theſe qualities is, that ſome are incapable of intenſion or remiſſion, as extenſion, ſolidity, inertia, mobility, figure, which are inſeparable from a body and its component parts; and others are relative and capable of encreaſe and decreaſe, as attractions and repulſions, whoſe intenſity depends upon the magnitude and diſtance of the attracting or repelling bodies, and as that magnitude and diſtance may vary without limit, the influence of theſe powers may, by remoteneſs, decreaſe without limit, and become evaneſcent. A knowledge of the exiſtence, and diſcrimination of theſe qualities, is entirely derived from experiment and obſervation, and conſtitutes the ſum of all that is known concerning matter. The internal conſtitution of it is unknown, and its effects cannot be inveſtigated without the aſſiſtance of experience; and whether any more, beſides thoſe eight general properties enumerated, belong to it, and what, or whether any, connection ſubſiſt between thoſe already diſcovered, and to what particular conſtitution ſpecific qualities are owing, muſt be derived from the ſame ſource, experiment, and if ever diſcovered, will probably reſult from the labour of the chymiſt.

CHAP.

CHAP. II.

OF EXTENSION.

71.* THERE are three kinds of extenſion, lineal, as the line *AB*; ſuperficial, having length and breadth, as *ABCD*; and ſolid, having length, breadth and thickneſs, as *AH*. Ideas of theſe three kinds of extenſion are undefinable, and only to be acquired by the ſenſes of ſeeing and feeling. The perceptions introduced to the mind by ſeeing or feeling two diſtant parts of the line *AB*, a ſurface *ABCD*, and material body *AH*, convey reſpectively the meaning of lineal, ſuperficial and ſolid extenſion; which are diſſimilar magnitudes, and incapable of compariſon with each other. A line, having no breadth, cannot, however repeated, conſtitute a ſurface; and a ſurface, having no thickneſs, cannot conſtitute a ſolid. The parts of a line, though divided without limit, are ſtill lineal extenſion, and the parts of a ſurface, or ſolid, though divided into a number of parts unaſſignably great, are ſtill ſuperficial and ſolid extenſion. This quality is ſo far eſſential to matter that it cannot be diveſted of it, or conceived to exiſt without it. FIG. VIII.

72. DEF. *A magnitude is ſaid to be finite when an equal to it can be aſſigned, or when its encreaſe and decreaſe are limited within aſſignable bounds.*

73. DEF. *A magnitude is ſaid to be infinitely great or ſmall, when no finite ratio obtains between it and a finite magnitude, or when its encreaſe or decreaſe is not limited by any aſſignable boundary.*

74. PROP.

* Keil's Phyſics. Muſchenbroek.

FIG. VI. 74. PROP. *A finite right line* EX *is divisible in infinitum.*

DEM. A mathematical point *A* may be taken between *X* and *E*, and another *B* between *A* and *E*, and this process may be continued without limit, because the intermediate points *A*, *B*, &c. having no length, can never coincide with *X* or *E*, or with each other. Q. E. D.

FIG. VII. Otherwise: Through *E* and *X* draw *EP*, *XC* parallel to each other, and, because they never meet, a number of points *a*, *b*, *c*, *d*, &c. greater than any assignable number may be taken in *XC*; and if right lines be drawn from them to any point *P* on the other side of *EX*, they will divide it into a number of parts equal to the number of points *a*, *b*, *c*, &c.: for if they did not, two lines must pass through the same part, that is, either intersect each other in *EX*, or coincide till they arrive at it, and then diverge, both of which are impossible. There is therefore no assignable limit to the divisibility of *EX*. Q. E. D.

FIG. VIII. 75. Cor. 1. The finite surface *ABCD* is infinitely divisible; for it is equally divided with *AB*, by drawing mathematical right lines *pf*, *qg*, *rh*, &c. from the points *p*, *q*, *r*, &c. parallel to *AD*, which having no breadth cannot coincide. If *ABCD* be a boundary of the solid *AH*, and mathematical planes be drawn through *pf*, *qg*, *rh*, &c. parallel to *AF*, they cannot coincide, having no thickness, and consequently will divide the solid into the same number of parts with the surface *ABCD*, or line *AB*.

76. Cor. 2. Every finite line, surface and solid, is therefore composed of an unlimited number of parts; and each of these parts
FIG. IX. is composed of an unlimited number, &c. For let *LM* be infinitely greater than *Ln*, and take *LM* : *Ln* :: *Ln* : *Lo* :: *Lo* : *Lp*, &c.; and drawing any finite line *MA*, making an angle with *LM*,

and

and nB, oC, pD, &c. parallel to MA and terminated by LA; MA is infinitely greater than nB; nB than oC; oC than pD, &c. But MA is infinitely diviſible, and right lines, drawn from L to the points of diviſion, will cut nB, oC, &c. into the ſame number of parts with it.

77. Cor. 3. Matter is therefore infinitely diviſible, becauſe extended. And this propoſition and corollaries are applicable to magnitudes of every kind, as velocities, forces, times, &c.

SCHOLIUM.

78. The terms infinitely great and ſmall are relative, and imply a compariſon with an aſſignable magnitude, and compared with it, all infinitely great or ſmall magnitudes are to each other in a ratio of equality; but, compared with each other, they admit of the ſame inequality of ratios with finite magnitudes. The line AM is infinitely diviſible, and if the points of diviſion be equidiſtant, any one part xy, multiplied into their number, is equal to AM; and $xy : AM ::$ unity: a number unaſſignably great, or xy is an infiniteſimal of the firſt order. The infiniteſimal xy is infinitely greater than wt, and wt than rs; for $xy : wt :: wt : xs :: LM : Ln$. But infiniteſimals of the ſame order may be to each other in any aſſignable ratio; for, let LM be to LF as 3, 4, 5, or or $m : 1$, and FG, being drawn parallel to MA, will be equally divided with it, and the infiniteſimal $vz : xy :: LM : LF :: 1 : 3, 4, 5,$ or any number m. This is called the mathematical diviſibility of matter, and is equally applicable to ſpace and all other magnitudes, which may be repreſented by extenſion; but that an actual diviſibility, or actual ſeparation of the parts of matter, is limited by certain inviolate bounds, is inferred from the identity of natural ſubſtances.

PHENOM. I. Salt diſſolved in a menſtruum, becomes the ſame ſalt when the menſtruum is ſuppoſed to dry gradually. Salt converted

verted, by a chymical proceſs, to an acid ſpirit, is, by reverſing the proceſs, regenerated into the ſame ſalt. Metals liquidated by fire, are reſtored to their priſtine ſtate by the attraction of coheſion. But were the decompounded particles of the ſalt, or metal, attenuated by the action of the menſtruum or fire, and again to coaleſce by their coheſive force, ſubſtances of a different texture and appearance, and of different ſpecific gravities, would reſult. The moſt powerful natural agent, with which we are acquainted, is fire; which, whether artificial or collected in the focus of a burning glaſs, only attenuates bodies to a limited degree by converting them into a thick ſmoke, glaſs, &c. Every ſpecies of matter fluctuates and decays by the ſeparation of its component parts, and is renovated by their acceſſion; but were the nutritious particles, adminiſtering to the encreaſe of natural ſubſtances, capable of diviſibility or diminution by attrition, &c. new ſpecies of ſubſtances would reſult with new properties and characters. Water and earth, compoſed of old particles, and fragments of particles produced by attrition, would have a different ſpecific gravity, from water and earth compoſed of entire and unbroken particles; and the nature and textures of theſe and all other ſubſtances would by repeated attritions, be perpetually changing, which is contrary to experience. Bodies therefore break, not in the midſt of ſolid particles, but where thoſe particles cohere in a few points; and the diviſibility of matter is only a ſeparation of its conſtituent parts, effected by a diſſolution of that coheſive force which unites them, and is limited by ultimate elementary particles which are a perfect repletion of ſpace, without pores and indiviſible.

CHAP.

CHAP. III.

OF SOLIDITY.

79. THE ſecond philoſophical quality of matter is that by which it occupies ſpace to the excluſion of all other matter, and is called its ſolidity; but the meaning of it cannot be conveyed by words, being undefinable and only to be acquired by the ſenſe of touching. The perception introduced to the mind by the inſuperable reſiſtance felt in a body is an idea of ſolidity. This reſiſtance, and total excluſion of matter from ſpace already occupied, is inſeparable from all matter with which we are acquainted, whether hard, ſoft, or fluid; and the little reſiſtance experienced in ſome bodies, is not owing to a want of ſolidity, but to their fluidity and ſoftneſs, by which qualities their component parts are eaſily diſplaced. That hard and ſoft bodies are ſolid, and cannot occupy the ſame part of ſpace, appears from uncontroverted experience; for the oppoſite ſides of the ſubſtance compreſſing them are never found to meet, but by removing the intervening parts. And the ſame is diſcovered to obtain in all fluids within the reach of obſervation; for no portion of water, air, mercury, &c. can occupy the ſame place, becauſe they afford an inſuperable reſiſtance preventing the coincidence of the oppoſite ſides of the ſubſtance compreſſing them; and their dimenſions are never found to be diminiſhed by preſſure without an adequate cauſe, viz. compreſſibility, and tranſmiſſion through the vacuities of the veſſel containing them. Numberleſs facts demonſtrate electric and magnetic effluvia, whoſe parts are immeaſurably minute, to be capable of impulſe and reſiſtance like other fluids; and it is inferred from analogy, that theſe fluids, circumſcribed and compreſſed by plane ſurfaces, would invariably oppoſe their junction, were the vacuities in thoſe ſurfaces leſs than the particles of

the fluids. Solidity therefore is an univerſal attribute of matter, but its cauſe is unknown and probably undiſcoverable by human faculties. The approach of one body to another is apparently limited by the actual contact of their neareſt parts, which abſolutely fill the ſpaces occupied by them, and conſequently a nearer acceſs and greater ſurfaces of contact, only reſult from their diſlocation; but this is not allowed without controverſion. Compreſſibility, contraction by cold, elaſticity, &c. prove that the minute parts of ſome bodies are not in mathematical contact, and their reſiſtance is an indication of repulſion; and the nearer acceſs therefore of two bodies is imagined to be impeded by a ſtrong repulſive power, which being overcome, a mutual penetration of parts enſues, and the bodies occupy the ſame part of ſpace. But this doctrine is ſtill only hypothetical; and though the minute parts of ſome bodies do exert an influence at a diſtance, and a repulſive power be confeſſed to obtain between them, it cannot be concluded, generally, to be the only cauſe preventing their nearer approach, nor admitted as a general principle in nature, contrary to the common apprehenſion of mankind, till eſtabliſhed by ſatisfactory and uncontroverted experiments. The tranſmiſſion of one body through another, and apparent penetration of parts, ſeem, and may be conceived, to reſult from empty unoccupied ſpace between thoſe parts; and the quantity of theſe vacuities is collected from the following phenomena.

80. PHENOM. Many vacuities or pores are actually viſible, through a microſcope, in every ſpecies of animals, vegetables and foſſils. The bottom of the ſea is viſible at a greater depth than ſixty feet. A man's finger, placed before the aperture of a dark chamber, is tranſparent by the paſſage of light through its pores: and light is tranſmitted in almoſt every direction through glaſs and water, and when condenſed three thouſand times in the focus of a burning glaſs, it ſeems to be admitted into water and glaſs without obſtruction. Electric effluvia paſs through gold with a velocity unaſſignably great, and the magnetic power is tranſmitted through every ſpecies of matter, except iron, without diminution.

The

The volatile ſpirit of ſulphur tinges, with a brown colour, ſilver ſurrounded with repeated coverings of cloth or paper; the ſcents of muſk, civet, &c. paſs through wood. Air and water imbibe each other; oils penetrate the vacuities of ſulphur and ſome ſtones; mercury penetrates the pores of gold, braſs, and is tranſmitted through human ſkin, leather, &c.; water is tranſmitted through the membranes of animals, and the fine tubes of vegetables, and may, by compreſſion, be forced through gold, ſilver, &c.* From numberleſs chymical experiments it appears, that all animals, vegetables, and foſſils, yield water plentifully by the force of heat; and all bodies, whether hard or fluid, admit the particles of fire into their pores, through which it paſſes and is diſſipated.

81. DEF. *The magnitude of a body is the magnitude of ſolid extenſion, that is, length, breadth and thickneſs, or the number of cubical inches contained in it.*

82. DEF. *The quantity of matter in a body is the number of equal particles contained in it. If the matter compoſing different bodies be reduced to equal particles without pores, the quantity of matter in each will be equal to one particle multiplied reſpectively into their number.*

83. DEF. *Denſity of a body is the contiguity, or cloſe adheſion, of its particles; but this term uſually implies the ratio obtaining between the number of equal particles, or quantities of matter, in bodies of the ſame magnitude.*

84. DEF. *Homogeneous bodies are thoſe which have the ſame denſity in every part; heterogeneous are thoſe which have not.*

85. DEF.

* In the celebrated Florentine experiment, a quantity of water was encloſed in a hollow ſphere of ſilver, and then forcibly compreſſed by ſcrews, till the fluid was ſeen to ooze through the pores of the metal and cover its ſurface like a dew.

85. DEF. *The poroſity of a body is the remoteneſs of its elementary particles, or the ratio of the quantities of vacuity in different bodies of equal bulk.*

86. PROP. *If* Q, D, B *repreſent reſpectively the quantity of matter, denſity, and magnitude, of a body, and be ſuppoſed to vary*; Q *will vary as* D × B.

DEM. If D, or B, be encreaſed or diminiſhed in the ratio of 2, 3, &c. to 1, Q will evidently be encreaſed or diminiſhed in the ſame ratio, and D and B are unconnected; therefore (64) Q is as $D \times B$. Q. E. D.

87. Cor. 1. Since Q is as $D \times B$, if q, d, b be cotemporary new values of Q, D, B; $Q : q :: DB : db$, and $\frac{Q}{D} : \frac{q}{d} :: B : b$, & $\frac{Q}{B} : \frac{q}{b} :: D : d$ (41); or if D be given, Q is as B; and if B be given, Q is as D; and if Q be given, D is $\frac{1}{B}$ and B as $\frac{1}{D}$. This cor. follows alſo from (65).

88. Cor. 2. If numbers, or lines, whoſe ratio is the ſame with that of D and B, be ſubſtituted for them, Q is properly expreſſed by the product of theſe numbers, or a rectangle whoſe ſides are theſe lines.

89. Cor. 3. If the denſity of a body be encreaſed in the ratio of 2, 3, &c. to 1, the poroſity (P) will be diminiſhed in the ratio of 1 to 2, 3, &c.; and conſequently P is as $\frac{1}{D}$, or (87) as $\frac{B}{Q}$. If B be given, P is as $\frac{1}{Q}$; and if Q be given, P is as B; and P being given, B is as Q.

EXAMP.

EXAMP. I. The denſity of gold is to that of water as $19\frac{1}{2} : 1$, and conſequently the relative quantity of pores in gold and water is as $1 : 19\frac{1}{2}$. The relative denſities of gold and cork are as $81\frac{1}{2} : 1$, and their quantities of pores therefore as $1 : 81\frac{1}{2}$. If one half of the magnitude of gold were vacuous, the relative quantities of pore and ſolid parts in the water and cork, would be reſpectively as 39 : 1 and 163 : 1.

EXAMP. II. If a body whoſe magnitude is A were conſtructed of particles cohering in ſuch a manner as to have $\frac{1}{2}$ of its magnitude vacuous, and theſe particles were ſimilarly conſtructed of other leſs particles having $\frac{1}{2}$ of their magnitude vacuous, and the third order of particles were elementary, and a perfect repletion of ſpace, the quantity of vacuity would be equal to $A \times \frac{1}{2} + A \times \frac{1}{2} \times \frac{1}{2} + A \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = A \times \overline{\frac{1}{2} + \frac{1}{4} + \frac{1}{8}} = A \times \frac{7}{8}$. And the magnitude of vacuity is to the magnitude of matter as 7 : 1.

SCHOLIUM.

90. What the figure and magnitude of the elementary particles of matter are, cannot be known from the ſenſes, which, with every microſcopical aſſiſtance, are unable to diſcern them. An elementary particle of matter hath probably never yet been ſeen. A number of elementary particles, uniting by the power of coheſion, form greater particles; and theſe, uniting again by the ſame power, form greater ſtill; and, this proceſs being made repeatedly, a corpuſcle is at length formed of a ſenſible bulk. All bodies ſeem to be compoſed of theſe derivative corpuſcles, which, formed of more or fewer repeated unions, compoſe bodies more or leſs denſe. Theſe derivative corpuſcles ſeem ſometimes to be ſimilar: if a beam of light be ſeparated by a priſm into ſmall coloured rays, and any ſlender ray of the ſame colour be minutely examined, its component parts ſeem to be ſimilar, becauſe they affect the ſight exactly in the ſame manner. Pure mercury ſqueezed through the pores of leather, or raiſed into fume, and received upon clean glaſs,

exhibit globules ſimilar and undiſtinguiſhable. This is alſo obſervable of the vapours of pure water. And, in other ſpecies of matter, the derivative particles are combined from others exactly ſimilar to them; a red globe of blood is obſerved, through a microſcope, to be compoſed of ſix yellowiſh ſerous globes, and every one of theſe is compoſed of ſix lymphatic globes; but farther the microſcope does not enable us to proceed. Every ſpecies of matter, however different in denſity, may be conceived to be formed of equal and ſimilar elementary particles. Thus, the particles
FIG. X. *A*, *B*, *C*, compoſed each of ſix equal elementary particles, are different; and theſe particles may, by a different combination, form different ſtill; and theſe repeated coalitions may, by changing the original and ſucceeding numbers and their poſitions, form maſſes of matter differing in endleſs variety.

UNIFORM MOTION.

91. From the ſolidity of matter, whether owing to repulſion or actual contact, reſults its capacity of impulſe or of being protruded, and conſequent mobility of body, as far as the cauſe of mobility ſeems to be known. If the body *A*, in motion, ſtrikes another body *B*, not retained in its place by any force, *B* will be protruded and move; becauſe, from their ſolidity, they cannot penetrate each other's dimenſions; and the communication of motion in this caſe is the neceſſary conſequence of ſolidity. Motion ſuppoſes the ſucceſſive exiſtence of the body moving in different parts of ſpace, and therefore cannot be underſtood without preſuppoſing a knowledge of ſpace and time; which therefore muſt be premiſed.

92. DEF. *Space is that which contains the whole ſenſible creation, is unlimited, and its parts are homogeneous, inſeparable, co-exiſting and unreſiſting.*

93. Cor. Since ſpace is unlimited in every direction, and its parts ſimilar and undiſtinguiſhable, any portion of it can only be aſcer-

aſcertained by its relation to ſome aſſumed ſenſible mark or object. Hence of place, which is a part of ſpace, there are two kinds, abſolute and relative.

94. DEF. *The abſolute place of a body is that portion of this unlimited ſpace, which is occupied by it when fixed and immoveable; and the relative place of a body is its ſituation with reſpect to ſome aſſumed mark or object, which itſelf may be moveable.*

95. Cor. The abſolute and relative place of a body coincide, when it and the aſſumed object are immoveable, or remain in the ſame part of fixed ſpace. But we cannot perceive the abſolute place of any object.

96. DEF. *Duration is that which flows uniformly, is unbounded, continuous, whoſe parts are ſimilar, and no two exiſt together.* An idea of duration is obtained by obſerving the interval between our ideas, or between the ſucceſſive appearances of any external object; and, as it is, ſtrictly, only meaſurable by a portion of duration, and no two parts exiſt together, conſequently cannot be compared by juxta poſition, time, which is a part of duration, is of two kinds, abſolute and relative.

97. DEF. *Abſolute time is a portion of duration, whoſe quantity is only known by a compariſon with another portion; and conſequently the relation between any two parts of abſolute time is undiſcoverable. Relative time is a part of duration which elapſes during any motion of body, or any ſucceſſion of external appearances.*

98. DEF. *An inſtant is the boundary between any two contiguous portions of time, as a point is the boundary of any contiguous lines; and a moment is any ſmall portion of time.*

99. DEF-

99. DEF. *Abſolute motion is a change of abſolute place, and abſolute reſt is a permanence in the ſame abſolute place.*

100. DEF. *The direction of motion is the poſition of the line upon which it is made. When the motion is rectilineal, its direction is this right line; when curvilineal, the direction is the tangent to that point of the curve where the moving point is.*

101. DEF. *Velocity is the quickneſs or ſlowneſs of motion, or the rate at which a body moves.*

102. DEF. *A body is ſaid to move with an uniform, accelerated, or retarded velocity, when it continues the ſame, encreaſes, or decreaſes. When the encreaſe, or decreaſe, of velocity is the ſame in any equal times, the acceleration, or retardation, is ſaid to be uniform; and when this encreaſe or decreaſe of velocity, encreaſes or decreaſes in any equal times, the acceleration, or retardation, encreaſes or decreaſes in the ſame ratio.*

103. DEF. *The line joining all the ſucceſſive places, through which a moving body paſſes, is called the ſpace deſcribed.*

104. PROP.* *If* S *repreſent the ſpace uniformly deſcribed with the velocity* V *in the time* T, *and theſe magnitudes be ſuppoſed to vary,* S *will vary as* V × T.*

DEM.

* This propoſition is proved, not inelegantly, by the following proceſs: Let S and s be reſpectively deſcribed uniformly, with the velocities V and v, in the times T and t; and $T : t :: S : \frac{S \times t}{T}$ (38) = the ſpace deſcribed in the time t with an uniform velocity equal to V; and $\frac{S \times t}{T}$; $s :: V : v$ and $S \times t \times v = s \times V \times T$ (37), and (38) $S : s :: V \times T : v \times t$. Q. E. D.

DEM. If either V or T be encreaſed or diminiſhed in the ratio of $2, 3, 4 \ldots . n : 1$, the ſpace will evidently be encreaſed or diminiſhed in the ſame ratio; and becauſe V and T are unconnected, or either of them may be changed without affecting the other, S will vary as $V \times T$ (64). Q. E. D.

105. Cor. 1. Becauſe S is as $V \times T$, V will be as $\frac{S}{T}$, T as $\frac{S}{V}$, and, if S be given, T as $\frac{1}{V}$ and V as $\frac{1}{T}$ (65).

106. Cor. 2. If V be given, S is as T, or the ſpaces deſcribed, with the ſame uniform velocity, are true meaſures of time, and may be ſubſtituted for it. And, T being given, S is as V, or the ſpaces deſcribed in the ſame time uniformly, are meaſures of the velocity: if V, or any other ſymbol, be called velocity, it denotes the length or number of feet deſcribed uniformly in one ſecond, or any other time. Let any values of the velocity be to each other as $A : B$, or as $3 : 2$, and any cotemporary values of the time be as $C : D$, or as $5 : 4$, and the ſpaces deſcribed will be to each other as $X : Z$, or as $3 \times 5 : 2 \times 4$. PLATE I. FIG. III.

107. Cor. 3. If lines therefore be ſubſtituted for V and T, the ſpace S will be as the rectangle, whoſe ſides are theſe lines; and, if numbers be ſubſtituted for them, the ſpace will be equal to the product of theſe numbers, the time equal to the quotient of the ſpace divided by the velocity, and the velocity equal to the quotient of the ſpace by the time. Let V be equal to n feet in one ſecond, and $T = p$ ſeconds; and S is actually equal to $n \times p$ feet, T actually equal to $\frac{S}{V}$ ſeconds, and $V = \frac{S}{T}$ feet in one ſecond.

VARIABLE MOTION.

108. PROP. *In variable finite velocities, the velocity during an infinitely small time, is uniform.*

DEM. The encreaſe, or decreaſe of velocity produced in any finite time, is finite, and, if the whole encreaſe or decreaſe be divided into any infinite number of increments or decrements, each will be infinitely ſmall and vaniſh, compared with the whole velocity, which therefore is uniform. Q. E. D.

109. Cor. If therefore S', V', T', repreſent correſponding increments of S, V, T, reſpectively; V' will be evaneſcent compared with V; S' will vary as $V \times T'$ (104), V as $\frac{S'}{T'}$ and T' as $\frac{S'}{V}$ (105).

PLATE I. FIG. XI. 110. PROP. *If the abſciſſa* AS *of any curve* DEF *repreſent the whole time, and the ordinates* AD, BE, CF, *&c. be as the velocities at the inſtants* A, B, C, *&c.. the ſpaces deſcribed in the times* AB, BC, *will vary as the areas* AE, BF, *&c.*

DEM. For let Am, mn, np, &c. be moments of time, and the ſpaces deſcribed in theſe moments, are as the rectangular parallelograms Aq, mr, nv, (107) &c.; and the whole ſpaces, deſcribed in the ſums of theſe moments, i.e. in the times AB, BC, are as the ſums of theſe parallelograms, or as the areas AE, BF. Q. E. D.

111. Cor. 1. If the relation between V and T, or the ordinate and abſciſſa, be given, the relation between the ſpaces, or the areas AE, BF may be found.

112. Cor.

112. Cor. 2. If the area AE be always as the ſpace deſcribed (S) in the time AB or T, the velocity is as the ordinate BE; for according to this ſuppoſition, S' is as $AD \times Am$, or $AD \times T'$, and (109) as $V \times T'$; therefore V is as AD. PLATE I. FIG. XI.

113. Cor. 3. If AS repreſent the ſpace deſcribed, and the ordinates AD, BE, CF, &c. be always inverſely as the velocities at thoſe points; the times of deſcribing AB, BC, &c. will be as the correſponding areas AE, BF, &c. For the time of deſcribing any ſmall ſpace Am is as $\frac{S'}{v}$, or as $Am \times AD$; and conſequently the ſum of the moments, or whole time of deſcribing AB, is as AE,

114. PROP. *The acceleration and retardation of velocity, vary as the change uniformly produced directly, and the moment of time in which they are produced inverſely.*

If the changes of velocity, uniformly produced in the ſame time, be as 2:1, the acceleration is as 2:1 (102); and if the times, in which the ſame change of velocity is effected, be as 1:3, the law of acceleration is as 3:1, or inverſely as the times; becauſe if the times were as 3:3, the changes of velocity uniformly produced in theſe equal times, which meaſure the ratio of acceleration, would evidently be as 3:1, therefore the acceleration is as $\frac{V'}{T'}$, and in the ſame manner the retardation is as $-\frac{V'}{T'}$. Q. E. D.

115. Cor. If the velocity vary as the time, the acceleration and retardation are conſtant; for $V : v :: T : t$, and, ſuppoſing $n =$ the number of moments of time, or changes of velocity, $\frac{V}{n}(V') : \frac{v}{n}(v') :: \frac{T}{n}(T') : \frac{t}{n}(t')$; or V' is as T' and $\frac{V'}{T'}$ is given.

NOTE.

PLATE I. FIG. XII.

*116. PROP. *In variable motions, if the velocity* (V) *in any point* L, *be as any power of the ſpace deſcribed, the time* (T) *of deſcribing that ſpace may be found.*

Let the body begin to move from A, and $AL = z$, and V be as z^n, and T the time of deſcribing AL. $\dot{T}$ is as $\frac{\dot{z}}{V}$ (109) as $\frac{\dot{z}}{z^n}$, and $T = \frac{z^{1-n}}{1-n} + C$ (correction). Q. E. D.

117. Cor. 1. If V be conſtant, or $n = o$; T will be as (z) the ſpace deſcribed.

118. Cor. 2. If V be as the ſpace deſcribed, or $n = 1$; $T = \frac{1}{o}$ or is infinite, and the body will never move from the point A. There are therefore no motions in nature, whoſe velocities are not in a leſs ratio than that of the ſpaces deſcribed, in the beginning of motion.

119. Cor. 3. If V be as the power of the ſpace deſcribed, whoſe exponent is $\frac{1}{2}$, $-\frac{1}{2}$, $-1-2$, &c. the time will be as that power of the ſpace deſcribed, whoſe exponent is $\frac{1}{2}$, $\frac{3}{2}$, 2, 3, &c.

FIG. XIII.

120. Cor. 4. Let the body move in AL, and the velocity (V) be as the ordinates of the curve line AM, which meets LA in A, and conſequently at A, $V = o$. It is evident, that if the time of deſcribing AL be finite, or (118) V be as ſome power of AL, whoſe exponent is leſs than unity, or fractional, the tangent at A will be perpendicular to AL. Let V be as AL^n and $\dot{V}$ as $n \times AL^{n-1} \times \dot{AL}$, and $\dot{V} : \dot{AL} :: n \times AL^{n-1} : 1$; and, if $\dot{V}$ be as any power of AL, whoſe exponent is leſs than unity, viz. $\frac{1}{2}$, $\dot{V} : \dot{AL} :: \frac{1}{2} \times AL^{-\frac{1}{2}} : 1 :: \frac{1}{2AL^{\frac{1}{2}}} : 1 :: \frac{1}{o} : 1$, when AL vaniſhes, and conſequently the fluxion of the ordinate LM is infinitely greater than that of the abſciſs AL.

121. EXAMP. If AMB be a ſemicircle, $AB = 2a$, $AL = s$, and the velocity at any point L, as LM, or ſuch as would deſcribe $n \times LM$ feet in 1″; $T =$ the fluent of $\frac{\dot{s}}{v}$, or of $\frac{\dot{s}}{n \times LM}$, or $\frac{\dot{s}}{n \times \sqrt{2as - s^2}}$, or $\frac{a\dot{s}}{na \times \sqrt{2as - s^2}}$, or $\frac{AM}{n \times AC}$ ſeconds. Therefore the whole time of deſcribing $AB = \frac{AMB}{n \times AC}$ ſeconds, which is a given quantity, and whatever be the length of AB, it will be deſcribed in the ſame time.

PLATE II. FIG. XIV.

122. PROP. *The times of deſcribing* AL *and* al (T *and* t) *with velocities which are always to each other as the ordinates* LM, lm *of ſimilar curves* AM, am, *are equal.*

* Euler's Mechanica.

DEM.

Plate I.

FIG. I.

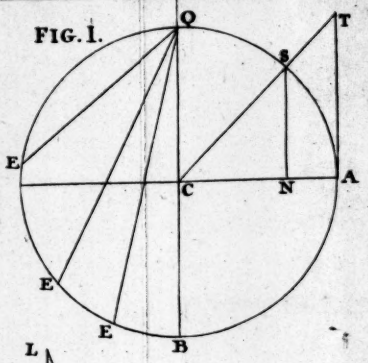


FIG. II.

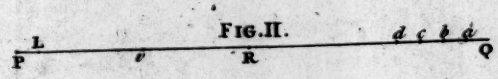


FIG III

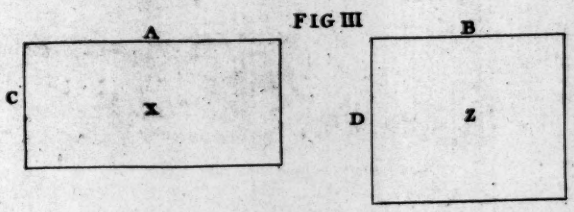


FIG. IV.

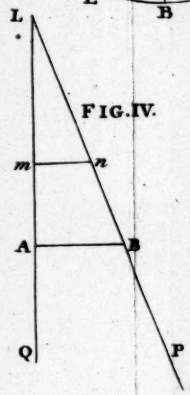


FIG. V.

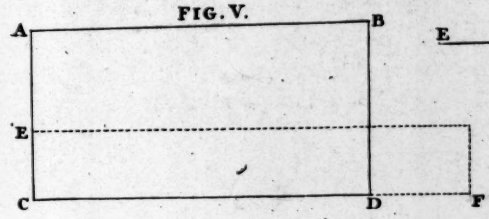


FIG. VI.

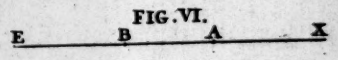


FIG. VII.

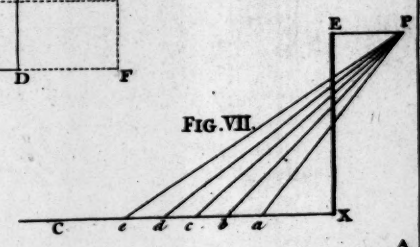


FIG. VIII.

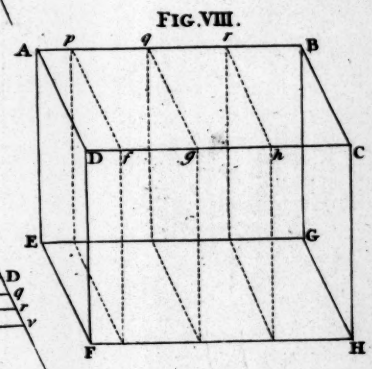


FIG. IX.

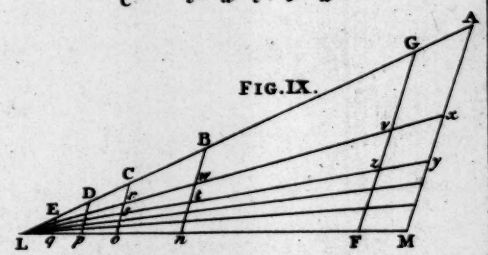


FIG. X.



FIG. XI.

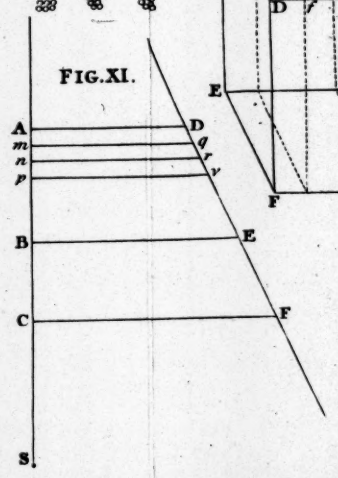


FIG XII

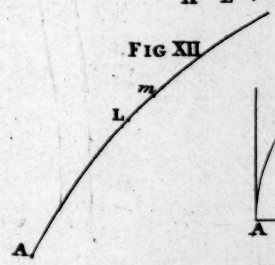
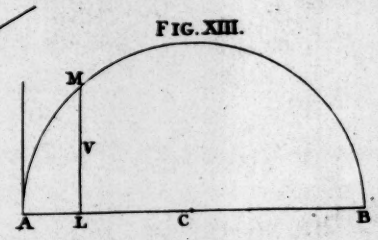


FIG. XIII.



DEM. Let AL and al be homologous ſpaces, whoſe ratio is that of $p : q$, and if $AL = s$, and $LM = v$; $al = \frac{AL \times q}{p}$, and $lm = LM \times \frac{q}{p}$. The time of deſcribing AL or $(T) =$ flu. $\frac{\dot{s}}{v}$, and $t =$ flu. $\frac{\dot{a}l}{lm} =$ flu. $\frac{\dot{AL}}{LM} =$ flu. $\frac{\dot{s}}{v} = T$. Q. E. D.

123. DEF. *The ſcale of velocity is a line whoſe ordinates are as the velocities at thoſe points from whence they are drawn. If the body move in* AL, *as in the laſt propoſition, &c.* AM *is the ſcale of velocity.*

124. DEF. *The ſcale of time is a line whoſe ordinates are as the time. If the ordinate* MQ *be always as the time of deſcribing* AM, *the line* AQ *is called the ſcale of time.* PLATE II. FIG. XV.

125. PROP. *If the ſcale of velocity* (V) *be given, the ſcale of the time* (T) *may be found.*

For $\dot{T}$ is as $\frac{\dot{S}}{V}$ and $T =$ flu. of $\frac{\dot{S}}{V} + C$, and if the relation of the ordinate and abſciſs, or V and S be known, the fluent of $\frac{\dot{S}}{V}$ may be found. Q. E. D.

126. Cor. If the ſcale of velocity AN be a circular arc, the time is that arc; for $\dot{T}$ is as $\frac{\dot{S}}{V}$ or $\frac{\dot{S}}{MN}$, or $\dot{AN}$, and T is as AN.

127. PROP. *The ſcale of time* AQ *being given to conſtruct the ſcale of velocity.*

$\dot{T}$ is as $\frac{\dot{S}}{V}$, and V as $\frac{\dot{S}}{\dot{T}}$ or $\frac{\dot{MQ}}{\dot{MO}}$, ſuppoſing QO to be perpendicular to the curve. Take therefore $MN = \frac{\dot{MQ}}{\dot{MO}}$, which is $= \frac{\dot{S}}{\dot{T}}$ or as V, and AN is the ſcale of velocity.

128. Cor. 1. If AQ be a right line, and T as $m \times S$; $\dot{T}$ is as $m \times \dot{s}$, and V is as $\frac{\dot{S}}{\dot{T}}$ as $\frac{\dot{S}}{m \times \dot{s}}$ as $\frac{1}{m}$, and AN is a right line.

129. Cor. 2. If T be as S^m and $\dot{T}$ as $m \times S^{m-1}\dot{s}$; V will be as $\frac{\dot{s}}{m \times S^{m-1}\dot{s}}$, or as $\frac{1}{m \times S^{m-1}}$. If AQ be the common parabola, or $m = \frac{1}{2}$; $\frac{1}{m \times s^{m-1}}$ will be equal to $s^{\frac{1}{2}}$, and AN is alſo a parabola; and the relation of $T\,(MQ)$ and the abſciſs AM or S being known, the relation of $V\,(MN)$ to S may always be found.

RELATIVE MOTION.

130. DEF. *Relative motion is a change of relative place, and relative reſt is a permanence in the ſame relative place.* Relative and apparent motion are ſometimes diſtinguiſhed from each other, the firſt being defined to be that which is attributed to a moving object by an obſerver in motion, and the ſecond that attributed to an object really quieſcent by an obſerver in motion.

131. DEF. *The relative velocity of two bodies is the velocity with which they accede to, or recede from, each other.*

132. Cor. 1. Relative motion and reſt of a body coincide with abſolute, when the aſſumed object, by which its ſituation is determined, remains in the ſame part of fixed ſpace. If the earth be quieſcent, every ſhip which moves, or is quieſcent, with reſpect to a fixed object upon the ſhore, moves alſo abſolutely, or is abſolutely quieſcent.

PLATE II. FIG. XVI. 133. Cor. 2. If the object, by which the ſituation of a body is determined, move, abſolute and relative motion and reſt do not coincide. If the body B, fixed at the point B in the line AB, be the aſſumed object with which the ſituation of A, placed in the ſame line and partaking of its motion, be compared, its abſolute velocity will be the ſum or difference of its own velocity and that of the line, according as they move in the ſame, or oppoſite, directions. If the line AB move in the direction YX, with an uniform velocity of 100 feet in 1″, and A alſo move in the ſame direction, with an uniform velocity of 50 feet in 1″; A's whole abſolute velocity in the direction YX is 150 feet in 1″, and its relative velocity, or uniform receſs from B, is 50 feet in 1″. If the line AB move uniformly in the direction YX with a velocity of 100 feet in 1″, and A move in the oppoſite direction XY, with a velocity of 100 feet in 1″, it will be abſolutely quieſcent, and its

relative velocity, or *B*'s acceſs to it, will be 100 feet in 1″ uniformly: if *A*'s velocity, in the laſt ſuppoſition, be 50 feet in 1″ uniformly, its relative velocity, or approach to *B* will be 50 feet in 1″, and its abſolute velocity will alſo be 50 feet in 1″ in an oppoſite direction.

134. Cor. 3. A ſpectator therefore, placed upon the moving line *AB* at the point *B*, only perceives *A*'s relative motion, which may be always different from the true.

*135. PROP. *If two bodies* A *and B move at the ſame time from* A *in the directions* AM *and* AL, *with uniform velocities* a *and* b *reſpectively, their relative velocity will be uniform, and to* a *as the ſine of* ∠ BAD : ∠ BDA, *and to* b *as ſin.* ∠ BAD : *ſin.* ∠ DBA *or* ∠ BAF, *ſuppoſing* B *and* D *to be cotemporary poſitions of* A *and* B, *and* AF *to be parallel to* BD. PLATE II. FIG. XVII.

DEM. Let *B*, *D*; *C*, *E*; *M*, *L*; be cotemporary poſitions of *A* and *B*; and (106) *a* : *b* :: *AB* : *AD* :: *AC* : *AE* :: *AM* : *AL*; and conſequently *BD*, *CE*, *LM*, which meaſure the relative velocity, are parallel to each other, and encreaſe uniformly becauſe *AM* does. But the relative velocity is to *a* :: *LM* : *AM* :: ſin. ∠ *MAL* : ſin. ∠ *ALM*, and it is to *b* :: *LM* : *AL* :: ſin. ∠ *LAM* : ſin. ∠ *LMA* or ∠ *MAH*. Q. E. D.

136. Cor. 1. The only ſenſible motion of *B* to a ſpectator placed at *A*, and ſuppoſing himſelf to be quieſcent, will be along *Ah* parallel and equal to *ML*; and when *A* hath really deſcribed *AM*, *B* will appear to have deſcribed *Ah* = *ML* uniformly.

137. Cor. 2. If *A* and *B* move uniformly with any equal velocities in parallel directions *AM*, *BL*, their relative ſituation is not changed, being always equal, and parallel, to *AB*; and if any FIG. XVIII.

ſpace

* Euler's Mechanica.

ſpace *AMBL*, containing any number of bodies, move, their relative ſituations are not affected by it. To a ſpectator therefore placed in *A*, in this moving ſpace, and ſuppoſing himſelf to be quieſcent, any body *B* will appear to be quieſcent.

138. Cor. 3. It is evident that the abſolute motion of a body may be changed into innumerable relative motions uniform and rectilineal, if its abſolute motion and that of the body, by which its ſituation is determined, be ſo.

PLATE II. FIG. XIX. 139. PROP. *If two bodies* A *and* B *move, at the ſame time, from the point* A, *in the directions* AM, AL, *with uniform velocities, as* AM, AL *reſpectively, and* B *be conſidered as quieſcent;* A's *apparent motion will be uniform and its direction and velocity as the diagonal of a parallelogram whoſe ſides are* AM, *and* AD *equal and oppoſite to* AL.

DEM. Let *m*, *l* be cotemporary poſitions of *A* and *B*, or let $Am : Al :: AM : AL$ and *lm*, *Lm* are parallel. But when *B* is at *l* and *L*, *A*'s diſtance is equal to *lm*, *LM*; or, ſuppoſing *B* to be quieſcent at the point *A*, and *AE*, *AD* be reſpectively equal and oppoſite to *Al*, *AL*, and *En*, *DN* be drawn parallel to *AM*, *A* will appear at *n* and *N*. But *An* varies as *Am*, and therefore encreaſes uniformly, and *AN* is the diagonal of a parallelogram, whoſe ſides are $AD = AL$, and *AM*. Q. E. D.

140. Cor. 1. The apparent and real velocities of *A* are to each other as $AN : AM ::$ ſin. $\angle MAD :$ ſin. $\angle NAD$.

141. Cor. 2. Suppoſing *A* and *B* to move as before, and *A* to be quieſcent, the diſtances and directions of *A* from *B* will be *Ah*, and *AH*, ſuppoſing them to be reſpectively equal and parallel to *ml*, *ML*. Whilſt *B* therefore really deſcribes *AL*, it appears to deſcribe $AH = AN$, and in the ſame right line with it; and the apparent

apparent motion of *A* and *B*, ſeen from each other, are equal and oppoſite.

142. Cor. 3. An uniform relative motion along the diagonal of any parallelogram *AMND* may be conſidered as reſulting from, and equivalent to, two uniform abſolute motions in the ſides *AM* and *AL* = *AD*; and theſe motions are in the ſame plane, and their velocities are to each other as the diagonal and ſides. FIG. XIX.

143. Cor. 4. If the motions of *A* and *B* be oppoſite, their relative motion is equal to the ſum of their real motions, *AN* being, in that ſuppoſition, equal to *AM* + *AD*. If the motions of *A* and *B* conſpire, their relative motion is equal to the difference of their real motions, or of *AM* and *AD*. And if the real velocities of *A* and *B* be variable, according to the ſame law, their relative velocity will vary according to that law.

144. Cor. 5. The real motions in *AM* and *AL* are ſaid to be equivalent to a motion in *AN*, becauſe they produce the ſame effect in the ſame time, as if *B* were quieſcent at *A*, and *A* were to deſcribe *AN* uniformly in that time. And in the ſame manner two motions in *AN* and *AD* are equivalent to a motion in *AM*, or its oppoſite and equal *AP*, according as the body deſcribing *AD*, or *AN*, is ſuppoſed to be quieſcent.

145. Cor. 6. If the real motion of *A* and *B*, or *AM* and *AL*, and their inclination be known, the relative motion of either of them may be found; for *AM*, *MN* and ∠ *MAD*, and conſequently its ſupplement *AMN*, being known, the baſe *AN*, and the ∠ *MAN*, or the inclination of *A*'s apparent to its real path, may be found.

146. Cor.

146. Cor. 7. The relative and real motion of A, or AM, AN, and the angle MAN, being known, the real motion of B, which is ſuppoſed to be quieſcent, or $AL = AD$ may be found.

PLATE II. FIG. XX.

147. PROP. *If two bodies* A *and* B *move at the ſame time uniformly in the directions* AM, BL, *with velocities equal to* a *and* b, *the relative motion of* B *will be uniform and rectilineal upon the line* BN, *ſuppoſing* AN *to be always equal, and parallel to right lines joining the cotemporary poſitions of* A *and* B.

DEM. Let m, l; M, L, be cotemporary poſitions of A and B, or let $a : b :: Am : Bl :: AM : BL$, and ml, ML will be the diſtances of B from A in the points m and M. And if A be conſidered as quieſcent at A, taking An, AN, reſpectively parallel and equal to ml, ML, B will appear at n and N in the line BnN, which is therefore the apparent path of B. But $a : b :: Am\,(nl) : Bl :: AM\,(NL) : BL$; therefore BnN is a right line, and encreaſes uniformly, becauſe it varies as BL. Q. E. D.

148. Cor. If A and B move as in this propoſition, and B be conſidered as quieſcent, A's apparent path will be in the right line ADE, parallel and equal to BnN, as is evident by taking BD and BE reſpectively parallel and equal to lm and LM. A's velocity will therefore be uniform and equal to B's velocity, when A was conſidered as quieſcent.

149. The abſolute and relative motions of B, and their directions being known, the direction and velocity of A's motion, which occaſioned B's relative motion, may be found; for, let A and B be cotemporary poſitions of A and B, and let Bl, Bn, be the ſpaces deſcribed in the ſame time by B's abſolute and apparent motion, and if ln be joined, AM drawn parallel to it, will be A's path, and A's velocity : B's velocity :: $nl : Bl$.

150. Cor.

150. Cor. 1. An abſolute uniform and rectilineal motion BL may be changed into any infinite number of uniform relative rectilineal motions, by changing the direction and velocity of A's motion; for BN may be drawn of any length, and in any direction, and the direction and velocity of A, moving uniformly ſo as to produce this relative motion, may be found by drawing AM parallel and equal to NL. FIG. XX.

151. Cor. 2. When two bodies move uniformly in right lines, and one of them is conſidered as quieſcent, the relative motion of the other is therefore uniform and rectilineal; and the ſpaces, deſcribed by it relatively, vary as the velocity multiplied into the time (104), or S varies as $V \times T$, T as $\frac{S}{V}$, and V as $\frac{S}{T}$.

152. Cor. 3. If the relative motion of B be variable, the abſolute motion of either A or B is variable.

153. PROP. *If a body* A *move in any curve line* AmM, *and another body* B *move in any other curve* BlL, B*'s apparent motion may be found.* PLATE II. FIG. XXI.

Let m, l, M, L, be cotemporary poſitions of A and B; and B's diſtance and the right lines in which it appears at m and M, are ml and ML. Or, if A be conſidered as quieſcent, and An, AN, be taken reſpectively parallel and equal to ml, ML, B will appear at n and N, and its path will appear to be BnN. Q. E. I.

154. Cor. It is evident that the curve BnN, the relative path of B, may be deſcribed by the motion of A in different curves; but all the curves deſcribed by A will be equal and ſimilar, and have their correſponding parts parallel, becauſe they are always ſubtended by right lines equal and parallel to nl, NL, &c.

FIG. XXII.

155. PROP. *If* B *be abſolutely quieſcent, and* A *move in any curve line* A m M *uniformly,* B *will appear to move uniformly in a curve equal and ſimilar to* A m M, *and theſe curves are ſimilarly ſituated with reſpect to the points* A *and* B.

Let *m*, *M* be any contiguous poſitions of *A*, which being conſidered as quieſcent at the point *A*, the cotemporary poſitions of *B* are *n* and *N*, ſuppoſing *An*, *AN* to be reſpectively parallel and equal to *Bm*, *BM*. And becauſe *An* = *Bm* and *AN* = *BM*, and the angle *NAn* = angle *mBM*, the ſmall arc *Nn* = *mM*, and they make equal angles with the diſtances *AN*, *BM*, and *An*, *Bm*. Q. E. D.

156. Cor. 1. If *B* move in the curve *BnN*, and *A* be quieſcent at the point *A*, it will appear to deſcribe the curve *AmM* ſimilar and equal to *BnN*.

PLATE III. FIG. XXIII.

157. Cor. 2. If *A* and *B*, placed in the ſame right line *ABC*, deſcribe the circles *AmM*, *BlL* in the ſame time uniformly, and *A* be conſidered as quieſcent, *B*'s apparent path is a circle whoſe center is *A* and radius *AB*. For let *m*, *l*, and *M*, *L*, be cotemporary poſitions of *A* and *B*, and *B* will appear at *l* and *L*; or, ſince *A* is conſidered as quieſcent, if *An*, *AN*, be drawn parallel and equal to *ml*, *ML*, it will appear at *n* and *N*, and its apparent diſtance from *A* always = *AB*, and the direction of *B*'s apparent motion is the ſame with that of *A* and *B*.

158. Cor. 3. If *B* be conſidered as quieſcent, and they move as in the laſt corollary, *A*'s apparent path will be a circle whoſe center is *B* and radius *BA*, and the direction of its motion is the ſame with that of *B* in that corollary. For taking any cotemporary poſitions *m*, *l*, and *M*, *L*; and, drawing *Bn*, *BN* reſpectively parallel and equal to *ml*, *mL*, *A* will appear at *n* and *N*, and its apparent diſtance from *B* always = *AB*.

159. Cor.

FIG. XIV.

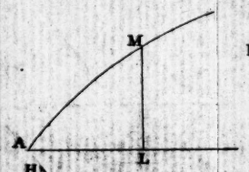


FIG. XV.

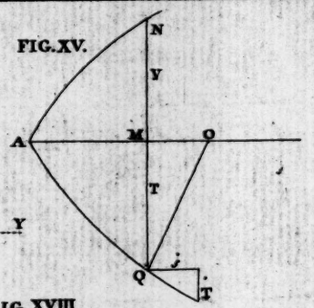


FIG. XVI.

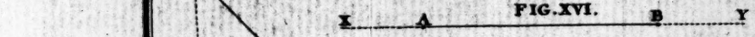


FIG. XVII.

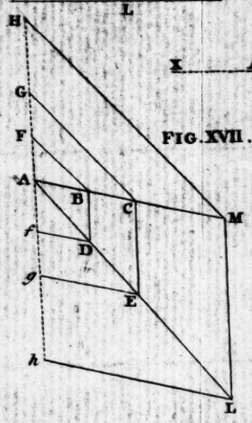


FIG. XVIII.

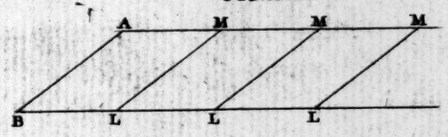


FIG. XIX.

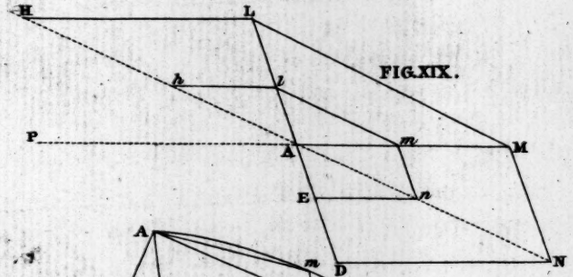
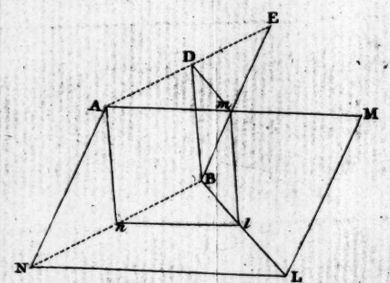


FIG. XX.



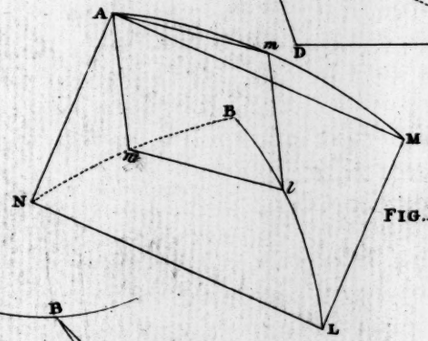


FIG. XXI.

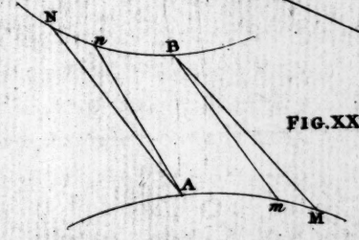


FIG. XXII.


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159. Cor. 4. The relative motions of A and B ſeen from each other, are equal and in ſimilar curves. For let m and M be any contiguous poſitions of A, and l, L correſponding poſitions of B; and joining ml, ML, B's relative motion, whilſt it really deſcribes lL, will make it deſcribe nN, ſuppoſing A to be quieſcent, and An, AN to be reſpectively equal and parallel to lm, LM. Suppoſing B to be quieſcent, and Bb, BH to be equal and parallel to ml, ML, or An, AN, A will appear to deſcribe bH, which is equal and ſimilar to Nn. FIG. XXIV.

160. Cor. 5. If B be immoveable and not placed in the plane of A's motion, it will appear to deſcribe a line equal and ſimilar to the line deſcribed by A, and they will be in parallel planes; for taking any two contiguous portions of A's path, the right lines joining the real and apparent places of A and B are equal and parallel, and conſequently the ſmall right lines joining them are equal and parallel: and this and the preceding corollaries are true wherever the eye's imaginary place is.

SCHOLIUM.

161. To diſtinguiſh real and abſolute, from relative motion, is an important, but very difficult problem; becauſe the ſame apparent motions may reſult from real motions combined in endleſs variety. Every apparent motion of a body reſults from, and may be explained by, its real motion and that of the ſpectator and v, v; and therefore its real motion, and the real motion of the ſpectator, or this, and the apparent motion of the body, muſt be preſuppoſed, and, with the aſſiſtance of mathematics, the other may then be detected. A ſpectator ignorant of the earth's annual and diurnal motion, and ſuppoſing himſelf to be quieſcent, muſt draw erroneous concluſions, in all his reaſonings, concerning the abſolute motion of any body, becauſe theſe motions of the earth will communicate an apparent motion to a body abſolutely quieſcent, and affect the abſolute motion of a body moving (134). The relative motions of bodies muſt therefore be aſcertained from obſervation, and their real motions, when

diſcoverable, are deduced from theſe, the properties of real motion, and the quantity of mechanical powers, or qualities in matter, that are found to generate motion. If the properties, invariably attached to all real motions, obtain in any obſerved motion; or if any mechanical cauſe be proved to exert an influence, which is exactly competent for the production of any obſerved motion, it may ſafely be inferred that this motion is not imaginary, but real.

EXAMP. I. The variation of diſtance between a ſhip and a remote object is owing to the motion of the ſhip, when the wind and current act and are competent for its production.

FIG. XXV. EXAMP. II. A ſhip ſailing from ſouth to north at the rate of five miles in an hour, obſerves another ſhip which ſailed from the ſame point *E* going to the ſouth-weſt at the rate of $7\frac{1}{10}$ miles in an hour nearly, and from hence it appears, that the laſt ſhip ſailed from eaſt to weſt at the rate of five miles in an hour nearly (139).

EXAMP. III. The magnitude of the earth's attractive power, or any other mechanical cauſe, is incompetent for the production of the diurnal revolution of the heavenly bodies, which is therefore probably apparent.

EXAMP. IV. The aberrations of the fixed ſtars are proved geometrically to reſult neceſſarily from the progreſſive motions of light and of the ſpectator combined, and are conſequently only apparent.

EXAMP. V. A real circular motion is inſeparably attended with a centrifugal force, and diſtinguiſhable therefore by this effect. Thus the diminution of gravity, which is obſerved in acceding towards the equator, affords a proof of the diurnal rotation of the earth.

CHAP.

CHAP. IV.

INERTIA OF MATTER.

162. DEF. *POWER and force are uſed ſynonimouſly to ſignify any action upon a body, which produces motion, or a tendency to motion, as animal exertions and the influence of phyſical cauſes, gravity, elaſticity, magnetiſm, &c. When a force acts always with the ſame intenſity, or produces the ſame effect in a given time, it is ſaid to be uniform or conſtant, and variable when it does not.*

163. DEF. *Any momentary impreſſion upon a body producing motion, or a tendency to motion, is called an impulſe, or percuſſion, of a power or force.*

164. DEF. *Whatever reſiſts the action of a force is called an obſtacle.*

165. DEF. *Inertia, or vis inertiæ, of matter, is that quality by whoſe influence a change, in the direction or quantity of its motion, cannot be produced without the application of a force.* The cauſe of a body's continuing in a ſtate of reſt, or of uniform rectilineal motion, is not any external force, but the nature and conſtitution of matter; and this internal cauſe or principle is called its inertia.

166. This property of matter, or inertia, is eſſential in the conſtitution of a ſyſtem, whoſe preſervation is neceſſarily dependent upon the regular obſervance of preſcribed, determinate, laws; but its

its exiſtence can only be collected from experience. *Firſt*, Every change in the ſituation of a body, from reſt to motion, and from motion to reſt and to an encreaſe or decreaſe of motion, indicates an inertia, which is found to pervade every ſpecies of matter acceſſible to obſervation, the particular facts, adduced in proof of it, being innumerable and concurring to eſtabliſh its univerſal exiſtence. A quieſcent body is never diſcovered to move, and motion, or change of motion, is never induced, without the actual impreſſion of a cauſe able to produce theſe effects; for matter not only continues in a ſtate of reſt, by an incapacity to give motion to itſelf, but never ceaſes to move, or changes the quantity and direction of its motion, without the application of ſome philoſophic influence adequate to their production, as magnetiſm, elaſticity, gravity, &c. or animal exertion, as percuſſion, protruſion, &c. The time of the motion of a wheel upon its axis, or braſs topp upon a poliſhed ſurface, is encreaſed with the diminution of friction upon the axis, or ſurface; and the motion of a body, placed upon the deck of a ſhip, and partaking of its motion, continues in the ſame direction after the ſhip has ceaſed to move; and theſe motions are evidently diſcontinued by the operation of a cauſe, i. e. friction, competent to deſtroy them. New motions are obſerved without any ſenſible material impulſe, reſulting apparently from an innate tendency to motion; thus, a body not ſupported deſcends towards the earth, and, projected in a direction not perpendicular to the earth's ſurface, deviates from the line of projection with a velocity perpetually variable; and new motions ariſe amongſt the minute particles of bodies; but theſe are the neceſſary reſult of eſtabliſhed natural powers, gravity, elaſticity, &c. *Secondly*, This quality is ſometimes called the vis inſita, or vis inertiæ, of matter, from the ſimilitude of its effects to thoſe of animal powers exerciſed upon a body, both producing a change of motion. If a body *A*, placed upon a poliſhed table, or ſuſpended by a rope, be quieſcent, it will continue in that ſtate till urged by ſome external cauſe; and if another body *B* be projected with any velocity, and impinge upon *A*, it will communicate motion to it, and be itſelf retarded. When the effect of *B*'s

impact is conſidered in relation to the change produced in *A*, *B* is ſaid to act upon *A*; and, when conſidered in relation to the change of motion produced in itſelf, it is ſaid to be reſiſted by *A*. Action is uſually aſcribed to a moving, and reſiſtance to a quieſcent, body; but they may both be conſidered as actions, becauſe they produce ſimilar effects, that is, a change of motion. *A*, by its vis inertiæ, deſtroys a part of *B*'s motion, and *B*, by endeavouring to retain its preſent ſtate, that is, by its vis inertiæ, protrudes, and communicates motion to, *A*. *Thirdly*, Every change in the ſituation of a body is therefore univerſally allowed to indicate ſome philoſophical influence, or animal exertion; and whether a body be quieſcent, or moving, the quantity of its inertia is invariably the ſame; for it is diſcovered from experiments, the only ſource of information, that a force, communicating a velocity equal to *a* to the quieſcent body *A*, will communicate an encreaſe of velocity, equal to *a*, to that body moving. The force of gravity, which communicates a velocity nearly equal to thirty-two feet in one ſecond of time, to a body falling from a ſtate of reſt, communicates an encreaſe or decreaſe of velocity equal to this, in one ſecond, to a body already moving, according as it conſpires with, or is oppoſite to, the direction of the body's motion. And if the body *B*, moving with a velocity equal to *b*, impinge upon *A* at reſt, and communicate a velocity to it equal to *a*, and loſe a velocity equal to *l*, it appears from experiments, that *B* moving with the velocities $b + m$, $b + 2m$, $b + 3m$, &c. will communicate an encreaſe of velocity equal to *a*, and loſe a velocity equal to *l*, by impinging on *A* moving in the ſame direction with velocities equal to m, $2m$, $3m$, &c. Conſequently the vis inertiæ of *A*, eſtimated by the velocity, and encreaſe or decreaſe, communicated to it, is the ſame in a ſtate of motion and reſt. *Fourthly*, As the whole vis inertiæ of a body is compoſed of that of all its parts, and as we cannot conceive the vis inertiæ of the ſame or equal particles to be encreaſed or diminiſhed, the whole vis inertiæ of different bodies will vary as the number of equal particles, or quantities of matter, contained in them. The invariable reſult of experiment is, that bodies equal

to

to A, $2A$, $3A$, &c. and heterogeneous bodies, whoſe weights are as 1, 2, 3, &c. receive the ſame velocity by the action of forces, whoſe magnitudes are as 1, 2, 3, &c.

167. DEF.* *The moment, quantity of motion, vis inſita, of a body, are uſed ſynonimouſly to denote the impetus, or force, with which it moves, or its capacity to communicate motion or preſſure.*

168. PROP. *If* M, Q, V, *repreſent moment, quantity of matter, and velocity, reſpectively, and be ſuppoſed variable,* M *will vary as* Q × V.

DEM. Preſuming that equal particles of matter have an equal inertia, it is evident that the preſſure of one particle upon an immoveable obſtacle, or the force requiſite to generate or deſtroy its velocity, will vary as this velocity, or as V; and becauſe the impetus of any body is compoſed of that of every particle or Q, and Q and V are independent, M will vary as $V \times Q$ (64). Q. E. D.

Another demonſtration.

The moments of different bodies vary as the uniform forces capable of generating or deſtroying them, in equal times, being their whole cotemporary effects; but forces equal to F, $2F$, $3F$, &c. will evidently communicate, in the ſame time, the ſame velocity to 1, 2, 3, &c. equal particles endued with an equal inertia, and velocities as 1, 2, 3, &c. to one particle. Therefore F, which is as M, is encreaſed in the ſame ratio, with the velocity or V, and number of equal particles or Q, and, theſe being independent, vary as $Q \times V$ (64). Q. E. D.

* Keil's Phyſics, Lect. IX.

SCHO-

SCHOLIUM.

169. Theſe, or ſimilar demonſtrations, are uſually adduced in ſupport of this propoſition, which is perhaps more properly and with more conviction, demonſtrated by experiments. The moments of different bodies are collected by meaſuring the magnitudes of the forces required to produce them, or the magnitudes of their cotemporary effects ſimilarly produced, in the ſimpleſt and moſt intelligible inſtances, which may be deemed the common meaſure of their moments.

EXP. I. If two unequal ſpherical bodies, moving in oppoſite directions, meet and after impact be quieſcent, their moments muſt be equal; but their quantities of matter are always found to be inverſely as their velocities. Or, if they move in the ſame direction, and one overtakes the other, the velocity gained by the ſtruck body, and loſt by the ſtriking body, are always found to be inverſely as their quantities of matter.

EXP. II. If a body A be placed on the ſame ſide of the fulcrum of a ſtraight lever, at the diſtance of 1, 2, 3, &c. feet from A, it will move with velocities as 1, 2, 3; and be reſtored to an equilibrium by the bodies A, $2A$, or $3A$, &c. placed, at the diſtance of one foot, on the other ſide. And, in general, a body, whoſe weight is $n \times A$ pounds, at the diſtance of one foot from the fulcrum, is found to equilibrate with bodies, whoſe weights are $\frac{n \times A}{2}$, $\frac{n \times A}{3}$, $\frac{n \times A}{4}$, &c. at the diſtance of 2, 3, 4, &c. feet from the fulcrum reſpectively. The preſſures or moments, are juſtly inferred from theſe experiments, to be as their weights multiplied into their diſtances, or as the quantities of matter multiplied into their velocities. And the ſame concluſion reſults from experiments upon every other, however complicated, machine, when allowance is made for friction, and the velocity is properly eſtimated.

170. Cor. 1. Since M varies as $\mathcal{Q} \times V$, V will vary as $\frac{M}{\mathcal{Q}}$, and $\mathcal{Q}$ as $\frac{M}{V}$ (55). If lines be taken in the ratio of $\mathcal{Q}$ to V, M will vary as the area of a rectangle, whoſe ſides are theſe lines; and if numbers be taken in that ratio, M will vary as their product.

171. Cor. 2. If S repreſent the ſpace deſcribed, in the time $\mathcal{T}$, with the velocity V, M, varying as $\mathcal{Q} \times V$, will vary as $\frac{\mathcal{Q} \times S}{\mathcal{T}}$ (105), S, varying as $V \times \mathcal{T}$, will vary as $\frac{M \times \mathcal{T}}{\mathcal{Q}}$, and $\mathcal{T}$ will vary as $\frac{S \times \mathcal{Q}}{M}$.

172. Cor. 3. If two bodies therefore A and B, moving in oppoſite directions with velocities reſpectively equal to a and b, meet, and after impact be quieſcent; or if their effects to produce motion upon any machine be oppoſite and equal, and their velocities, when properly eſtimated, be a and b; $A \times a = B \times b$, and $A : B :: b : a$ (38), or A and B are to each other inverſely as their velocities. The converſe of this is true, and if the bodies be inverſely as their velocities, or $A : B :: b : a$, their moments are equal, or they are in equilibrio upon any machine, and $A \times a = B \times b$ (37).

SCHOLIUM.

173. Forces are diſtinguiſhed by ſome foreign writers into two kinds; 1ſt, of bodies at reſt, and 2dly, of bodies in motion.

Firſt, *The force of a quieſcent body, ſuch as is conceived to reſide in one lying upon a table, ſuſpended by a ſtring, ſuſtained by ſprings, &c. is called its preſſure, tenſion, force, ſolicitation, vis mortua, &c. and is eſtimated by the weight with which the table is preſſed, the ſtring ſtretched, ſpring bent, &c. To this kind are referred centripetal and centrifugal forces, as their effects are ſimilar to thoſe of weights, preſſures, tenſions.*

Secondly,

Secondly, *The force of a moving body, arising from its inertia, by which it communicates motion, or a change of motion, to another body by impact, overcomes gravity, friction, and other preſſures, and is only deſtroyed by an extinction of its motion, is called the vis motrix of that body, or its vis viva, to diſtinguiſh it from the vis mortua.*

Concerning the meaſure of the firſt of theſe forces no doubts are entertained, the preſſure of one particle, upon the arm of a lever, being univerſally allowed to be as its velocity, and of a number of particles, as that number and velocity conjointly; but the meaſure of the vis viva hath long been a ſubject of warm contention between the adherents of Newton and Leibnitz, the former maintaining it to be the product of the quantity of matter and velocity, and the latter the product of the quantity of matter and ſquare of the velocity. The term force being defined to be that, which, acting upon a body, communicates motion, or a tendency to motion, the magnitude of the force of a moving body, at any inſtant of time, may be beſt, and indeed can only be clearly, conceived by meaſuring the motion communicated by it; and in this menſuration three things are to be conſidered, viz. the magnitude of the body or quantity of matter to be moved, the velocity communicated, and the time in which it is communicated. Some of theſe circumſtances may be, as they have been, omitted, and different meaſures of the ſame force have reſulted. The followers of Newton conceive forces to act, for the ſame time, with their reſpective intenſities continued uniformly, and eſtimate their magnitudes by their cotemporary effects. If two forces F and f be ſuppoſed to act, at the ſame inſtant, upon two bodies A and B, with uniform intenſities, and communicate velocities to them, equal to V and v reſpectively, in the ſame time, the magnitudes of theſe forces are to each other as $A \times V : B \times v$. And in the communication of motion by the impact of one body upon another, this law is found to obtain, wherever the effects are diſtinctly underſtood, and can be aſcertained, the quantity of motion, if meaſured by the product of the quantity of matter and velocity, remaining invariable, when eſtimated in the ſame direction. But the followers of Leibnitz, adopting a different definition of force, derive a different concluſion: they do not ſuppoſe the force to

act with uniform intenſity, as it may decreaſe gradually to evaneſcence, which happens in the colliſions of bodies, and actions of ſprings; and the time of action, and ſometimes the direction, is diſregarded, and not deemed to affect the reſult. Though it be true in ſome particular caſes of the communication of motion from one body to another, that the force, according to their acceptation of the term, varies as the product of the quantity of matter and ſquare of the velocity, or that this product is the ſame before and after impact, this concluſion cannot be affirmed to obtain generally. The doctrine advanced by Newton is univerſally true, according to his meaning of the term force; but whether the opinion of Leibnitz be true or not, is beſt known from experiments, the reſult of which is generally repugnant to it: and it would therefore, perhaps, create leſs confuſion to adhere to the old definition of force, include the time of action, and ſuppoſe the intenſity of the force, during that time, to be the ſame. The diſtinction between theſe two kinds of forces hath often been urged to be ſuperfluous, for the following reaſons. *Firſt*, The force of a moving body is a vague and undefinable term; for there is no force in a body conſidered abſolutely, except its inertia, which is always the ſame, whether the body be quieſcent or moving; but if a moving body impinge upon another body either moving or quieſcent, its inertia exerts itſelf as a force, whoſe magnitude is relative and depends upon its velocity, and the magnitude and velocity of the ſtruck body. If the body A, moving with the ſame velocity a, impinge upon the quieſcent bodies B, $2B$, $3B$, &c. or upon the ſame body B quieſcent, and moving with different velocities equal to b, $2b$, $3b$, &c. it will in every caſe produce a different effect, and the change of ſtate, both in the impinging and ſtruck body, will be different. The force of ſprings, and animal exertions, is alſo relative, and it ſeems therefore improper to talk of the abſolute force of a moving body, ſprings, &c. becauſe it is relative. *Secondly*, No idea can be formed of an inſtantaneous communication of motion; for, in the colliſion of bodies, the parts which come into contact firſt, are diſplaced and move, and ſome time elapſes before motion be communicated to the whole body. This interval of time between the firſt contact of the neareſt parts of the impinging

ing bodies, and their motion, is the time of colliſion, in which the bodies exert a mutual, and perpetually variable preſſure, by which their ſtate is gradually changed. This is moſt obſervable in ſoft bodies, or thoſe whoſe parts yield to percuſſion; and, as the parts of all bodies do yield, obtains univerſally, in a greater or leſs degree, according to the different contexture of their parts. The forces of percuſſion are therefore preſſures, exerted with variable intenſity for a ſhort time; and, to comprehend their magnitude, this time ought to be aſcertained, the quantity of preſſure, for every inſtant of this time, computed, and the whole collected into one ſum. This diſcrimination of forces ſeems therefore to be unneceſſary, becauſe the action of colliſion is nothing more than the operation of a continued preſſure, and preſſures and percuſſions are ſimilar, and properly expreſſed by the ſame term, force. The arguments adduced, in ſupport of their opinion, by the advocates of Leibnitz, are derived from experiments upon, the colliſion of bodies, the action of elaſtic ſprings, the compoſition and reſolution of motion, the indentings or cavities formed in clay, tallow, &c. by falling bodies, and the velocity of fluids in hydraulics. I will only produce a few experiments, which may convey ſome uſeful knowledge, are moſt ſtrongly urged in ſupport of this hypotheſis, and appear moſt to militate againſt the old opinion.

EXP. I. If one ſpring be requiſite to deſtroy the motion of a body, whoſe velocity is 1, four ſprings, equal to it, are requiſite to deſtroy its motion when its velocity is 2, nine ſprings when its velocity is 3; and generally the number ſprings varies as the ſquare of the velocity of the moving body.

EXP. II. The number of men, horſes, &c. of equal ſtrength requiſite to communicate velocities, as 1, 2, 3, &c. to any body, are as 1, 4, 9, &c. or as the ſquare of the velocity.

Theſe ſimilar experiments may be true, and indeed ſeem to be eſtabliſhed.

eſtabliſhed by Graveſand, Deſaguliers, &c.; but they prove nothing againſt the common opinion of the force of bodies, which is not meaſured by effects produced in any unequal times. And beſides, the effect of any number of agents, as ſprings or animal exertions, cannot properly be ſaid to meaſure their force; becauſe their whole force is not exerted, that of each, conſidered individually, being greater than when acting in conjunction with the reſt, and will be greater or leſs according as the number of the reſt is leſs or greater.

EXP. III. If the velocity of water, ſtriking upon the float-board, or ladle-board, of a mill, be as 1, 2, 3, &c. the effect of the wheel is found to be as 1, 4, 9, &c. or as the ſquare of the velocity of the ſtream; and the force of the water, being meaſured by its effect, varies therefore as the ſquare of the velocity. In this experiment the number of particles of water ſtriking the board in a given time, encreaſes as the velocity, and, the force of each being encreaſed in the ſame ratio, the effect ought to be as the ſquare of the velocity. Theory is very ſeldom confirmed by practice without allowances for the operation of collateral cauſes, which, perhaps, cannot be made with demonſtrative accuracy. Such experiments as this are too imperfectly underſtood to juſtify any general concluſions againſt experimental proofs clearly comprehended, or theoretic demonſtrations, reſulting from data, which, perhaps, do not take place in the experiment.

EXP. IV. Mr. Smeaton has proved, by ſome very ingenious experiments (Phil. Tranſ. Vol. LXVI. pag. 469.), that the mechanic power to be expended, varies as the ſquare of the velocity to be generated, and the velocity as the impelling power multiplied into the time of action. But the mechanic power, according to Mr. Smeaton, is meaſured by the weight multiplied into the ſpace deſcended, and from this definition, his concluſion is inevitable: but this power is different from what is uſually meant by moment, force, &c. In N°. 2, 3, the weight *S* deſcends through ſpaces

ſpaces equal to 10, and $2\frac{1}{2}$, turns, in $28\frac{1}{4}''$ and $14\frac{1}{4}''$ reſpectively, and communicates velocities nearly as 2 : 1 ; and in this experiment the weight S acts with the ſame intenſity for times, whoſe ratio is 2 : 1, but its capacity to communicate motion, if meaſured by the ſpace, muſt be as 4 : 1.*

EXP. V. Equal cavities are formed in clay, tallow, &c. by equal bodies falling through ſpaces inverſely as their quantities of matter; but the cavities being equal, the cauſes, or forces of the bodies, are equal, or as $\frac{S}{S}$ (ſuppoſing S to repreſent the ſpace) or as $S \times 2$, from the ſuppoſition, or as $2 \times V^2$.

EXP. VI. A ball ſhot with velocities as 1, 2, 3, &c. is found to form cavities in wood, clay, &c. whoſe depths are as 1, 4, 9, &c. or as the ſquares of the velocities.

The two laſt experiments prove nothing againſt the common doctrine; and though they are of practical utility, yet admitting the propriety of meaſuring forces by their whole effects produced in any unequal times, they do not vary, in theſe experiments, as the ſquare of the velocity: becauſe the cavities formed are not the whole effect, a greater or leſs motion being communicated to the parts contiguous to the cavity in the ſtruck body, according to the magnitude and velocity of the ſtriking body. A ſmall body, moving with a great velocity, may by impact overcome the coheſive force of the particles of the ſtruck body, and effect a ſeparation between them; and from its great velocity, or ſhort time of action, little motion will be communicated to the contiguous parts; but

* Theſe experiments are immediately deducible from mechanical principles. Let $F =$ the radius of the barrel M; $V =$ velocity generated in K in the time T; X and $Z =$ the reſpective ſpaces paſſed through by S and K in the time T; and $1 = K$'s diſtance from the axis Be. And from eſtabliſhed principles, the force impelling K is .8 oz. $\times F$, or as F, and, becauſe F is conſtant, $F \times T$ is as V. $F \times T^2$ is alſo as Z; but $X : Z :: F : 1$, therefore X (or the ſpace deſcended by S, which is as the mechanic power) $= Z \times F$, and is as $F^2 \times T^2$, or as V^2. (Phil. Tranſ. Vol. LXVI. p. 469.)

but a great body, moving ſlowly, will form no cavity in the obſtacle; becauſe the moment of thoſe parts of its ſurface in contact, is leſs than the coheſive force of the parts of the obſtacle, and the parts ſtruck will, by their unknown connection with the parts adjoining, communicate motion to them, and theſe to the next, till the whole obſtacle move. Two bodies therefore moving with the ſame moment, may produce quite diſſimilar effects; and as lines, ſurfaces, &c. are meaſured by the application of a common meaſure, the magnitudes of forces are to be meaſured by their effects, which ought to be ſimilar in every reſpect, and this ſimilitude is moſt obſervable and beſt underſtood in experiments upon the lever and other ſimple machines.*

LAWS OF MOTION.†

174. The laws of motion are general rules, or conſequences, reſulting from the nature and conſtitution of matter, and its relation to mechanical forces, the cauſes of motion, by which it is governed, and from which it cannot deviate without ſuffering a total change of its nature. Their truth is eſtabliſhed by a number of concurring and uncontroverted obſervations, all information upon natural powers and motions derived from them being only deducible from thence; and, when thus eſtabliſhed, they are eſteemed to be uniform characteriſtic marks obtaining in all material motions.

FIRST

* Let s repreſent the effect, or ſpace through which any obſtacle is impelled by a moving body, v be the velocity of the body: and if s be as v^2, $\dot{s}$ is as $2v\dot{v}$, or as $v\dot{v}$, or as $\frac{\dot{v} \times s}{t}$ (if t be the time), and $\dot{v}$ therefore is as $\dot{t}$. The decrement of velocity is therefore as the time in which it is produced, and the reſiſtance muſt be conſtant; which ſeems to be a neceſſary property of the vis viva; but as this ſeldom happens, and the vis viva is really a preſſure, and may be meaſured in the ſame manner with the vis mortua, the term force may expreſs them both.

See Deſagulier, Vol. II. p. 51. Maclaurin's Account of Sir I. Newton's Diſcoveries, p. 178. Diſcours ſur les Loix de la comm. du Movement, Oper. Tom. III. and Diſſertat. de vera notione virium vivarum. Act. Petropol. Tom. I. p. 131. Muſchen. Int. ad Phil. Nat. Vol. I. p. 83, &c. Phil. Tranſ. No. 371, 375, 376, 396, 400, 401, 459.

† Maclaurin, Book II. Chap. II. Newt. Prin. p. 13. Keil's Phyſics, Lect. XI, XII. Helſham, Lect. III. IV.

FIRST LAW OF MOTION.

175. *Every body perſeveres in a ſtate of reſt, or of uniform rectilineal motion, unleſs affected by ſome external cauſe, as animal exertion or phyſical power.*

The truth of this law is collected by obſervation. It is nearly ſynonimous to the inertia, or that quality, of matter, by which it continues in every new ſtate; for, this being eſtabliſhed, it is a neceſſary reſult that it cannot begin to move from a ſtate of reſt, nor, if in motion, can it accelerate, retard, or change the direction of that motion.

176. Cor. Every body, moving in a curve line, or in a right line with a perpetually varying velocity, is acted upon without intermiſſion during its motion, by ſome external force; and every body, moving in a curve line, has a perpetual tendency to move in a right line, and to leave the curve in the direction of a tangent to that point of the curve where the body is.

SECOND LAW OF MOTION.

177. *Motion, and change of motion, are proportional to, and in the direction of, the force impreſſed.*

The motion communicated to a quieſcent body is proportional to the cauſe producing it; and when a body is accelerated or retarded, during its motion, the acceleration and retardation are proportional to the force producing them. If a body move upon an inclined plane AB, or in any curve line LM, by the action of a force tending to any point S, the change of motion is not proportional to the whole force directed to that point, but to that part which acts in the direction AB, or NT touching the curve where the body is; for this part, only, accelerates or retards the FIG. XXVI.

body's motion. When water or air act upon the vanes of a mill, or ſails of a ſhip, the change of motion is not proportional to the whole force of the water or air, but to that part which is actually impreſſed upon them; for, if the velocity of the water or air be equal to that of the vanes or ſails, they will move on together without any acceleration; and, if the direction of the air or water be oblique to the motion of the vanes or ſhip, ſome force will be ineffectual, and the change of motion will be proportional to the remainder, acting in the direction of their motion.

The truth of this law is demonſtrated by numberleſs experiments upon the motion, and change of direction and motion, communicated by material impulſe. Every experiment upon the effects of conſpiring and oppoſite forces, and the effects of forces, acting obliquely, or reſulting from the compoſition and reſolution of forces, demonſtrated experimentally, exhibit concurring proofs of its truth; for were motion, and change of motion, not proportional to, and in the direction of, the force impreſſed, it may be proved geometrically that theſe effects would not take place. The concluſion is therefore inevitable, and, being thus eſtabliſhed by uniform experience, may be deemed a fixed principle in nature, and applied not only to the explication of thoſe particular experiments from whence it was deduced, but of all other natural phenomena. This law is alſo a proof of the inertia of matter, or, that being eſtabliſhed, a corollary from it; for if matter can produce no change in itſelf, it muſt move in the direction of the force impreſſed; and, ſince the vis inertiæ of a body is the ſame, whether moving or quieſcent, it is evident that the geneſis, encreaſe, and decreaſe, of velocity equal to V in any body, require exactly the ſame force. If the force of colliſion, or impulſe of any force ſuch as gravity, be capable of generating a velocity, equal to V, in the body A, it will in the ſame time generate an encreaſe, or decreaſe, of velocity equal to V in that body, according as it conſpires with, or oppoſes its motion.

178. Cor. 1. The velocity generated by one ſingle impulſe equal to nF, n being a number, is equivalent to that generated by n ſucceſſive

ceſſive conſpiring impulſes, each of which is equal to F; and conſequently the velocity communicated to a body by any number of forces acting in the ſame direction is the ſame, whether they act together or ſeparately; becauſe the geneſis, and encreaſe, of a velocity $= V$, require exactly the ſame force.

179. Cor. 2. The velocity generated, in any body, by any number of unequal conſpiring impulſes X, Y, Z, &c. is as their ſum; for let x, y, z, &c. be the velocities reſpectively generated by them in any body A, and (177) $X : Y :: x : y$, and $X : X+Y :: x : x+y$, and $X : X+Y+Z :: x : x+y+z$, &c. And the velocity, communicated to any body by a force $= F$, in a given time, is as the magnitude of that force; for ſuppoſing this force to act by impulſes, whoſe magnitudes are X, Y, Z, &c., $F = X+Y+Z$, &c.

180. Cor. 3. The magnitudes of any forces are therefore as the ſpaces uniformly deſcribed in a given time, by the velocity which they communicate to the ſame body (106).

181. Cor. 4. The velocity generated in a body in the ſame time by two forces X, Y, acting in oppoſite directions, is as their difference; for (177) $X : Y :: x : y$ and $X : X-Y :: x : x-y$.

THIRD LAW OF MOTION.

182. *Action and reaction are always equal and oppoſite; or the mutual actions of two bodies are always equal, and in oppoſite directions.*

This is another rule obſervable in all the motions of nature, reſulting, like the two firſt, from the inertia of matter. Were matter diveſted of this property, motion would be communicable without reſiſtance, and conſequently without effecting any change in the force communicating it. The action of all forces, whether operating by the viſible impact of one body upon another, or the

inviſible agency of gravity, magnetiſm, &c. conſiſts in producing preſſure and motion; and reaction in ſupporting this preſſure, or reſiſting the production of motion. And theſe this law aſſerts to be equal, when eſtimated in oppoſite directions; which, being proved experimentally, like the two firſt, in all communications of motion with which we are acquainted, may be eſteemed a general principle pervading the whole material ſyſtem.

In the communication of preſſure upon any immoveable plane, whether ariſing from the protruſion, gravity, or force of impact, of a body; the meaning of the law is, that the reſiſtance of the plane, and an oppoſite force equal to that producing the preſſure, have preciſely the ſame effect, as they only deſtroy the force of protruſion, weight, or impact. In the communication of motion by viſible impact, the meaning of the law is, that action is mutual, equal and oppoſite, or that the quantities of motion loſt and gained, which meaſure this action and reaction, are equal when eſtimated in oppoſite directions. It is always ſuppoſed, that the quantity of motion is eſtimated by the product of the quantity of matter and velocity. No idea can be formed of the loſs of motion, except by communication; and that the quantity, loſt by the impinging body, is gained by the ſtruck body, appears from innumerable experiments upon the colliſion of bodies. If a perfectly hard, or a ſoft body, A, moving with the velocity V, impinge upon an equal unelaſtic body B, they will move together, after impact, with a velocity equal to $\frac{V}{2}$; and if A, moving with the velocity $V+u$, impinge upon B, moving in an oppoſite direction with the velocity u, they will move together, after impact, with a velocity equal to $\frac{V}{2}$. And whatever be the magnitudes of A and B, it is the invariable reſult of experiments, that the quantity of matter in A, multiplied into the velocity loſt by it, is equal to the quantity of matter in B, multiplied into the velocity which it gains by impact; or if l and g repreſent the velocities loſt by A and gained by B reſpectively, $A \times l = B \times g$. When A and B are perfectly elaſtic, the velocities loſt and gained are doubled, and the law ſtill obtains, $A \times 2l$ being equal to $B \times 2g$. If motion be communicated to any body A, by any

any force X protruding, pulling, &c. the meaning of the law is, that the reaction, or reſiſtance, of A deſtroys ſuch a part of X's force, as would be deſtroyed by a ſimilar force, capable of generating that motion in A, acting in oppoſition to X. In the communication of motion by unknown means, as by gravity, magnetiſm, repulſion, &c. the law aſſerts that the body, attracting or repelling, moves in an oppoſite direction to that of the body attracted or repelled, and with an equal quantity of motion. The attraction between the earth, and any body upon its ſurface, is mutual and equal; for, were it not, a rectilineal motion would enſue from the ſtronger attraction, which is contrary to experience. And, ſince gravity is an innate principle, it, and its effects will remain the ſame when that body is detached from the earth, and conſequently their attractions continue to be mutual and equal, and they will meet with equal moments. If two magnets A and B, whoſe weights are unequal, be placed upon two pieces of wood, floating in water within the reach of attraction, they will meet with velocities inverſely as their quantities of matter; and, if a reed be inſerted between them to prevent their junction, they will be quieſcent, which they would not be, were their attractions unequal. When the weight of A is equal to 2, 3, 4, &c. times that of B, its velocity, in an oppoſite direction, is equal to $\frac{1}{2}$, $\frac{1}{3}$, &c. of B's, the products of their weights and velocities being always found to be equal. And if A is repelled, B is alſo found to be repelled in an oppoſite direction, and their velocities are always inverſely as their weights. This law, being found to obtain in all actions of bodies within the reach of experiments, is inferred to obtain univerſally through the material ſyſtem.

183. Cor. In the impact of bodies therefore the quantity of motion, eſtimated in the ſame direction, is the ſame before and after impact. If A, moving with a velocity equal to a, impinge upon B, moving in the ſame, or an oppoſite, direction, with a velocity equal to b, the ſum of their moments after impact, is equal to $Aa + Bb$, or $Aa - Bb$, according as they move in the ſame, or oppoſite directions; for, if they move in the ſame direction, the encreaſe

creaſe of moment communicated to *B* is deſtroyed in *A*, and their ſum continues the ſame; and, if they move in oppoſite directions, the leaſt moment, and a part of the other body's moment equal to it, are deſtroyed, ſo that their ſum after impact continues to be equal to $Aa - Bb$, the ſame as before impact.

COMPOSITION AND RESOLUTION OF FORCES.

184. * The ſame motion may be communicated to a body *B* by a ſingle force *Z*, and any number of conſpiring forces, into which it is ſaid to be reſolvable, and they are ſaid to be compounded into *Z*; thus the body *B* may deſcribe the line *BP*, in the ſame time, by a force *Z* acting in that direction, or by two, three, &c. forces *X*, *Y*, &c. inclined to it. For it is evident, that *B*, acted upon at the ſame time, by two forces *X*, *Y*, in directions making an angle, which ceaſe to act after the body has left the point *B*, will move in ſome intermediate rectilineal direction, and this, by changing the inclination and magnitude of *X* and *Y*, which are variable without limit, may be any intermediate line whatever *BP*. The force *Z* is ſaid to be reſolvable into the two, *X* and *Y*, producing the ſame effect with it; and, vice versâ, *X* and *Y* are ſaid to be compounded into one, *Z*, which produces the ſame effect with them.

PLATE III. FIG. XXVII.

185. PROP. *A body* B *urged at the ſame time, by two forces* x, y, *whoſe action ceaſes when the body has left the point* B, *and whoſe magnitudes and directions are as two right lines* BM, BN, *making any angle, will move as if it were impelled by one force* z, *whoſe magnitude and direction are as* BP, *the diagonal of a parallelogram, whoſe ſides are* BM *and* BN.†

FIG. XXVIII.

DEM.

* Newt. Princip. p. 14. Muſchen. Ch. X. Helſham, Lect. IV. Maclaurin's Newt. Ch. II. Graveſ. Lect. I. Ch. XIII. Hermannus, Lect. I. Ch. II. Varignon. Tom. I. Sect. I.

† The magnitudes of forces, whether animal exertions as percuſſions, protruſions, &c. or philoſophic powers, as gravity, magnetiſm, &c. can only be compared by meaſuring their effects, which are ſuppoſed to be produced uniformly in the ſame time. And, in this ſuppoſition, it is evident, that the concluſions will be the ſame, whether the forces *X*, *Y* be impulſes, or the ſame number of impulſes always equal to themſelves.

DEM. The acceſs of B to, or receſs from, MP, is not affected by the action of y; nor its acceſs to, or receſs from, NP, by the action of x (ſecond law of motion): therefore they will carry B to MP and NP reſpectively, in the ſame time, whether they act together or ſeparately. But if they act together, B, being carried to both MP and NP, muſt be found at P their interſection, and deſcribe BP uniformly (firſt law of motion); which is the magnitude and direction of z (180). Q. E. D.

Otherwiſe,

Let BM, moving parallel to NP uniformly, arrive at NP in the ſame time that B deſcribes BM uniformly, which conſequently at the end of that time will arrive at P. And if npm, be any new poſition of BM, and the body at p; $Bn : np ::$ velocity of BM: velocity of the body $::$ BN: BM (106); therefore p is always in the diagonal BP; Bp encreaſes uniformly becauſe Bn does; and BP is the magnitude and direction of z (180). Q. E. D.*

186. Cor. 1. A force z, producing the ſame effect with two x and y, which are as BM, BN, is as twice Bs, ſuppoſing s to be the biſection of BP.

187. Cor. 2. If any two forces act upon a body, in directions which make an angle, it cannot be quieſcent.

188. Cor. 3. A force repreſented by a ſide of a triangle BP, may be ſubſtituted for two repreſented by the remaining ſides BM, MP; and it is ſaid to be equivalent to them, becauſe they produce

* This propoſition, which is deduced from the ſecond law of motion, may be demonſtrated not inelegantly from the third. The forces x and y, urging the body B obliquely, partly oppoſe each other, and ſince, from the third law of motion, the oppoſite parts are equal, the path of B will be equally diſtant from any cotemporary poſitions of it, M and N, when they act ſeparately; which therefore is Bs, s being the biſection of BP. And becauſe each of the forces x, y carry B through Bs, they will both carry it through $2Bs$ or BP, the diagonal of a parallelogram, whoſe ſides are BM and BN.

duce the ſame effect, or make a body deſcribe BP in the ſame time. And theſe three forces BM, BP, BN are in the ſame plane (EUC. B. XI. P. II.)

FIG. XXVIII. 189. Cor. 4. A force BM is equivalent to BA and AM; and BN to BC and CN; and therefore theſe forces BA, AM, BC, CN, are equivalent to, and may be ſubſtituted for, the force BP. And any number of forces, whoſe directions are in the ſame plane, may be reduced to one in that plane producing the ſame effect with them.

FIG. XXIX. 190. Cor. 5. A force AH is equivalent to two repreſented by AB and AG; and AD to AH and AF; therefore any number of forces AG, AB, AF, &c. in different planes, may be reduced to one AD producing the ſame effect with them.

FIG. XXVIII. XXIX. 191. Cor. 6. The converſe of this is true, and one force BP, or AD, may be reſolved into any number, either in the ſame, or different planes, producing the ſame effect with it.

FIG. XXX. 192. Cor. 7, The velocity generated by z: velocity generated by $x + y :: BP : BM + BN$; and the quantity of motion is therefore diminiſhed by compoſition, and encreaſed by reſolution; but, when eſtimated in any given direction BL, it remains invariable. For let BP, BM, BN, be each reſolved into two, one in the direction BL, and the other perpendicular to it; and from ſimilar triangles

$$Py : Dy :: yL : yn,$$
$$\text{and comp. } PD : nL :: Dy : yn :: BE : Bm,$$
$$\text{therefore } BL = Bm + Bn.$$

193. Cor. 8. The ſame force z, and conſequently the ſame velocity, may be generated by an infinite number of pairs of forces, becauſe

becauſe the ſame right line may be the diagonal of an infinite number of parallelograms.

194. Cor. 9. The parts, of the two forces, directly oppoſed to each other, are to the parts, acting in the diagonal, as $Mm + Nn$ to $Bm \pm Bn$, according as the angle MBN is not greater, or greater than a right angle. If x and y are given, and the angle MBN be ſuppoſed variable, and become equal to two right angles, then N will come to N', P to P', s to s' (s always biſecting BP and MN) and $2Bs'$, or the conſpiring parts of x and y, $= BP' = BM - BN$; if this angle vaniſh, then N will come to N'', P to P'', s to s'', and $2Bs''$, or the conſpiring parts of x and y, $= BP'' = BM + BN$. FIG. XXXI. FIG. XXXII.

195. Cor. 10. If BP and the angle BMP be given, the curve line paſſing through M, in different poſitions of M, is a circular arc ſubtended by BP; if BP and $BM + MP$ be given, this curve line is an ellipſe, whoſe focal diſtance is BP; and if BP and the difference of BM and MP be given, the curve is an hyperbola and BP its focal diſtance. PLATE IV. FIG. XXXIII.

196. PROP. *Any three forces* x, y, z, *whoſe directions are* Bx, By, BN *in the ſame plane, acting upon a body* B *without producing motion, are to each other as the three ſides of a triangle reſpectively parallel to their directions.* FIG. XXXIV.

DEM. Let BN be the magnitude of z, and be reſolved into two, in the directions yB, xB; viz. BM, BP; which muſt be reſpectively equal to x and y, becauſe in equilibrio with them, and acting in the ſame directions; therefore x, y, z, are to each other as BM, BP (MN), BN reſpectively. Q. E. D.

197. Cor. 1. x, y, z are to each other as the three ſides of a triangle reſpectively perpendicular, or equally inclined to their directions; for this is ſimilar to the former.

198. Cor. 2. Any two of these forces are to each other inversely as the sines of the angles, which their directions make with the direction of the third. For,

$z : y :: BN : BP ::$ sin. $\angle BPN$ or PBM : sin. $\angle BNP$ or NBM,
$z : x :: BN : BM ::$ sin. $\angle BMN$ or MBP : sin. $\angle MNB$ or NBP,
and $x : y :: BM : MN ::$ sin. $\angle BNM$ or NBP : sin. $\angle MBN$.

199. Cor. 3. The angle xBy, and magnitude of x and y, being given, the magnitude and direction of z may be found; for BM, MN and the angle BMN, the supplement to the angle xBy, being known, BN, and the angles MBN and MNB or NBP, may be found by trigonometry.

FIG. XXXV. 200. Cor. 4. Any number of forces in the same plane, acting upon a body B which remains quiescent, may be reduced to two equal and opposite: if BM, BN, BD, act upon B, they are equivalent to BP and BC in opposite directions, which must be equal, because B is quiescent.

201. Cor. 5. Any number of forces in different planes, urging a body without producing motion, may be reduced to two in the same plane, equal and opposite. If BD and DC be elevated above the plane BNM, and PBC be the common intersection of the planes BMN and BDC; BP, which is equivalent to BM and BN, must be equal to BC, which is equivalent to BD and DC, because they are opposite and destroy each other.

202. Cor. 6. If more than three forces BM, BN, and BD, each of which is combined from two, act in different planes, upon a body B without producing motion, and are estimated in any direction BL by letting fall parallel lines upon it, the forces resulting will produce no motion in that direction; for the parts of BM, BN, resulting from this resolution, which are in the direction BL, are equal and opposite to the part of BD reduced to the same direction, and the sums of the opposite parts, on each side of BL, must also be equal, because B is supposed to be quiescent.

203. Cor.

Plate III

FIG. XXIII.

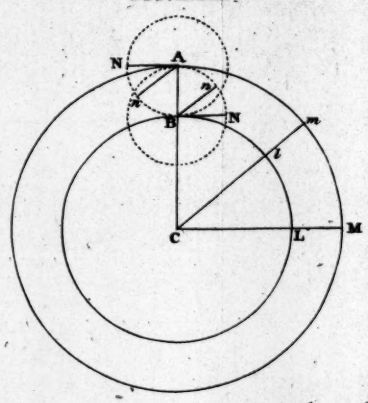
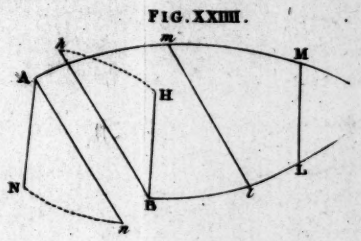


FIG. XXIIII.



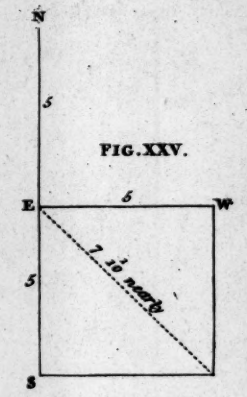
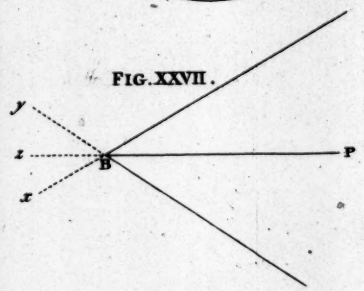


FIG. XXVI.



FIG. XXVII.



S.

FIG. XXVIII.

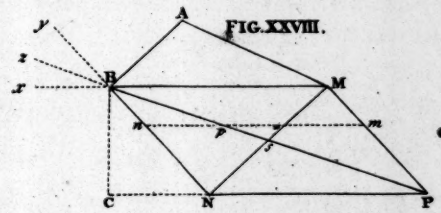


FIG. XXIX.

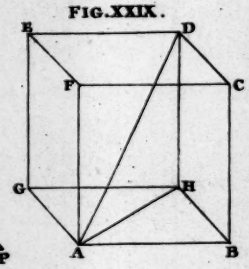


FIG. XXX.

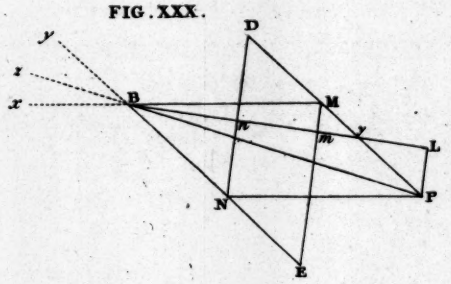


FIG. XXXI.

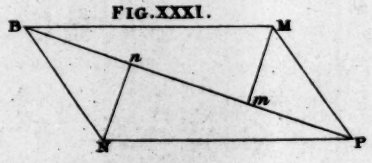


FIG. XXXII.

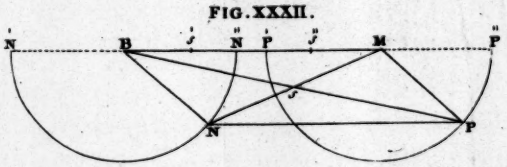
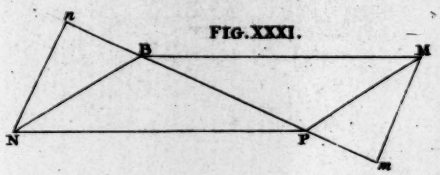


FIG. XXXI.




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203. Cor. 7. If the forces *BM, BN, BC,* which act upon a body without producing motion, be reduced to any plane *FH,* by reſolving each into two, one perpendicular to it, and the other parallel to the lines joining the points, where the perpendiculars meet it, the forces reſulting will produce no motion in that plane. For *BP,* being taken equal and oppoſite to *BC,* will be equivalent to *BN* and *BM,* and *BC, BP* will be projected into equal ſtraight lines *bc, bp*; but *BN, MP,* being equal and parallel, are projected into equal and parallel lines *bn, mp,* and the figure *bmpn,* is a parallelogram. And *bm, bn,* being equivalent to *bp,* will be equivalent to its equal and oppoſite *bc.* FIG. XXXVI.

204. PROP. *If a body be acted upon by two ſimilar variable forces, in directions parallel to* BP, BQ, *making any angle, which act when the body hath left the point* B, *it will deſcribe a right line.* FIG. XXXVII.

DEM. Let the forces act by impulſes at the beginning of equal particles of time, and let *BD, DE, EF,* and *BG, GH, HI* be the relative magnitudes of correſponding impulſes. By the action of the two firſt impulſes *BD, BG,* the body will move in the direction *BK* (185); by the two next *DE, GH,* it will deſcribe, in the ſame direction, *KL,* &c.; and, becauſe the forces are ſimilar, *BD* : *DE* :: *BG* : *GH,* and componendo *BD* : *BG* :: *BE* : *BH,* and conſequently *BKL* is a right line. Q. E. D.

205. Cor. 1. Since this demonſtration does not depend upon the magnitude of the particles of time, into which the whole time is ſuppoſed to be divided, it will obtain when theſe particles are evaneſcent and the forces act inceſſantly.

206. Cor. 2. If the forces be conſtant, the velocity in *BM* is uniformly accelerated; for, in this ſuppoſition, *BD, DE,* &c. are equal, and *BD* : *DE* :: *BK* : *KL*; but *BK, KL* which are therefore equal, &c. meaſure the increments of velocity communicated at *B* and *K* (106).

207. Cor. 3. Every body moving in a curve line, muſt be acted upon by, at leaſt, two diſſimilar forces.

208. Cor. 4. The whole force in the direction *BP* is the ſum of the impulſes *BD*, *DE*, *EF*, &c. and the force in the direction *BM*, is the ſum of the impulſes *BK*, *KL*, *LM*, &c. and theſe forces are ſimilar; for *BD*, *DE*, &c. meaſure the intenſity of the force in the direction *BP* at the points *B* and *D*; and *BK*, *KL*, &c. meaſure the intenſity of the force in the direction *BM*, at *B* and *K*, and $BD : DE :: BK : KL$.

FIG. XXXVII. 209. PROP. 86. *If a body be urged at the ſame time by two conſtant, or ſimilar variable, forces* x, y, *whoſe magnitudes and directions are the two ſides of a parallelogram* BF, BI, *it will move in the ſame manner as if it were urged by a conſtant or ſimilar variable force* z, *repreſented by the diagonal.*

DEM. The impulſe *BK* is equivalent to *BD* and *BG*; *KL* to *DE* and *GH*; *LM* to *EF* and *HI*, &c. conſequently the ſum of theſe impulſes *BM* is equivalent to *BF* and *BI*. Q. E. D.

210. Cor. 1. A force repreſented by one ſide of a triangle *BM*, may be ſubſtituted for two ſimilar forces repreſented by the remaining ſides *BF*, *FM*.

211. Cor. 2. A force repreſented by *BM* may be reduced into any number of forces *BR*, *RS*, *SM*, ſimilar to it, and vice verſâ.

212. Cor. 3. Three ſimilar variable forces, urging a body without producing motion, are to one another as the three ſides of a triangle, parallel or perpendicular to their directions; and they are to each other inverſely as the ſines of the angles, which their directions

directions make with the direction of the third (198); and any number of ſimilar forces in the ſame or different planes, in equilibrio, may be reduced to two, in the ſame plane, equal and oppoſite.

SCHOLIUM.

213. Prop. (204) may be demonſtrated per ſaltum, though perhaps leſs intelligibly, by the following proceſs. Let a body B be urged, at the ſame time, by any forces x and y, in the directions BM, BN, which carry it reſpectively through Bm, Bn and BM, BN, in equal times; and it is evident that, at the end of thoſe times, B will be in d and D. If x and y act only at the point B, or if $Bm : BM :: Bn : BN$, the path of B is the diagonal; if they act at B, m, n, M, N, &c. and Bm be to BM as the times of deſcription, and Bn be to BN as the ſquares, cubes, &c. of thoſe times; $Bn : BN :: Bm^2 : BM^2$ or $Bm^3 : BM^3$, &c.; and the path of B is a parabola, &c. FIG. XXXVIII.

214. PROP. 87. *A body* B *impelled by two forces tending to two equidiſtant points*, Q, R, *which at equal diſtances are equal, and at unequal diſtances are variable according to any law whatever, will deſcribe a right line.* FIG. XXXIX.

DEM. Let theſe forces act by impulſes at the beginning of equal particles of time, and BE, BF being the magnitude of the two firſt, will make B deſcribe BD, which produced biſects the baſe QR; and DL, DM, the two next, will make it deſcribe DN, biſecting the baſe QR: therefore BN is a right line; and the demonſtration obtains when the particles of time are evaneſcent, and the action of the forces inceſſant. Q. E. D.

215. Cor. The direction of the combined attraction of all the particles, compoſing a ſphere, paſſes through the center: for let it be divided into thin laminæ parallel and contiguous to each other, and

and let Q, R, be any corpuſcules in a lamina, equidiſtant from PB paſſing through the center, and what has been proved of theſe two may be proved of every two equidiſtant from P, which compoſe the whole lamina; and the ſame may be proved of every lamina, and conſequently of the whole ſphere.

SCHOLIUM.

216. Three forces, acting upon a body without producing motion, are to each other as the three ſides of a triangle parallel to their direction, which conſequently meet in the ſame point of the body, and are in the ſame plane (EUC. B. XI. P. II.). This may be evinced otherwiſe; for ſince a body, acted upon by any number of forces, cannot be at reſt, unleſs they oppoſe each other, and the oppoſite parts be equal, and in the ſame direction; it is clear that a plane AB may paſs through P, acted upon by x, y, z in different planes, ſo that they ſhall be on the ſame ſide of AB, and therefore, as they partly conſpire, they muſt communicate motion to the point P, and to the whole body. And if any two of theſe forces, x, y, act againſt the ſame point p of the body AB, and x acts againſt q; x and y will communicate motion to p (187) and z to q, and AB cannot therefore be quieſcent. If the directions of x and y meet in a point p without the body, and z act at the point q, AB cannot be at reſt; becauſe x and y have the ſame effect with, and are equivalent to, ſome force in an intermediate direction pd, which is not oppoſite to the direction of z, and cannot therefore deſtroy it. It is ſuppoſed, that the directions of theſe forces are inclined to each other; for, if they be parallel, BA may be quieſcent, when they act upon different points.

PLATE V. FIG. XL. FIG. XLI. FIG. XLII.

FIG. XLIII.

217. PROP. *If a body* B *be acted upon by any number of forces, at the ſame time, whoſe magnitudes and directions are* BC, BA, BF, BH, *&c. and from the biſection,* m, *of the diagonal* BD *of a parallelogram, whoſe ſides are* BC, BA, *a right line be drawn to the extremity* F, *of* BF, *cutting the diagonal* BE, *of a parallelogram whoſe ſides are* BD, BF, *in* n, *and from* n *a right line be drawn to the extremity,* H, *of* BH,

cutting

cutting the diagonal BG *of a parallelogram, whoſe ſides are* BE, BH, *in* p, Bp : BG :: *unity* : *number of forces.*

DEM. The triangles Bpn and GpH, Bnm and nEF, are ſimilar, and conſequently $Bp : pG :: Bn : GH(Bn+nE) :: Bm : Bm+EF(Bm+BD$ or $3Bm)$; therefore $Bp : BG :: Bn : 4Bm :: 1 : 4$. The proceſs is ſimilar when there are more forces. Q. E. D.

218. Cor. 1. It is evident, from the conſtruction of the figure, and ſimilar triangles, that $CA : Cm :: 2 : 1$,

$mF : mn :: 3 : 1$,

$nH : np :: 4 : 1$; and if the number of forces were equal to any number, m, nH would be to $np :: m : 1$.

219. Cor. 2. The force BG, reſulting from the action of any number of forces BC, BA, BF, BH, &c. either in the ſame, or a different, plane, is to their ſum as $BG : BC+BA+BF+BH$, &c. and if the magnitudes and directions of theſe forces be given, the magnitude and direction of BG may be found; for CB, BA, and the $\angle CBA$ being given, BD, and the $\angle$s CBD, DBA, are known; and DB, BF, and the $\angle DBF$ (which may be found) being given, BE is alſo given, and in the ſame manner any other diagonals may be inveſtigated.

220. Cor. 3. If a body therefore be acted upon by any number of forces BA, BC, BD, BE, BF, BG, the directions of the forces reſulting from the action of two, three, four, &c. are found by joining AC, biſecting it in m, and taking $mD : mn :: 3 : 1$, $nE : nP :: 4 : 1$, $pF : pq :: 5 : 1$, $qG : qr :: 6 : 1$, and the directions are Bm, Bn, Bp, Bq, Br. Their magnitudes are $2Bm$, $3Bn$, $4Bp$, $5Bq$, $6Br$, &c. FIG. XLIV.

221. Cor.

FIG. XLV. 221. Cor. 4. The ſame force BP may be generated by the action of 1, 2, 3, &c. forces, either in the ſame, or different, planes, whoſe directions and quantities are variable in infinitum: for let it reſult from the combined action of four forces, and taking $BP : Bq :: 4 : 1$, through q draw any line whatever qA, and take $qA : qP :: 3 : 1$; through p draw any line pC, and let pC be to $pn :: 2 : 1$; and through n draw any line nD, and taking nE equal to it; the forces BA, BC, BD, BE, are equivalent to BP. The lines qA, pC, nD, &c. may be drawn in any directions, in any different planes, and of any different lengths.

FIG. XLVI. 222. PROP. *The directions of three forces* x, y, z, *acting upon a body* B *without producing motion, will meet in the center of gravity of a triangle, whoſe diſtances from the angular points, are as the magnitudes of the forces.*

DEM. Produce AB, and taking $BE = AB = z$, complete the parallelogram $BDEC$ by drawing from E lines parallel to BC, BD; BE is equivalent to BC and BD, and equal to z, which is equivalent to x and y, which muſt therefore be equal to BC, and BD, reſpectively; but $BG = \frac{BE}{2} = \frac{AB}{2}$, and B is therefore the center of gravity of the triangle. Q. E. D.

Otherwiſe:

Let BA, BC, BD, be the magnitudes of z, x, y, reſpectively, and joining DC and biſecting it in G, and joining GA, $AB = 2BG$ (217). Q. E. D.

FIG. XLVII. 223. PROP. *If a body* B, *be acted upon by four forces, in different planes,* x, y, z, w, *or* BA, BC, BE, BF, *and remain at reſt, they are to each other as the three ſides and diagonal of a parallelopiped parallel to their directions reſpectively.*

DEM.

Plate III

FIG. XXXIII.

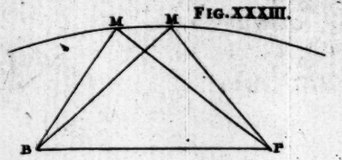


FIG. XXXIII.

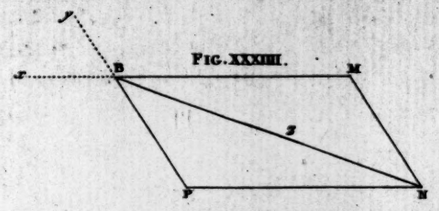


FIG. XXXV.

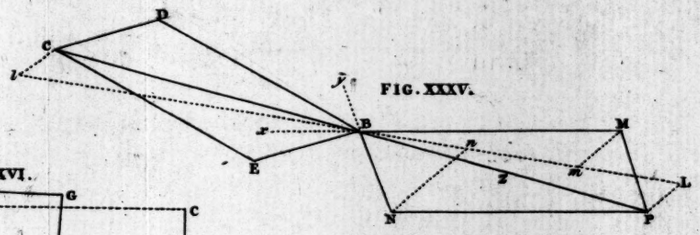


FIG. XXXVI.

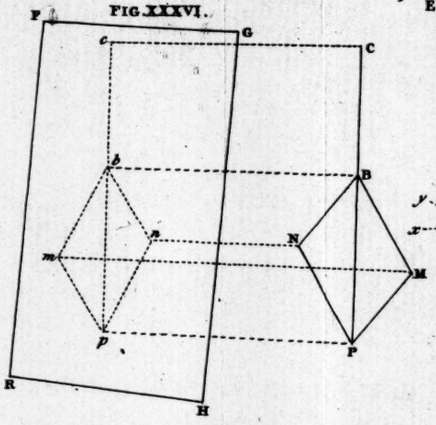


FIG. XXXVII.

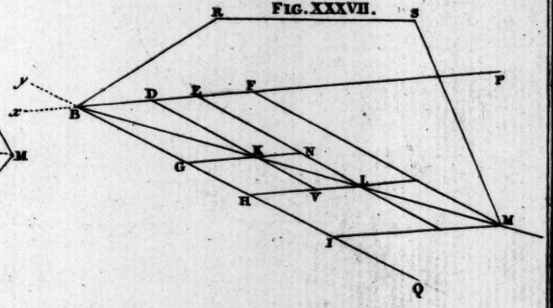


FIG. XXXVIII.

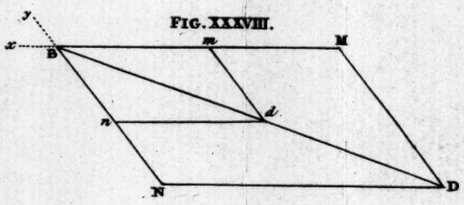
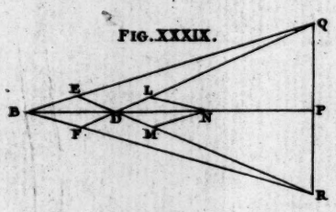


FIG. XXXIX.




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DEM. The forces *BA*, *BC* are equivalent to *BI*, and *BI*, *BE* to *BD*, which muſt be equal and oppoſite to *w* or *BF*, becauſe *B* is quieſcent. Q. E. D.

224. Cor. An infinite number of parallelopipeds may be deſcribed, whoſe ſides and diagonal are the ſame: for let *x*, *y*, *z*, *w*, be invariable, and taking *BC*, *BA*, equal reſpectively to *y* and *x*, making any angle whatever *ABC*, and *BD*, *ID* in a different plane, reſpectively equal to *w* and *z*, draw *BE* parallel to *ID*, and *DE* to *BI*. Biſect *BI* in *H*, and join *EH*, interſecting *DB* in *G*; and becauſe the triangles *BGH*, *EGD* are ſimilar, *EG* : *GH* :: *ED* (2*BH*) : *BH* :: 2 : 1; therefore theſe two forces combined with *BE* produce a force paſſing through *G* (217); and becauſe *DG* : *GB* :: *EG* : *GH* :: 2 : 1 and *DB* : *GB* :: 3 : 1, the force *w* is equal to *DB*. But the angle *CBA* is infinitely variable, and *BD*, *FD* may be drawn in any planes whatever. FIG. XLVII.

225. PROP. *If four forces*, x, y, z, w, *which are as* BA, BD, BE, BF *reſpectively, act in different planes, upon the body* B *without producing motion,* B *is the center of gravity of a triangular pyramid whoſe baſe is* EAD *and vertex* F. FIG. XLVIII.

DEM. Join *AD*, and biſect it in *m*, and, taking *mn* to *nE* :: 1 : 2, *n* will be the center of gravity of the triangle *EAD*, and *nB* is the direction of the force reſulting from *x*, *y*, *z* combined, and its magnitude is equal to 3 × *Bn*, which muſt be equal and oppoſite to *w* or *BF*, becauſe *B* is quieſcent: *BF* is therefore equal to 3*Bn*, and *B* conſequently the center of gravity of a pyramid, whoſe baſe is *EAD* and vertex *F*. Q. E. D.

226. Cor. 1. It is evident that *BF* = 3*Bn*, becauſe the force reſulting from *x* + *y* + *z* is to thoſe forces as 3*Bn* : *BA* + *BD* + *BE* (220) and *w* : *x* + *y* + *z* :: *BF* : *BA* + *BD* + *BE* (hypoth.) therefore *w* = *BF* = 3*Bn*.

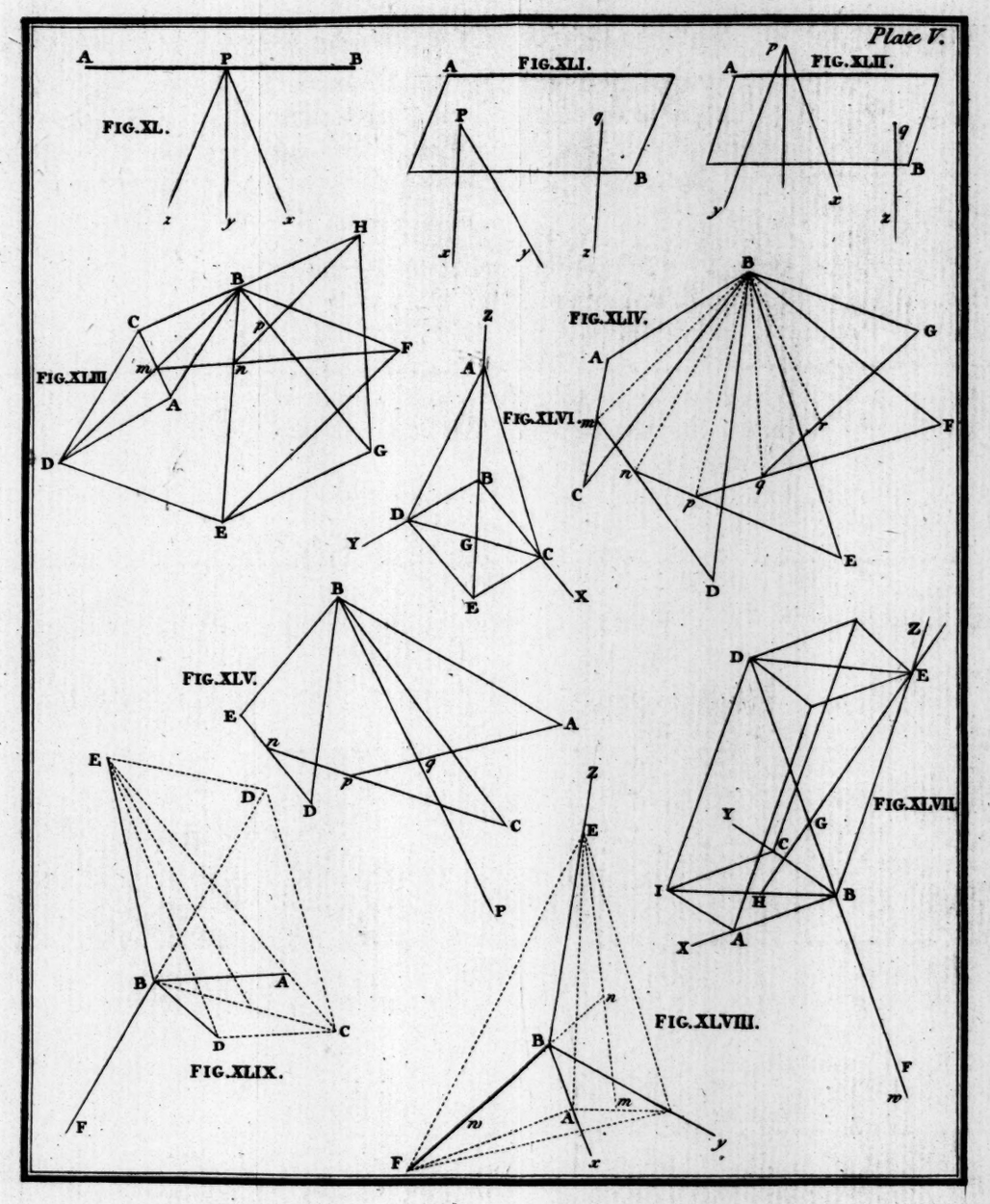
227. Cor. 2. The force reſulting from the combined action of *BA*, *BD*, *BE* paſſes through the center of gravity of a triangle, which is formed by joining *A*, *E*, *D*; and the direction of a force, equivalent to them, paſſes through the ſame point.

228. Cor. 3. An indefinite number of pyramids may be deſcribed, having the diſtance of their angular points from the center of gravity always the ſame; for let *BA*, *BD*, *BE*, *BF*, be invariable, and making the angle *ABD* of any magnitude whatever, draw *CD*, *BD*, in any plane whatever, equal reſpectively to *BE*, *BF*, and complete the parallelogram *BCDE*, and the proceſs is the ſame as that in (224).

FIG. XLIX.

CHAP.

Plate V.


A
P
B
FIG. XL.
z
y
x
A
FIG. XLI.
P
q
B
x
y
z
p
A
FIG. XLII.
q
B
y
x
z
H
B
C
p
F
FIG. XLIII.
m
n
A
D
G
E
Z
A
FIG. XLVI.
B
D
Y
G
C
E
X
B
FIG. XLIV.
G
A
m
F
C
n
p
q
r
E
D
B
FIG. XLV.
E
A
n
q
p
D
C
P
E
D
B
A
C
D
F
FIG. XLIX.
Z
E
B
n
A
m
w
F
x
y
Z
D
E
FIG. XLVII.
Y
G
C
I
H
B
X
A
FIG. XLVIII.
F
w


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CHAP. V.

OF ATTRACTION.

229. DEF. *THAT power or principle in nature, by whoſe influence bodies, or the conſtituent parts of bodies, accede, or have a tendency to accede, to each other, without any ſenſible material impulſe, is called attraction.*

230. DEF. *That natural power, by whoſe influence different bodies, or different parts of the ſame body, recede, or have a tendency to recede, from each other, without any ſenſible material effect, is repulſion.*

When the changes of motion in the body *A* are uniformly obſerved to depend upon the ſituation and diſtance of another body *B*, a connexion is underſtood to obtain between them, expreſſing ſome quality or mechanical affection of matter, ſuch as gravity, coheſion, elaſticity, magnetiſm, electricity, which appears to reſide in matter, and by whoſe agency this change of motion is conceived to be produced. If the direction of *A*'s motion, or tendency to motion, be towards *B*, it is ſaid to be attracted by *B*; and if it be from *B*, it is ſaid to be repelled by *B*.

231. That there are in nature motions and tendencies to motion, conatus accedendi & recedendi, both in aggregate bodies, and in their minute conſtituent particles, without any ſenſible cauſe, and indeed inexplicable by any known properties of matter, is unqueſtionably certain. Theſe demonſtrate a ſource of motion diſtinct from, and repugnant to, any material impulſe with which

we are acquainted, which is denominated attraction, a term indifferently applied to the mechanical affection and its effects, as all inexplicable powers are only measurable by their effects. These effects being found by observation to be different at different distances, the intensity of the power producing them is inferred to be variable, and the law of variation is discovered by an actual mensuration of these effects at different known distances. By this process, these material affections, though intelligible only in their effects, are considered as quantities capable of mathematical comparison, and their ratios are compared with those of lines and numbers. Attraction is usually divided into two kinds: *first*, that which operates at sensible distances, as gravity, electricity, magnetism; and, *secondly*, that, whose effects are limited to almost insensible distances, which is called the attraction of cohesion.

ATTRACTION OF GRAVITY.

232. The existence of this species of attraction is evidenced by uniform experience. Matter when supported is heavy, and, the support being removed, accedes towards the earth; when projected obliquely, it deviates from the line of projection, and vibrates when suspended by a string inclined to the earth's surface. And the direction of this motion of matter, and tendency to motion, is invariably the same, towards the center of the earth nearly. A mechanical affection is justly inferred from these phenomena (230), which is conceived to produce this motion and tendency to motion, and is promiscuously called gravity, gravitation, or the attraction of gravity. But more correctly the tendency of a body towards the earth, which is measured by the velocity acquired in a given time, is called the accelerating force of gravity; the weight or moment of a body, which is measured by the product of the quantity of matter and the accelerating force, is called its vis motrix; and the power conceived to be in the earth, which produces this weight and tendency towards it, is called the absolute force or attraction of gravity; and a body influenced by it, is said to be attracted by, or gravitate towards, the earth. All hard and soft

bodies

bodies are ſubject to this attractive power, and have weight; air, water, and other fluids, and the vapours ariſing from them, and all odorous ſubſtances, and exhalations from all terreſtrial bodies, gravitate towards the earth, as they may be collected in a balance and actually weighed; and the weight of theſe laſt ſubſtances is alſo rightly inferred from the decrement of weight ſuſtained by the evaporable matter. Even fire and light, and the moſt volatile vapours, ſeem to be under the controul of this univerſal principle, and no matter in the vicinity of the earth, acceſſible to experiments, hath yet been diſcovered to be uninfluenced by it, and conſequently all matter may be conſidered as gravitating towards the earth, till future ſatisfactory experiments induce a different opinion. This attractive power is not confined to bodies in the vicinity of the earth, but extends alſo to the moon; for the moon deſcribes equal areas in equal times about the earth's center, and is therefore urged by a force directed to that center (NEWT. Sect. II. P. II.) and coincident in direction with gravity: and it appears, from aſtronomical obſervations, that it is equal in quantity to the force of gravity at that diſtance, and they are conſequently the ſame power. And becauſe the revolutions of their ſatellites round jupiter and ſaturn, and of the primary planets round the ſun, are phenomena ſimilar to that of the moon round the earth, both being acted upon by forces directed to their reſpective centers, which vary according to the ſame law; the ſatellites therefore gravitate towards their primaries, and theſe towards the ſun (7). The primary planets gravitate towards each other; for jupiter and ſaturn, in conjunction or at their leaſt diſtance, are diſcovered to produce very ſenſible irregularities in each other's motions; and the motions of their ſatellites are ſaid alſo to be ſubject to irregularities, which are ſenſible at their leaſt diſtance where their action is greateſt. All the great bodies, compoſing the ſolar ſyſtem, are therefore impreſſed with this principle of attraction; which, being attached to the whole of any body, muſt pervade every conſtituent portion; and the minuteſt particles of matter gravitate, though perhaps inſenſibly, towards each other. The operation of this principle between two ſmall bodies, at the ſurface of the earth, is only rendered inſenſible from the predominant influence of the

earth;

earth; for a pendulous body was obſerved to be conſiderably deflected from its vertical ſituation by the attraction of the mountain Chimborazo in Peru; and, by ſome ingenious experiments made with great preciſion, Dr. Maſkelyne aſcertained the quantity of attraction of the mountain Schehallien in Scotland.* Every mountain therefore, and every leſs portion of the earth, poſſeſſes this quality; which might have been inferred from (215). For if a particle of matter be attracted by every part of a ſphere of matter, equally at equal diſtances, the direction of the combined attraction will paſs through its center and $v.v$; but it appears, from experience, that all bodies deſcend in directions perpendicular to the earth's ſurface, and, becauſe the earth is nearly ſpherical, the whole force, producing this deſcent, is directed nearly to the center, and is conſequently combined of the force of every particle.

233. PROP. *The accelerating force of gravity at equal diſtances from the earth's center, is the ſame in all bodies, whether quieſcent or moving, and whatever be their magnitude, figure, denſity.*

DEM. This propoſition is only demonſtrable from experiments. Two equal wooden boxes, ſuſpended by threads of eleven feet in length, one of which was filled with wood, and an equal weight of gold fixed in the center of oſcillation of the other, were diſcovered by Sir I. Newton, to perform all equal vibrations in the ſame time; and numberleſs experiments demonſtrate that all bodies,

* If a mountain do poſſeſs an attractive power, a body ſuſpended near it will be deflected from its vertical poſition, and point to a falſe zenith, and the arc, meaſuring the diſtance of this from the true, is the meaſure of the mountain's attraction. In an obſervation on the north ſide of Schehallien, the plumb line, being deflected towards the mountain, pointed too much to the ſouth, and gave the diſtance of a ſtar from the zenith too much to the north; and from an obſervation made on the north ſide, the diſtance was too much to the ſouth. The difference of the latitude of the two ſtations, collected by theſe obſervations, muſt be greater than it really is, by the ſum of the arcs meaſuring the deflections from the two zeniths. From obſervations of ten ſtars near the zenith, Dr. Maſkelyne found the difference of the latitude of the two ſtations to be 54″6; and, from a menſuration of their diſtance, it was only 43″; the difference of theſe is 11″6, and the half of it 5″8, meaſures the attraction of the mountain. This experiment is moſt convincing, and deciſive in ſupport of the univerſality of gravitation.

dies, however different their magnitude, figure, and denſity, deſcend in an exhauſted receiver, whatever be its height, exactly in the ſame time; and conſequently the tendency of bodies towards the earth, or their accelerating force, is the ſame whether quieſcent or moving, &c.* Q. E. D.

234. PROP. *The weight of bodies, at the ſurface of the earth, are proportional to their quantities of matter.*

DEM. The weight of a body is evidently as the number of equal particles or quantity of matter contained in it, multiplied into the tendency towards the earth, or accelerating force of each, and this being given (233) varies as the quantity of matter. Q. E. D.

Otherwiſe:

The weight of any two bodies A and B, or the forces producing them, are clearly proportional to their vis inertiæ; for if the vis inertiæ of A were to that of B, as 1, 2, 3, &c. : 1, it is evident that the weights or forces, producing the ſame velocities in A and B in the ſame time, will be as 1, 2, 3, &c, : 1; and conſequently the weight, being as the vis inertiæ, will be as the quantity of matter (166). Q. E. D.

235. Cor. If the accelerating force of gravity were encreaſed in any ratio, the weight of a given body would be encreaſed in the ſame ratio. Subſtituting therefore W, Q, F, for the weight, quantity of matter, and force of gravity, reſpectively, and ſuppoſing them to be variable; W will be as $Q \times F$; F as $\frac{W}{Q}$; Q as $\frac{W}{F}$, and, if W be given, F and Q are inverſely as each other.

SCHO-

* All bodies, whether great or ſmall, denſe or rare, acquire a velocity in falling 1", which would carry them uniformly through 32.2 feet in 1", and an encreaſe of velocity, equal to this, is found to be added in every ſucceeding ſecond of time.

SCHOLIUM.*

236. In the ſame place, and at the ſame diſtance from the æquator, bodies are found to deſcend through the ſame ſpace in the ſame time always; but, the force of gravity being diminiſhed unequally by the diurnal rotation of the earth, they do not, and ought not to, deſcend through equal ſpaces, or perform equal vibrations, in the ſame time, at different diſtances from the æquator. However, this unequal diminution of gravity, ariſing from the different centrifugal forces, does not afford a ſolution for the different ſpaces fallen through; and there muſt be ſome other cauſes of inequality, which are probably the difference of diſtance from the center, and different ſpecies of matter near the places of obſervation.

237. PROP. *The force of gravity varies as the ſquare of the diſtance from the center of the earth inverſely.*

PLATE VI. FIG. LI.

DEM. Gravity acts in right lines, and, whatever be the cauſe, is equally diffuſed over every point of the ſurface $ABDC$, equidiſtant from the center of the earth S; and that ſame influence is alſo equally diffuſed over the ſurface $abdc$, ſimilar to the former and ſimilarly ſituated. Taking $almn$ equal and ſimilar to $ABDC$, the influence of gravity upon $almn$: influence upon $abdc$ or $ABDC :: almn\,(ABDC) : abdc :: SA^2 : Sa^2$. Q. E. D.

Another

PLATE VI. FIG. L.

* PROP. *Suppoſing the earth to be ſpherical, the diminution of gravity, ariſing from the centrifugal force, varies as the ſquare of the coſine of latitude.*

DEM. Let Pp be the earth's axis, and $\text{Æ}\mathcal{Q}$ its æquator, and the centrifugal force at $\mathcal{Q}$: centrifugal force at $A :: \mathcal{Q}C : AD$; the centrifugal force at A: that part oppoſite to gravity :: $AN : AL$ (ſuppoſing NL to be perpendicular to AC) :: $AC : AD$; and conſequently the centrifugal force, or diminution of gravity at $\mathcal{Q}$: the diminution of gravity at $A :: \mathcal{Q}C^2 : AD^2$. This diminution of gravity varies therefore as AD^2, or as the ſquare of the coſine of the place's diſtance from $\mathcal{Q}$. Q. E. D.

Another demonſtration:

The magnitude of the earth's attractive force, at different diſtances, varies as the ſpace through which it impels a body in equal times; but a body at the earth's ſurface falls through 16.1 feet in 1″, and the moon, at her mean diſtance, nearly 60 ſemidiameters of the earth, falls through $\frac{16.1}{60 \times 60}$ feet in 1″ (from obſervation): preſuming therefore that the moon is retained in her orbit by the earth's attractive power, the force of gravity at the earth's ſurface, is to the force at the moon, as $16.1 : \frac{16.1}{60 \times 60} :: 60 \times 60 : 1$. Q. E. D.

238. Cor. 1. The firſt of theſe demonſtrations is applicable to all forces diffuſed by rectilineal effluvia of matter from a center, as heat, ſmells, &c.

239. Cor. 2. Putting d for the ſemidiameter of the earth, and W for the weight of a body at the earth's ſurface, its weight at any other diſtance nd (n being a number) will be equal to $\frac{W}{n^2}$. A weight, of 3600 lb. at the earth's ſurface, is equal to $\frac{3600}{60 \times 60}$ or 1 lb. at the diſtance of the moon.

240. Cor. 3. If BC and bc be homogeneous and ſimilar, their weights at unequal diſtances are equal; for they are to each other as $\frac{BC}{SA^2} : \frac{bc}{Sa^2}$ (237) as 1 : 1. If they be not homogeneous, their weights will be as their quantities of matter, that is, as their weights at equal diſtances, divided by the ſquares of their diſtances.

SCHOLIUM I.

241. The variation of the force of gravity was collected from obſervation in Peru by Meſſ. Condamine and Bouguer. Condamine found that the ſame pendulum upon mount Pichinca, at Quito, and upon the banks of the river of the Amazons, performed reſpectively 98720, 98740 and 98770 vibrations, in twenty-four hours. And Bouguer obſerved upon the ſummit of Pichinca, and upon the ſea-ſhore, being $\frac{1}{1349}$th part nearer to the center of the earth, that the lengths of iſocronous pendulums were to each other as 438.71 to 439.7. Theſe obſervations demonſtrate a diminution of the force of gravity in receding from the earth's ſurface; but the law of variation cannot be collected from them, or any other ſimilar experiments, with preciſion; becauſe the different ſtates of the atmoſphere, of heat and cold, and different denſities of the earth contiguous to the different places of obſervation, would proportionably affect the experiments, and theſe cannot be aſcertained with ſufficient accuracy. From the obſervations of Bouguer, the force of gravity upon the mountain and upon the ſea-ſhore, is as 99575 : 99802; and, ſuppoſing it to vary inverſely as the ſquare of the diſtance, it is as 99575 : 99722; which differs leſs from the true law than could be expected.

SCHOLIUM II.

242. An inveſtigation of the cauſe of attractions and their mode of operation, and particularly of this mechanical affection of matter, its gravity, ſo univerſally prevalent, hath long been the object of the philoſopher's reſearches, but without effect; the ſubject is ſtill involved in impenetrable darkneſs, and his wiſhes are unſatisfied. Dr. Halley refers it to the immediate agency of the Creator; Mr. Cotes deems it eſſential to matter, like extenſion and mobility, &c. and Sir I. Newton ſeems to entertain a different opinion, and attributes it to ſome undiſcovered and inviſible mechanical affection of matter. Electrical and magnetic experiments prove the exiſtence

ence of a ſubtle fluid pervading the pores of the denſeſt bodies, and the gravity of bodies may ariſe from the action of a fluid, whoſe particles are ſo ſmall as to paſs through all bodies, and does not therefore reſiſt their motion; and, if endued with a ſtrong repulſive power, it will by expanding itſelf preſs upon groſs bodies. This æther, Newton ſuppoſes, is, from its repulſive force, much rarer within the denſe bodies of the ſun and planets, than in empty ſpace, and in receding from them becomes perpetually denſer and more elaſtic, and therefore occaſions their gravity, every body being impelled from the denſer towards the rarer parts of the fluid. This however is conjectural, and its truth or falſehood can only be eſtabliſhed by future experiments. All that is certainly known about this myſterious cauſe, is, that it cannot ariſe from the preſſure of a fluid ſimilar to thoſe with which we are acquainted, whether ſoft or elaſtic, quiet or agitated, for the following reaſons. *Firſt*, Becauſe a fluid, ſubtle enough to penetrate the inmoſt receſſes of the denſeſt bodies, and ſo rare as not to impede their motion, cannot be conceived capable of communicating motion to them. *Secondly*, Becauſe the attraction of gravity penetrates the inmoſt receſſes of bodies, and their weights are proportional to their quantities of matter, not magnitude of ſurface; but the preſſure of every fluid with which we are acquainted, varies as the ſurface oppoſed to it. *Thirdly*, Becauſe the attraction of gravity acts with equal intenſity upon bodies, whether quieſcent or moving; but all known fluids act upon quieſcent and moving bodies with different forces. But, however unſearchable the efficient cauſe of gravity may be, its final cauſe is moſt conſpicuous, being the preſervation of the earth and other planets, and of their periodical revolutions, which are only continued by an uninterrupted exertion of this mechanical affection of matter.

CHAP. VI.

ATTRACTION OF COHESION.

* A SECOND ſpecies of attraction, diſtinguiſhed from that of gravity by the intenſity of its action, and the ſmall diſtances to which its influence extends, is that obſervable between the minute particles of bodies, whether hard, ſoft, or fluid. Hard and ſoft bodies have an appearance of endleſs compoſition; but they are compoſed of inſenſibly minute particles which cohere together and form greater, and theſe, being ſucceſſively compounded by the influence of the ſame principle, form ſucceſſively greater, till, by repeated coalitions, they become objects of ſenſe; and it appears, from many phenomena, that the particles of all known fluids have alſo a tendency to cohere with others. This mutual tendency to accede to each other, of the component parts of hard, ſoft, and fluid bodies, not ariſing from any ſenſible material impulſe, indicates that philoſophical relation denominated attraction (230); which, being great at the point of contact, and evaneſcent at the leaſt ſenſible diſtance, coincides, in theſe marks of ſimilitude, with innumerable phenomena, and the material power, from whoſe agency they are conceived to reſult, is called by the ſame name, the attraction of coheſion, becauſe they are all effected by the mutual action of particles, whoſe force is great when contiguous, and, when removed to any ſenſible diſtance, evaneſcent. But whether the phenomena, which have theſe marks of coincidence and ſimilitude, reſult from a common cauſe, varying at different diſtances, like the attraction of gravity, according to the ſame law of the diſtance, can only be aſcertained by experiments, which are yet too undeciſive to allow of accurate diſcrimination. The laws and limits of the action of theſe minute particles are ſtill unknown,

* Helſham, Lect. I, II. Muſchenbroek, Ch. XVIII. Newt. Opt. p. 380. Hamilton's Tracts, Lect. II.

known, and, from their almoſt inſenſibly minute ſphere of action, perhaps undiſcoverable; but its intenſity ſeems to vary according to ſome higher inverſe law of the diſtance than the duplicate, and therefore ſoon evaneſcent, that it may not interfere with the attraction of gravity.

243. PROP. *If the particles compoſing two ſpherical ſurfaces* AFC, BDE, *attract with forces, equal at equal diſtances, and at unequal diſtances, varying as the* n^{th} *power of the diſtance inverſely, a particle* P *will be attracted by them with a force varying inverſely as that power of the diſtance, whoſe exponent is* $n - 2$. FIG. LII.

DEM. The attraction of P towards theſe ſurfaces, is as the number of particles and power of each, or as $\frac{AFC}{PA^n} : \frac{BDE}{PB^n}$, or as $\frac{PA^2}{PA^n} : \frac{PB^2}{PB^n}$, or as $\frac{1}{PA^{n-2}} : \frac{1}{PB^{n-2}}$. Q. E. D.

244. Cor. 1. If $n = 2$, the attractions of theſe laminæ are as 1 : 1, or equal to each other; and the attraction of the ſolid $BECA$ is equal to that of one lamina multiplied into their number. The attraction therefore of this ſolid is to that of $PAC :: BA : PA$; and conſequently the attraction of a portion of matter, in contact with P, is not much greater than when removed to a ſmall diſtance.

245. Cor. 2. If $n = 3$, the attractive powers of any laminæ BDE, AFC, are to each other as $\frac{1}{PB} : \frac{1}{PA}$, or as the ordinates BI to AL of an equilateral hyperbola; and the whole attractive power of the body $BECA$ is to that of $PAC ::$ area BL : area AR :: a finite quantity : a quantity infinitely greater. According to this ſuppoſition, the attraction of P is not much affected by the addition, or deduction, of new matter at a ſmall diſtance; for the encreaſe, FIG. LIII.

encreaſe, or decreaſe, of force reſulting from it, would be finite, and conſequently infinitely leſs than that reſulting from contact. But if the ſurface of P were enlarged, and in contact with new matter, attracting according to this law, it's coheſive force would be proportionably encreaſed.

246. Cor. 3. If this attraction, at a finite diſtance, had a finite ratio to that of gravity, at the point of contact it would be infinitely ſuperior to that of gravity; and, if at the point of contact, the ratio between this attraction and gravity were finite, at any aſſignable diſtance it would be evaneſcent.

247. Cor. 4. The attraction of coheſion, being great in contact and inſenſible at a ſmall diſtance, varies in a higher than the inverſe duplicate ratio of the diſtance.

FIG. LII. LIV. 248. Cor. 5. Since the coheſive force of any particle P, in contact with the body PAC, varies as the quantity of ſurface, it will be greateſt when the ſurfaces of contact are plane. Two ſpecies of matter conſtructed of diſſimilar particles, a, b, c, d, e, f, g, &c. l, m, n, o, p, q, &c. have different degrees of hardneſs; for the ſurfaces of contact, and coheſive forces of a, b, c, d, &c. are greater than thoſe of l, m, n, &c.

249. Cor. 6. Of unequal ſimilar particles, the ſmalleſt are capable of the firmeſt coheſion, becauſe they have the greateſt ſurfaces of contact compared with their magnitudes, the ſurfaces being diminiſhed only as the ſquares, and the magnitudes as the cubes, of any lines ſimilarly placed in them.

250. Cor. 7. Hard bodies therefore may be conceived to be conſtructed of very ſmall particles, or ſuch as are terminated by plane ſurfaces; for collections of theſe particles cemented by this principle

principle of coheſion, with which they are proved to be impreſſed, would conſtitute hard bodies. And ſoft bodies may be conceived to be combined from large ſpherical particles, or ſuch as have many angular points, and do not allow of great ſurfaces of contact.

SCHOLIUM.

251. The exiſtence of this natural power is collected, and a general idea of it formed, from the following familiar obſervations and experiments. *Firſt, An attraction ſubſiſts between the component parts of hard and ſoft bodies.* The force required to ſeparate the contiguous parts of hard bodies, which is equal to the force of coheſion, is much ſuperior to their gravity, though they be full of pores and conſiſt of portions collected into one maſs, and touching only a few points; and it muſt be much greater between the ſmalleſt particles, whoſe ſurfaces are in contact, without any great intermediate vacuities. This attraction is very ſtrong in all tenacious and viſcid bodies, as pitch, roſin, &c. which adhere to every thing in contact with them; and if a body very ſoft and viſcid be extended, it will again contract its dimenſions by the tendency of the parts to each other. Large ſhips, floating near each other in a calm ſea, have a great tendency to each other, and are with difficulty kept from coming together. If the ſurfaces of the ſegments of two leaden bullets, whoſe diameters are not greater than $\frac{1}{4}$th of an inch, be poliſhed, and compreſſed together by a gentle twiſt, it is ſaid that a weight of 100 lb. is frequently required to ſeparate them. And if two poliſhed plates of glaſs, marble, braſs, or any metalline ſubſtances, be compreſſed together and ſuſpended in an exhauſted receiver, the weight of the lower plate will not diſſolve their coheſion; and if any fine thread be wrapped round one of them ſeveral times, at conſiderable intervals, after compreſſion, their coheſion is ſtill ſenſible, though they be removed from actual contact by the thickneſs of the thread. The coheſive forces of theſe ſubſtances is augmented by moiſtening their ſurfaces with water, oil, greaſe, &c. which expels the air from their pores, or, if very tenacious, keeps it confined, and prevents the

action

action of its repulsive power. The cohesive force of two polished plane surfaces of metal, whose diameters were two inches, heated in boiling water, and besmeared with grease, oil, &c. was overcome by the following weights:

	Cold grease.	Hot grease.
Planes of glass by	130 lb. by	300 lb.
brass	150	800
copper	200	850
marble	225	600
silver	150	250
iron	300	950.

When these surfaces were moistened with water, oil, &c. and compressed together, their cohesion was overcome by the weights in the following table:

With water	by 12 oz.
oil	18
Venice terpentine	24
rosin	850 lb.
tallow-candle	800
pitch	1400.

Secondly, An attraction subsists between the particles of any portions of water, oil, mercury, and all other fluids, except air, fire, and light. Small portions of these fluids form themselves into globular drops both in the open air and in vacuo; and if a drop of mercury be poured upon clean paper, or a drop of water upon the leaf of a plant, their spherical figure is not changed, the gravity of their parts being unable to dissolve their cohesion; and two drops of any fluid, not much attracted by the surface on which they are placed, will coalesce into one when in contact, as is often observable in drops of water lodged upon the leaves of plants. If mercury, well purged of air, be poured into a clean glass tube 70 inches long, so that its parts be contiguous to each other and to the glass, after inversion the whole column will remain suspended; but the pressure of the atmosphere sustains only 29 or 30 inches, and the remainder must be supported by some other agent, which is chiefly the mutual adhesion of the parts; for if they be discontinued

tinued by a bubble of air intervening, or by ſhaking the glaſs, the column ſubſides to the height of 29 or 30 inches.

Thirdly, This ſpecies of attraction ſubſiſts between the particles of different fluids, and between them and other ſubſtances. If a piece of fir wood, whoſe ſurface is one ſquare inch, be ſoaked in water and float upon its ſurface, a weight of 50 grains, beſides an equivalent to its own weight, is required to detach it from the fluid; and when its ſurface is enlarged, a proportionably greater weight is required. Water riſes near the ſides of the veſſel containing it, round a glaſs bubble floating upon its ſurface, up capillary tubes of glaſs, plants, &c. A remarkable inſtance of this attraction occurs in the new invented water-pump. If two wheels, or pullies, *B* and *D* in the ſame plane, be made to revolve with great velocity, and *D* be immerſed in water, a column of water will aſcend with the rope; and, if there be two or more grooves in *B* and *D*, and as many ropes paſs over them, at the diſtance of about an inch, the columns raiſed by each rope will cohere, and the quantity of water be much augmented.* Air is incorporated with moſt hard bodies, poſſeſſes their interſtices, and probably ſerves as a bond of union to their conſtituent parts. It appears from many experiments, that the quantity of air, detached from ſome hard bodies in which it was conſolidated, by the action of fire, or ſome particular fermentation, is very conſiderable; and Dr. Hale diſcovered that $\frac{1}{3}$ of a calculus humanus were air. It is attracted by water, and perhaps all fluids, into whoſe pores it inſinuates itſelf, is intimately mixed

FIG. LV.

* There are two of theſe machines, at Windſor, of different dimenſions. The depth of the well at the round tower is 178 feet: *D* is the wheel in the water, its diameter = 12 inches, the thickneſs of the rope *DB* = $\frac{1}{2}$ inch nearly; diameter of the upper wheel *B* = 13 inches; diameter of *C* = 11 inches; diameter of *E* = 4 feet 6 inches; a power, applied at *F* turns the wheel *E* round, and that, by means of the ſtring *EC*, communicates motion to *G*, which has the ſame axis with *B*. FIG. LV.

In the other machine the depth of the well is 95 feet, and, in this, the quantity of water, raiſed by the utmoſt efforts of a man, was at the rate of nine gallons in a minute. At Paris 500 pounds of water were elevated through an altitude equal to 240 feet, in ten minutes, when the diameter of the rope, ſurrounding the pullies, did not exceed ſix French lines. In the beginning of the motion, the column, adhering to the rope, is always leſs than when it has been worked for ſome time, and continues to encreaſe, till the ſurrounding air partake of its motion.

mixed with them, and coheres too ſtrongly to be ſeparated without the agency of fire, or ſome more attractive ſubſtance. If ſal ammoniac, or corroſive mercury ſublimate, be diſſolved in water, bubbles of air will diſengage themſelves from the water, and, adhering very tenaciouſly to the thin particles of the ſalt, aſcend with them, and burſt upon the ſurface. When water, beer, &c. are poured into a glaſs, or other veſſel, many bubbles of air are detached from the fluid by the ſtronger attraction of the veſſel, and adhere to its ſides and bottom; and, if theſe be rough, or have more points of contact, the number of bubbles will be encreaſed. Light is attracted by glaſs, water, and all tranſparent mediums, as appears by its refraction and reflection; and alſo by all other ſubſtances, as is obſervable in an experiment of Sir I. Newton.* A beam of light admitted through an aperture into a dark chamber, and paſſing by the edge of any ſubſtance, will be deflected from a rectilineal courſe, and by repeated attractions and repulſions deſcribe an undulating line in its paſſage.

OF HARDNESS, SOFTNESS, AND ELASTICITY.

252. DEF. *A hard body is that whoſe parts are not eaſily moved from their places; as wood, metal, ſtones, &c.*

Bodies conſtituted in ſuch a manner that their component parts do not give way to compreſſion, and whoſe coheſive force is inſuperable, may be called perfectly hard. The hardeſt bodies with which we are acquainted, as adamant, flint, gems, tempered ſteel, &c. are full of vacuities, and contiguous portions of them, not being in contact all round, are penetrable, and ſeparable by the action of a ſufficient force. Fire inſinuates itſelf amongſt the vacuities of the denſeſt and hardeſt bodies, and produces fuſion or expanſion, which cannot be effected without an intire ſeparation, and change of place, of their component parts. Perfect hardneſs therefore ſeems to be confined to the elementary particles, or leaſt parts into which bodies can be divided, whoſe figure and dimenſions cannot be ſeparated by any known power, and are impenetrable.

253. DEF.

* Newton's Optics, p. 317.

253. DEF. *A ſoft body is that whoſe parts change their poſition by the action of a ſmall force, and retain it when the force is removed; as butter, ſnow, wax, &c.*

Bodies conſtructed in ſuch a manner that their vacuities are not replete with any fluid, and whoſe parts do not repel each other, and are only retained in their places by their inertia, may be called perfectly ſoft; but no ſpecies of matter is perfectly ſoft, for the particles of all bodies, not poſſeſſed of a repelling power, require a much greater force to ſeparate them, than what is equivalent to their inertia. The degrees of hardneſs and ſoftneſs are infinitely variable, and the limit where hardneſs ends, and ſoftneſs begins, cannot be defined. In the congreſs of hard and ſoft bodies, there is no cauſe of ſeparation after they come into contact, and they will therefore either move on together, or be quieſcent, after impact; and, the effects of colliſion being aſcertained when there is no repulſive power, allowance muſt afterwards be made for their different degrees of repulſion.

254. DEF. *An elaſtic body is that which changes its figure, or the poſition of its parts, by the action of a force, and recovers, or has a tendency to recover, its figure.*

Elaſticity is ſaid to be perfect, when the parts of the body return to their firſt ſituation with a force equal to the force diſplacing them, and imperfect, when they do not.

255. PROP. *If a perfectly hard body* A *impinge upon a perfectly elaſtic immoveable body* B, *it will be reflected with a velocity equal to that of impact.*

DEM. The particles, compoſing the ſurface of *B*, are continually removed from their places by the compreſſion of *A*, till it and their reſiſtance become equal; and then, returning to their firſt ſituation with a force equal to that of compreſſion, and

in a direction oppoſite to *A*'s motion, *A* muſt evidently be repelled with a velocity equal to that of impact. Q. E. D.

256. Cor. 1. If *A* and *B* be both perfectly elaſtic, the particles, compoſing their ſurfaces, recede equally, though the receſs is leſs than when *A* was perfectly hard; but the effect is the ſame, the whole force of reſtitution being equal to that of compreſſion, and conſpiring to repel *A*.

257. Cor. 2. If *B* be moveable, the velocity loſt by *A* is double of that loſt by impact only; for the parts of *B*'s ſurface, contiguous to the point of impact, reſtoring themſelves with a force equal to that which diſplaced them and acting againſt *A*, it will be equally retarded by the forces of impact and reſtitution; and, for the ſame reaſon, the velocity communicated to *B* is double of what it would be, were both bodies perfectly hard.

258. Cor. 3. The relative velocity of *A* and *B* is the ſame before and after impact, or the velocity with which they accede to each other before impact, is equal to the velocity with which they recede from each other after impact; for the forces of impact and reſtitution, being equal and oppoſite, produce the ſame effects in oppoſite directions, and the relative velocity, which is deſtroyed by impact, muſt be reſtored by the force of elaſticity.

SCHOLIUM.

259. The whole time of contact of *A* and *B*, may be divided into two equal periods, the former between their firſt contact and the ceſſation of *A*'s compreſſion, and the latter between this point and their laſt contact, when they are ſeparated by the reſtoration of their relative velocity. And, if the time of *A*'s compreſſion be divided into very ſmall equal parts, *A*'s decrements of velocity produced in them will perpetually encreaſe, from the firſt contact

contact to the end of compreſſion, where its force of protruſion vaniſhes, and it is ſtationary for a moment, if *B* be fixed, or, if *B* be moveable, its velocity is equal, for a moment, to that of *B*. The particles then returning, by their elaſticity, to their firſt ſituation, will communicate, in equal times, perpetually decreaſing increments of velocity to *A*, equal to the correſponding decrements during its compreſſion, which vaniſh when the ſurface has regained its natural ſtate, when *A* leaves *B*, and is reflected with a velocity equal to that loſt by impact. When *A* and *B* are both perfectly elaſtic, the particles, contiguous to the point of impact, give way equally; and when *A* is perfectly hard, and *B* imperfectly elaſtic, or when they are both elaſtic in different degrees, the particles yield to impact unequally, and a pit or cavity muſt be formed in one of them.

260. Cor. 4. If the times of compreſſion and reſtitution be divided into the ſame number of equal moments, and the decrements and increments of velocity, in correſponding moments of compreſſion and dilatation, be as $n : 1$, the velocity loſt by impact is to that communicated in an oppoſite direction, in the ſame ratio of $n : 1$ (EUC. B.V. P.XII.). And, if this ratio between theſe velocities always obtain, their correſponding increments and decrements will always be to each other in that ratio.

SCHOLIUM.

261. *The exiſtence of elaſticity is demonſtrated by numberleſs experiments. When two bodies impinge, they muſt coaleſce if the ſurfaces of contact be immoveable, or, after receding, remain immoveable; and their ſeparation is a proof of elaſticity. Metals, ſemimetals, ſtones, gems, foſſils, cartilages, moſt fluids, as air and even water, exert an influence oppoſite to the direction of the force compreſſing them, and diſcover a tendency to return to their natural ſtate, which is, in all of them, imperfect and leſs than the force impreſſed, but moſt perfect in glaſs, ivory, hardened ſteel and cartilages.

* Muſchenbroek, Ch. XVI. Deſaguliers, Lect. VI.

lages. Elaſticity is encreaſed by encreaſing the denſity of a body; for metals are rendered more elaſtic by being beaten with a hammer, and their elaſticity, which was not perceptible before, becomes after this very ſenſible. Steel is more elaſtic when tempered, and its denſity is then encreaſed in the ratio of 7809 : 7738. It is alſo ſometimes encreaſed by cold, as the range of a cannon ball is greater when the cannon is cold, than when heated, and the ſtring of a violin, or a ſteel lamina, is inflected, and recovers its ſituation with leſs force in hot than in cold weather. The ſphere of action of the component parts of an elaſtic body, and the moments of time in which they loſe, and recover, their ſituation, are too ſmall to allow of deciſive experiments for aſcertaining their intenſity, and laws of operation, at different diſtances from their natural ſtate; and the reſult of experiments is only the relative magnitude of the velocity loſt by impact, or the whole effect of compreſſion, and that communicated by the whole aggregate force of reſtitution. Metal fibres, and thin ſteel laminæ, exhibit no elaſticity unleſs ſtretched to a certain degree, and inflected by a certain force, as appears from lax chords, which, if a little ſtretched and removed from their natural ſtate, diſcover no tendency to return to it; and, when the inflection of a fibre is very great, the influence of elaſticity ſeems to be annihilated, as appears from fibres of wood, which, if inflected beyond a certain limit, remain quieſcent and have no tendency to recover their ſituation. This is alſo obſervable in elaſtic bodies, for their elaſticity is only diſcovered by impact, and the force of impact may be ſo ſmall as to excite no ſenſible motion of the conſtituent parts, or ſo great as to deſtroy their elaſticity; but the limits where it begins and terminates are unknown. A general idea of elaſticity may be formed by conſidering the moſt ſimple caſes of the vibrations of fibres, or thin ſteel laminæ, and conceiving elaſtic bodies to be compoſed of them.

FIG. LVI. EXP. I. If any fibre, metalline chord, or thin lamina of ſteel, whoſe length is *AB*, be ſtretched and fixed to two immoveable points *A* and *B*, and inflected into the poſition *ACB* by a power which ceaſes to act at *C*, it will return by its elaſticity to its natural ſtate *AB*,

AB, and, proceeding with the velocity acquired, continue to perform nearly equal vibrations on each ſide of AB till its motion be deſtroyed by friction, and the reſiſtance of the air.

Exp. II. If the diſtance of this fibre from a table, to which it is parallel, be equal to D, and a ſpherical ball P, whoſe diameter is $2D$, be rolled againſt it, the fibre will be inflected, and this inflection will encreaſe till P be quieſcent, and, then returning to its firſt ſituation, P will be repelled and detached from it, when arrived at AB, with a velocity nearly equal to that of impact.

Exp. III. Very ſmall inflections PD, PC, of the ſame fire AB, are found to be nearly as the inflecting forces; but when the inflections are conſiderable, they vary as ſome power of the force, whoſe exponent is leſs than unity or fractional. If AB be a ſmall braſs wire tended by a weight W of 3 lb. and inflected at P by weights equal to $\frac{1}{5}$ oz. and 1 oz. ſucceſſively, $PD : PC :: \frac{1}{5} : \frac{4}{5} :: \frac{1}{2} : 1$; or the inflections are as the inflecting forces.

Exp. IV. If the lengths of the wire be multiplied by 2, 3, &c. and the ſmall inflections PD, PC be always to each other as $\frac{1}{2} : 1$; the inflecting forces are found to be as 1 : 2 and 1 : 3, &c. or inverſely as the lengths of the wire.

Exp. V. If the lengths of the wire be the ſame as in Exp. III, W be equal to 3 lb. and 6 lb. and $PD : PC :: 1 : 2$, the ſame as before; the inflecting forces are as 1 : 2, or they vary directly as W.

262. Cor. 1. If F repreſent the inflecting force, L the length of the wire, I a ſmall inflection PD or PC; I is as F (Exp. III.); F is as $\frac{1}{L}$ (Exp. IV.); and F is as W (Exp. V.). And conſequently

if

if I, F, W be ſuppoſed to vary, the inflecting force F will vary as $\frac{I \times W}{L}$.

263. Cor. 2. Fibres of unequal thickneſſes may be conceived to be compoſed of a greater or leſs number of finer fibres of the ſame thickneſs, and if W, L, I, be given, it is evident that F will be as the number of ſmaller fibres, or as the area of a ſection of the fibre compoſed of them, or as the ſquare of its diameter (D^2); and conſequently F will vary as $\frac{I \times W \times D^2}{L}$.

264. Cor. 3. It is collected from theſe and other ſimilar experiments, that the elaſticity of a ſtretched fibre appears as ſoon as it is inflected by the impact of the ball, and continues to encreaſe to the limit of inflection, where, the moment of the ball and reſiſtance of the fibre becoming equal and oppoſite, the protruſion ceaſes; and, becauſe the velocities of impact and reſilition are always nearly equal whatever be the velocity of impact, the velocities of P are equal, at equal diſtances from the limit of inflection, both in its progreſs and regreſs.

FIG. LVII. 265. Cor. 4. Elaſtic bodies may be conceived to be formed of elaſtic fibres or ſtrata, ſuch as AB; for let the ſphere DBE be imagined to be compoſed of ſuch ſtrata, and ſtricken at D by a body perfectly hard, and the parts neareſt to D, receding by the force of impact, will communicate motion to the contiguous parts, and theſe to the next, till the different ſtrata be inflected as is repreſented by the dotted lines in the figure. When the motion of the impinging body is extinguiſhed, and the particles, compoſing the ſeveral ſtrata, are no longer protruded, they will return, by their elaſticity, to their firſt ſituation; and, if their velocity be perfect, with the ſame force in acceding to, and receding from, D; and conſequently the impinging body will be reflected with a force equal

equal to the force of impact. And that this is not merely hypothetical, but that motion is diffuſed from the point of impact to the remote parts of elaſtic bodies, is preſumed from the following experiments.

EXP. VI. If a ſpherical ball of ivory *A* be preſſed againſt another *B*, whoſe ſurface is freſh painted with any colour, it will receive a ſmall point of that colour upon its ſurface; but if *A* impinge upon *B* with any velocity, the breadth of the ſpot will be magnified, and become ſtill greater as the velocity of impact is encreaſed. And, becauſe the ball retains its ſpherical figure after impact, the parts of its ſurface muſt have loſt, and recovered, their firſt ſituation.

EXP. VII. If two glaſs balls impinge with a proper degree of velocity, the interior parts of the balls will be broken, though the exterior, contiguous to the point of impact, be unbroken.

EXP. VIII. If two ivory balls *A* and *B* be ſuſpended from the ſame point by two ſtrings of the ſame length, and the leſs ball *A* impinge upon *B* at reſt with a given velocity, *A* will be reflected always to the ſame height, and *B* will be impelled to the ſame height, upon the graduated periphery of a circle whoſe radius is the length of the ſtring. But if either *A* or *B* be hollowed and lead inſerted in the center, or nearer to the poſterior ſurface, neither ball, though their weight be the ſame, will aſcend as high as before the inſertion of lead.

266. Cor. 1. Motion is therefore communicated from the point of impact to the contiguous parts (EXP. VI.), and diffuſed from thence to the remote parts of every elaſtic body (EXP. VII. & VIII.)

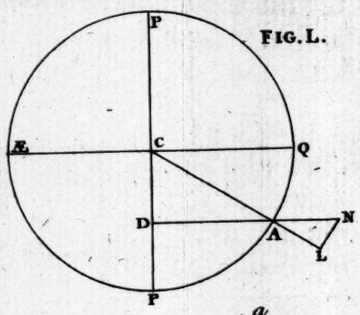
267. Cor. 2. It is evident, from Exp. VIII., that the progreſſive motion of the parts, from the point of impact, is ſtopped by the inſertion of lead, and conſequently that the force of reſtitution, and change of figure, is leſs, than before it was inſerted.

Exp. IX. A ſtroke or friction upon the edge of a glaſs, filled with water, communicates a tremulous motion to the parts of the glaſs, which is viſibly communicated to the water. A reed or ſtick, placed acroſs the bottom of a large glaſs bell, will fall when the bell is ſtruck, the ſtroke producing a change of figure; and, if a piece of metal be fixed near the brim or lip of the bell without touching it, and the bell be ſtricken by any hard body, it is ſeen to touch the metal, and a ſucceſſive of ſounds, perpetually decaying, may be heard. If the edge of the bell be pinched, and the fingers ſuddenly withdrawn, the ſame ſound is heard without producing any ſenſible motion towards the metal, or diſplacing the reed acroſs it.

268. Cor. The motion diffuſed from the point of impact, to the remote parts of an elaſtic body, is continued for ſome time, and diminiſhed gradually till it vaniſhes. And there ſeem to be two kinds of vibrations of the parts of an elaſtic body, one of which is quick, and called a tremor of its minute parts, and the other ſlower and longer, by which its figure is changed, and an impinging body repelled.

Plate VI

FIG. L.



a	*b*	*c*	*d*
e	*f*	*g*	*h*

FIG. LIV.

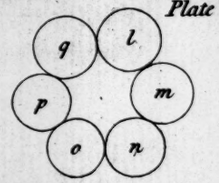


FIG. LI.

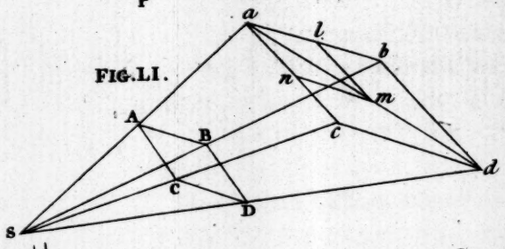


FIG. LV.

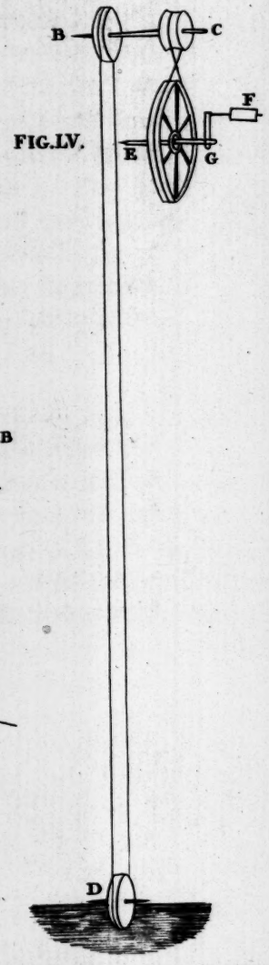


FIG. LVI.

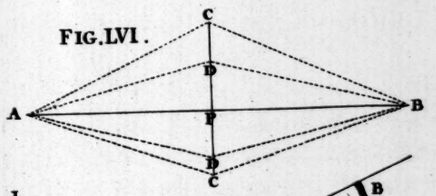


FIG. LIII.

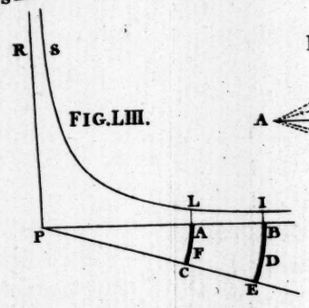


FIG. LII.

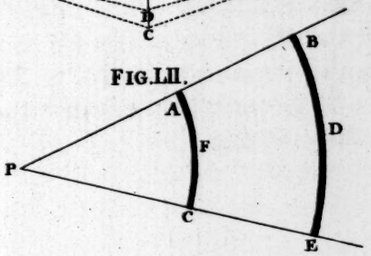
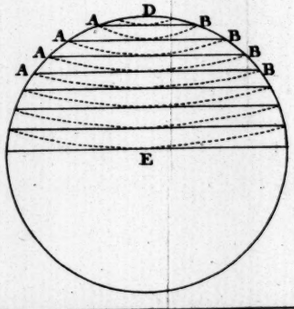


FIG. LVII.




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CHAP. VII.

MECHANICAL POWERS.

*THE exiſtence, and intenſity of operation, of the mechanical affections, gravity, coheſion and elaſticity, and the nature of the other qualities of matter, being aſcertained experimentally, they are aſſumed as eſtabliſhed principles, and their efficacy in the production of preſſure, motion, and other phenomena, is the next object of mechanical philoſophy. There are ſix ſimple machines, commonly ſtiled mechanical powers, from the effects produced, with their intervention, by the action of gravity and animal exertions, viz. the lever, wheel and axis, pulley, wedge, inclined plane and ſcrew. They are all calculated to communicate motion to bodies, and ſuſtain their preſſure, for which, the power unaſſiſted by them, is incompetent; and the artifice in all conſiſts in diſtributing the weight amongſt ſuch a number of agents, that the part ſuſtained by the power may bear a ſmall ratio to the whole. Thus, a power incapable of communicating motion to, or ſupporting the preſſure of, a body, without mechanical aſſiſtance, may effect its purpoſe by transferring a part of the weight upon a fulcrum, diſtributing it amongſt a number of pulleys, or placing it upon an inclined plane or ſcrew; and, by this artifice, a power *P* may keep a weight ſuſpended which exceeds it in any aſſigned ratio, though without any acquiſition of moment in a given direction; for motion is only communicable according to the eſtabliſhed natural relations ſubſiſting between matter and motion, and the magnitudes of two powers, in equilibrio, are always inverſely as their velocities.

* Keil's Phyſics, Lect. X. Helſham, Lect. VI. Emerſon, Prop. 18, &c. Graves, L. I. C. X. Muſchenbroek, Ch. VIII. Varignon, pag. 305. Maclaurin's Newton, B. II. Ch. III. Hamilton's Eſſay on the Principles of Mechanics. Morgan's Notes to Rohault.

LEVER.

269. DEF. *A lever is a bar of wood or metal, and is uſually repreſented by an inflexible line, without gravity, revolving about a fixed point, called the fulcrum, by the action of a power upon its arms.*

PLATE VII. FIG. LVIII. The points *W*, *P*, where the weight and power act, are the points of ſuſpenſion, and the immoveable point *F*, about which every point of the lever revolves, is called promiſcuouſly the fulcrum, hypo-mochlion, and center of motion. There are three kinds of levers: 1. the fulcrum is between the power and weight, as in the common balance, ſciſſars, ſnuffers, &c. 2. The weight is between the power and centre of motion, as the oars and rudder of a boat, cutting knives fixed at one end, doors, &c. 3. The power is between the weight and fulcrum, as a ladder raiſed againſt a wall, a weight raiſed by the arm, where the center of motion is at the ſhoulder, &c.

FIG. LIX. 270. Cor. 1. Forces whoſe magnitudes are to each other as *PA*, *PB*, *PC*, &c. acting at *P* and terminated by a line *AC*, parallel to *PF*, have the ſame effect: for each may be reſolved into two forces, one perpendicular, and the other parallel, to *PF*; of which, the perpendicular parts are equal and entirely employed in producing a rotation of the lever round *F*, or in ſupporting a body *W* placed on the other ſide of *F*, and acting perpendicularly to *FW*, and the parallel parts only produce a motion in the direction *PF*, and do not produce any rotation, or contribute to the ſupport of *W*.

FIG. LX. 271. Cor. 2. If a given power, repreſented by *PB*, act at the ſame point *P* in any direction *PD*, its efficacy to turn the lever round *F*, or ſupport any body *W*, is as the chord *PC* of the circle whoſe diameter is *PB*; for *PD* being taken equal to *PB* and reſolved

reſolved into two forces, one *DE* perpendicular, and the other *PE* parallel, to *PF*, it appears from ſimilar triangles, that *DE*, the only effective part of the force, is equal to *PC*.

272. PROP. *Two powers* W, P, *acting upon a lever* PW, *whoſe center of motion is* F, *at the points* P *and* W *in the directions* WM, PL *in the ſame plane, and in equilibrio, are to each other inverſely as the perpendiculars let fall from* F *upon their directions.* PLATE VII. FIG. LXI.

DEM. From *F* as a center, with the longer perpendicular *FL*, deſcribe a circular arc cutting the direction of *W* in *D*, and, becauſe the efficacy of theſe forces is the ſame to whatever points of their directions they are applied, let them be applied at *L* and *D*. Let *DE* repreſent the magnitude of *W* and be reſolved into two forces, one *DG* in the direction *FD*, and the other *EG* perpendicular to it; and, becauſe *DG* has no effect in making *DF* revolve, nor conſequently the lever which makes an invariable angle with it, and *EG* and *P* act at equal diſtances from *F*, in directions perpendicular to thoſe diſtances, and are in equilibrio, *P* is equal to *EG*, and $W : P :: DE : EG :: DF\,(FL) : FM$, from ſimilar triangles. Q. E. D.

273. Cor. 1. In any lever *FP* or *FHK*, thoſe parts of *P* and *W*, which are oppoſite to each other, are inverſely as their rectilineal diſtances from *F*; for let *WA* and *PD*, be the reſpective magnitudes of *W* and *P*, which act at *W* and *P*, and reſolving each into two forces, *WB*, *PE*, coincident with the lever, or arm *FP* which makes an invariable angle with it, and *AB*, *DE*, parallel and oppoſite, and drawing the perpendiculars *AC*, *DG*, to *FP*, and *FM*, *FL*, to the directions; it appears, from ſimilar triangles, that $DE : AB :: DG : AC :: \frac{DP \times FL}{FP} : \frac{WA \times FM}{FW}$ $(:: \frac{1}{FP} : \frac{1}{FW}$, becauſe $DP \times FL = WA \times FM$ from this propoſition. FIG. LXII.

274. Cor.

274. Cor. 2. If the directions of P and W, acting upon the arms of a ſtraight lever, and in equilibrio, be parallel, they are therefore inverſely as their diſtances, or inverſely as the portions of any line drawn through F and terminated by their directions (273). This follows alſo from the propoſition, becauſe the diſtances, or the ſegments of any right line on each ſide of F and terminated by their directions, are, in this ſuppoſition, as the perpendiculars let fall from F upon their directions.

276. Cor. 3. The velocity of any point either in a ſtraight, or curved lever, varies as its rectilineal diſtance from the center of motion F; for all points deſcribe ſimilar circular arcs, having their centers in F. If P and W act upon the ſame right line, and their directions be perpendicular, or inclined in the ſame angle, to their rectilineal diſtances, their velocities will be as thoſe diſtances, being meaſured by the baſes of ſimilar triangles which are deſcribed in the ſame time: their velocities are therefore to each other inverſely as the oppoſite parts of P and W (273).

FIG. LXIII. 277. Cor. 4. If a lever be moveable about an axis AB, or fixed to an axis which is moveable about two centers A and B, and perpendiculars PF, WF, be drawn to this axis from P and W, to which their directions are perpendicular and in the ſame plane, an equilibrium obtains when $P : W$ inverſely as their perpendicular diſtances from the axis.

FIG. LXII. 278. Cor. 5. There will be an equilibrium upon the lever when $P \times FP \times \text{ſin.} \angle FPL$ (or the angle which P's direction makes with FP) is equal to $W \times FW \times \text{ſin.} \angle FWM$ (or the angle which W's direction makes with FW; for $P : W :: FM : FL :: \frac{FW \times \text{ſin.} \angle FWM}{\text{rad.}} : \frac{FP \times \text{ſin.} \angle FPL}{\text{rad.}}$; and $F \times FP \times \text{ſin.} \angle FPL = W \times FW \times \text{ſin.} \angle FWM$, ſuppoſing the radius to be given.

279. Cor.

279. Cor. 6. The intenſities or moments of any powers A, B, C, &c. whoſe directions are parallel, vary as their magnitudes multiplied into their diſtances from the center of motion; for let Q, R, S, &c. acting at the ſame point E, in directions parallel to thoſe of A, B, C, &c. be in equilibrio with them reſpectively, or (Cor. 2.) let $A \times AF = Q \times EF$, $B \times BF = R \times EF$, $C \times FC = S \times EF$, &c.; and the efficacy of A, B, C, &c. to make the lever revolve, or ſupport any power acting againſt them at E, is evidently as $Q + R + S$, &c. or as $\overline{Q + R + S} \times EF$, or, ſubſtituting their equals, as $A \times AF + B \times BF + C \times CF$, &c. FIG. LXIV.

280. Cor. 7. The intenſities or moments of A, B, C, &c. the ſines of whoſe directions with their rectilineal diſtances to the ſame radius are a, b, c, &c. reſpectively, will be as $A \times a \times AF + B \times BF \times b + C \times CF \times c$, &c.; for if Q, R, S act at the ſame point E, in the ſame direction, the ſine of whoſe inclination to $EF = z$, and be in equilibrio with them, $\overline{Q + R + S} \times EF \times z = A \times AF \times a + B \times BF \times b + C \times CF \times c$, &c. (278).

281. Cor. 8. If more than two powers act upon a lever, there will be an equilibrium when the ſum of the products ariſing from multiplying each into the perpendicular diſtance of its direction from the center of motion; or, if their directions be parallel and the lever ſtraight, into its diſtance from F, on one ſide, is equal to the ſum of the products on the other ſide. Whatever be the form of the lever, the value of the perpendicular may be ſubſtiſtituted for it, and an equilibrium obtains when the ſums of the products on each ſide of F are equal.

282. Cor. 9. Becauſe the efficacy of P and W is the ſame to whatever part of their direction they are applied, a bended or curved lever may be reduced to a ſtraight one, making an invariable angle with it, and P and W may therefore be always ſuppoſed to act in the ſame right line.

283. PROP.

PLATE VIII. FIG. LXV. 283. PROP. *When any number of levers* WC, CE, EP, *&c. are combined together in the ſame direction, the ratio of* P *to* W, *acting in the ſame plane at their extremities, in parallel directions, and in equilibrio, is that of* WB × CD × EF : BC × DE × FP.

DEM. Let the forces Q, R, P, acting at the points C, E, P, in directions parallel to thoſe of W and P, be reſpectively in equilibrio with W, Q, R, and conſequently (274) $W : Q :: BC : WB$
$Q : R :: DE : CD$
$R : P :: FP : EF$;
and, by compoſition of ratios, $W : P :: BC \times DE \times FP : WB \times CD \times EF$. Q.E.D.

284. Cor. 1. If P and W act in different directions, and Q and R act alſo in any other different directions, and Q, R, P be reſpectively in equilibrio with W, Q, R, perpendiculars, let fall from the centers of motion, B, D, F, upon their directions, are to be ſubſtituted for the diſtances.

285. Cor. 2. If any of the forces, in this and the preceding propoſition and corollaries, act in different planes, they are to be reduced to the ſame plane by reſolving each into two forces, one in that plane and the other perpendicular to it, of which the former only are effective, and are to be uſed in the ſeveral analogies.

FIG. LXVI. 286. PROP. *If lines be drawn from* F *parallel to the directions of* P *and* W *which meet in* A; P, W, *and preſſure upon* F (Pr) *are to each other as the ſides and diagonal* AB, AC, AF, *of the parallelogram* CABF, *reſpectively.*

DEM. From F let fall the perpendiculars FL, FM, upon the directions of P and W, and the triangles FCM, FLB (having a right angle in each, and the angles FCM, FBL, either equal to or

the

Plate VII.

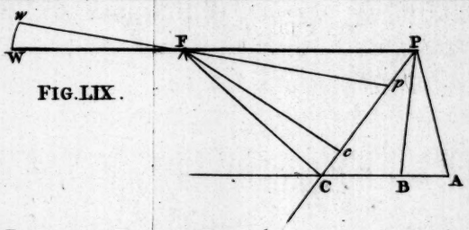


FIG. LIX.

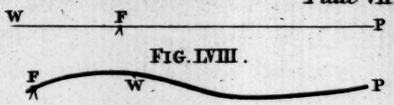


FIG. LVIII.

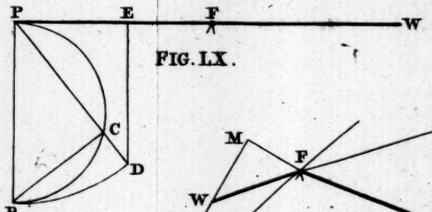


FIG. LX.

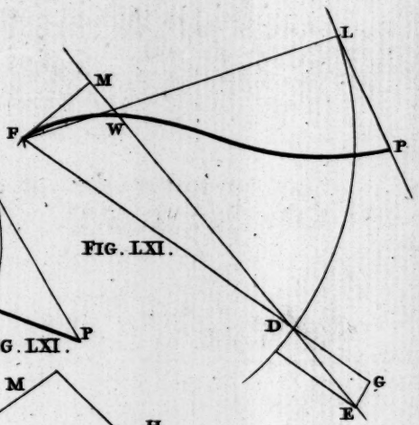


FIG. LXI.

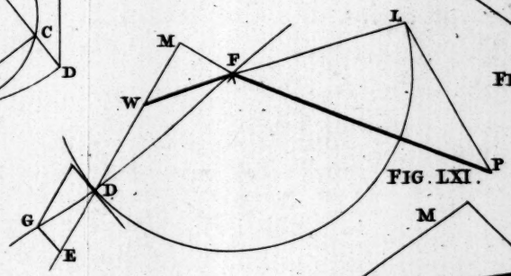


FIG. LXI.

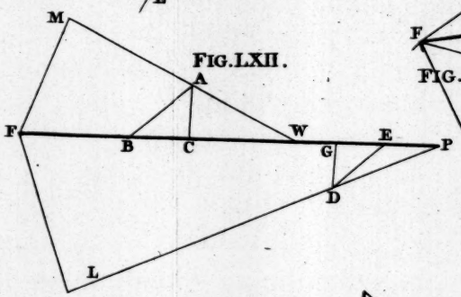


FIG. LXII.

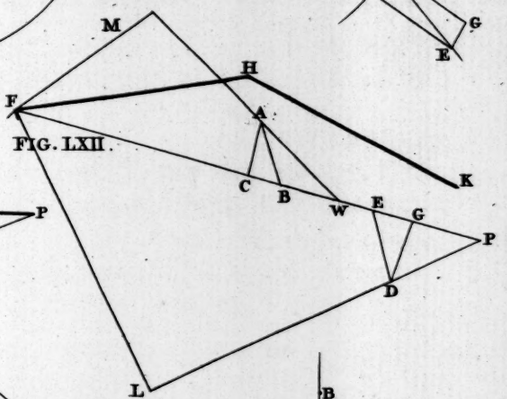
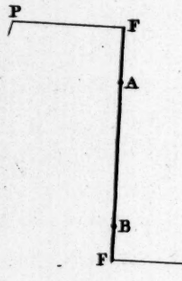


FIG. LXII.



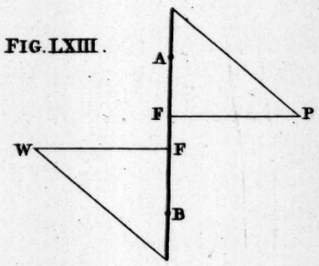


FIG. LXIII.

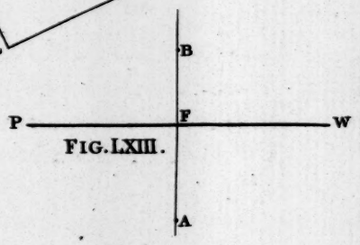


FIG. LXIII.

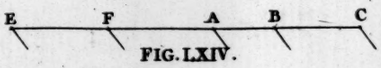


FIG. LXIV.


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the ſupplements of CAB) are ſimilar; therefore $P : W :: FM : FL :: FC : FB$, and a force, whoſe quantity and direction are AF, is equivalent to P and W (185). Q. E. D.

287. Cor. 1. A power therefore acting at F, whoſe magnitude is to $P + W$ as AF to $AB + AC$, and whoſe directions are FA, AB and AC, reſpectively, will prevent all motion.

288. Cor. 2. The magnitude of P, W or Pr, is as the ſine of the angle formed by the directions of the other two; for

$Pr : P :: FA : AB$ :: ſin. of $\angle FBA$ or BAC : ſin. of $\angle AFB$ or FAC;

$Pr : W :: FA : AC$ :: ſin. of $\angle FCA$ or $\angle CAB$: ſin. of $\angle CFA$ or FAB;

and $W : P :: AC : FC$ :: ſin. of $\angle CFA$ or FAB : ſin. of $\angle CAF$.

289. Cor. 3. Any two of theſe forces, preſſure upon F, P, and W, are to each other inverſely as the perpendiculars let fall upon their directions from any point F in the direction of the third; for $Pr : P$ :: ſin. of $\angle FCA$ or WAH : ſin. of $\angle CAF :: WH : WI$ (ſuppoſing WH and WI to be perpendicular to the directions of P and Pr. and AW to be the radius); and $Pr : W$ :: ſin. of $\angle FBA$ or PAK : ſin. of $\angle BAF$ or $BAN :: PK : PN$, ſuppoſing PK and PN to be perpendicular, reſpectively, to the directions of W and Pr, and AP to be the radius, &c. And if the lever be ſtraight and the directions of P and W parallel, the magnitudes of P, W and Pr, are as the diſtances of the other two, the perpendiculars then becoming the diſtances.

290. Cor. 4. If the extremities of the perpendiculars FL, FM, be joined by a right line LM, Pr, P and W are to each other reſpectively as LM, FM and FL, for this triangle is ſimilar to FAB, as eaſily appears by deſcribing a circle upon FA as a diameter.

FIG. LXVII. 291. PROP. *The distance from* F, *quantity and direction, of any two forces* Q *and* R *acting upon a straight lever in the same plane, being given, to find the magnitude, direction and distance of a force equivalent to them.*

Let the directions of Q and R meet in A, and taking $AB : AD :: Q : R$, and completing the parallelogram, the diagonal AE is the magnitude and direction of a force equivalent to them (185). But QR and the angles AQR, ARQ being given, AQ, AR, and the angle QAR may be found; and AB, AD, and the angle BAD being known, AE, the magnitude of the combined force, and the angle BAE may be found; and QA, and the angles AQN, QAN being found, the angle QNA, or inclination of AN, to FN and QN, are known. Q. E. I.

FIG. LXVIII. 292. Cor. 1. If any number of forces Q, R, S in the same plane, whose magnitudes and directions are AB, AC, ER, act upon the lever FW, and be in equilibrio with any other forces T, V, W, whose quantities and directions are HG, HI, LN, they may be reduced to two which are in equilibrio; for AB and AC are equivalent to AD, and taking EF equal to AD, EF and ER are equivalent to EQ; and in the same manner HG, HI, LN are equivalent to LP, which is in equilibrio with EQ, because the lever is at rest. And the directions, and distances from F, of LP and EQ, are found as in the proposition.

FIG. LXIX. 293. Cor. 2. Any number of forces in the same plane, whose magnitudes and directions are AB, AC, EX, HG, HI, LN, may be reduced to one, which is equal to the pressure upon the fulcrum; for, supposing the forces EQ, and LP, resulting from the other forces combined, to act at their intersection S, a third force equivalent to them, and consequently to the pressure upon F, must pass through S (185), and, taking SU, SV, respectively equal to EQ, LP, and completing the parallelogram, SY will be its magnitude and direction.

294. Cor.

294. Cor. 3. If any number of forces, whoſe quantities and directions are given, act upon a lever, the poſition of a fulcrum, about which they will be in equilibrio, and the quantity and direction of preſſure upon it, may be found; for, let the directions of Q and R meet in A, and taking $AB : AC :: Q : R$, and, completing the parallelogram, its diagonal AD produced, will cut the lever in a point, which being ſupported, Q and R will be in equilibrio (286); and combining AD with S, and the force, reſulting from theſe, with another, &c. the diagonal will always interſect the lever in a point F, about which they will be in equilibrio. FIG. LXVIII.

295. LEMMA. *If right lines be drawn from any point* P *to the extremities of the diagonal, and ſides, of the parallelogram* ABCD, *the triangle* PAC, *having the diagonal for its baſe, is equal to the difference, or ſum of the triangles* PAB, PAD, *having the ſides for their baſes, according as* P *is ſituated between the lines forming the angle* BAD, *or thoſe which form its ſupplement to two right angles.* FIG. LXX.

DEM. CASE I. Let P be ſituated between the lines forming the angle BAD; and, drawing Pnm perpendicular to AB or CD, the triangle $APC = ADPC - ADP$; but $ADPC = ADC + DPC = AB \times \frac{mn}{2} + DC \times \frac{Pn}{2} = AB \times \frac{Pm}{2} = APB$; therefore $APC = APB - APD$.

CASE II. Let P be placed between the lines forming the ſupplemental angle to BAD, and the triangles $ADC \pm DPC = \frac{AB}{2} \times \overline{mn \pm Pn}\,(Pm) = ABP$; and, adding APD to both, $APC = ABP + ADP$. Q.E.D. FIG. LXXI.

PLATE IX. FIG. LXXII. 296. Cor. 1. If P be in one of the ſides containing the angle BAD, the triangle $APC = ADC + DPC = \frac{AB}{2} \times \overline{mn + Pn}$ $(Pm) = PAB$.

FIG. LXXIII. 297. Cor. 2. If P be in the diagonal, the triangle $PAB = ABC \pm BPC = BC \times \frac{mn}{2} \pm BC \times \frac{Pn}{2} = BC \times \frac{Pm}{2} = PAD$. The upper ſign is to be uſed when P is without the figure, the lower when it is within.

298. Cor. 3. If perpendiculars be drawn from P to the diagonal and ſides, Pd, Pm, Pq; $AC \times Pd = AB \times Pm \pm AD \times Pq$; and when P is in the diagonal AC, $AB \times Pq = AD \times Pm$.

FIG. LXXIV. 299. PROP. *If there be any number of forces* Q, R, S, T, *in the ſame plane, which are combined as before, the ſums of the products, ariſing from multiplying each into the perpendicular diſtance of its direction from any point* F, *in the diagonal* CL, *are equal on each ſide of it.*

DEM. Let CD, CG, CH, CK be the relative magnitudes of Q, R, S, T, reſpectively, and compounding them, and drawing perpendiculars, from any point F in the laſt diagonal, upon their directions, viz. Fa, Fb, Fc, Fd, Fg, Fb; from the preceding lemma, $CK \times Fb = CI \times Fg = CE \times Fc - CH \times Fd = CD \times Fa + CG \times Fb - CH \times Fd$, and conſequently CK or $T \times Fb + CH$ or $S \times Fd = CD$ or $Q \times Fa + CG$ or $R \times Fb$. Q. E. D.

300. Cor. A lever therefore paſſing through any point F in the diagonal LC, produced in any direction, and in the ſame plane with the forces which act upon it, will be in equilibrio.

301. LEMMA.

Plate VIII.

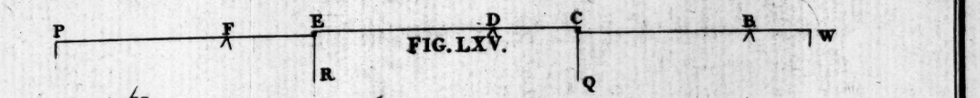


FIG. LXV.

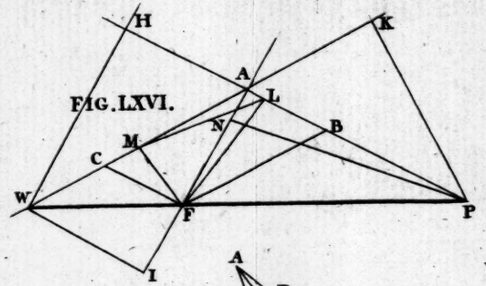


FIG. LXVI.

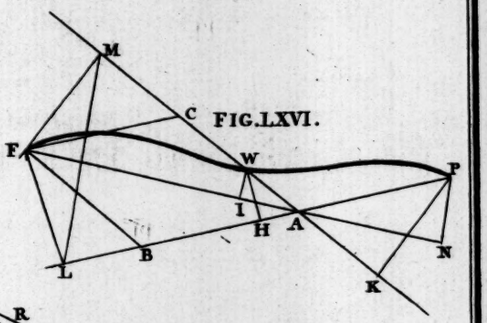


FIG. LXVI.



FIG. LXVII.

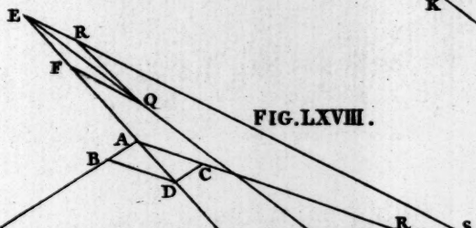


FIG. LXVIII.

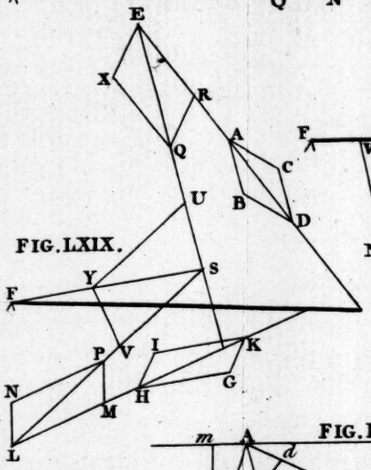


FIG. LXIX.

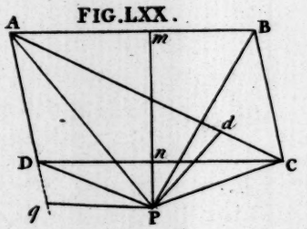


FIG. LXX.

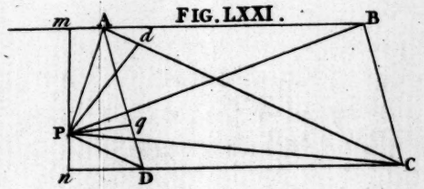


FIG. LXXI.

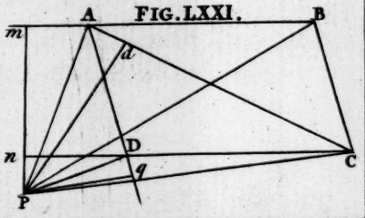


FIG. LXXI.


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301. LEMMA. *If a right line* PW *revolve round a fixed point* F, *and lines are drawn from its extremities to another fixed point* Q, *to which perpendiculars* FL, FM, *and* Fl, Fm *are drawn from* F, *and* W *is ſuppoſed to aſcend,* FL *has to* FM *a greater ratio than* Fl : Fm. FIG. LXXV.

DEM. Take $Pr = py = FW$, and, drawing the perpendiculars rv, yz; $FL : rv :: \text{ſin.} \angle FWL : \text{ſin.} \angle rPv :: PQ : WQ$; $rv : FM :: Pr\,(FW) : FP$, and comp. $FL : FM :: PQ \times FW : WQ \times FP$. By a ſimilar proceſs, $Fm : Fl :: wQ \times FP : pQ \times FW$; and adding theſe analogies together, $FL \times Fm : FM \times Fl :: PQ \times FW \times wQ \times FP : WQ \times FP \times pQ \times FW :: PQ \times wQ : WQ \times pQ$. But PQ is greater than pQ, and wQ than WQ, and conſequently $PQ \times wQ$ is greater than $WQ \times pQ$, and $FL \times Fm$ than $FM \times Fl$, and the ratio of $FL : FM$ is greater than that of $Fl : Fm$. Q. E. D.

302. PROP. *If any two powers* P and *W*, *whoſe directions always meet in the ſame point* Q, *be in equilibrio upon the lever* PW *in any one poſition of* PW, *they cannot be in equilibrio when it is in any other poſition* pw.

DEM. Let the lever revolve, and be in any other ſituation *pw*, and becauſe $P : W :: FL : FM$, or in a greater ratio than $Fl : Fm$ (301) P will preponderate. Q. E. D.

303. Cor. 1. It is evident that the lever cannot reſt till it paſs through the points *F*, *Q*.

304. Cor. 2. If the directions of P and W be parallel to each other, they will be in equilibrio in any poſition of the lever, becauſe the perpendiculars drawn from F to their directions are always as the diſtances, and conſequently in a given ratio to each other. And, for the ſame reaſon, if there be ever ſo many forces, acting in parallel directions upon the arms of a ſtraight lever,

and in equilibrio in any one ſituation of the lever, they will be in equilibrio in every ſituation of it.

FIG. LXXVI. 305. PROP. *If the center of motion* F *be placed above the ſtraight lever* PW, *and* P *and* W, *acting always in parallel directions, equilibrate in any poſition* PW, *they do not equilibrate in any other,* pw.

DEM. From *F* draw *FM* and *Fm* perpendicular to the lever, and $P : W :: WM : PM :: wm : pm$, or in a leſs ratio than that of $Lw : Lp$ or $LV : LR$, and conſequently much leſs than that of the perpendiculars from the center of motion *F* upon the directions, or $MV : MR$. Q. E. D.

306. Cor. 1. Becauſe $P : W$ in a leſs ratio than that of $MV : MR$, $P \times MR$ is leſs than $W \times MV$, and conſequently *W* will deſcend.

PLATE X. FIG. LXXVII. 307. Cor. 2. If *F* be placed on the other ſide of the lever, the deſcending body will preponderate; for $P : W :: WM : PM :: mw : mp$, and conſequently $P : W$ in a greater ratio than that of $Lw : Lp$ or $LV : LR$, and therefore much greater than that of the perpendiculars upon the directions or $MV : MR$; and $P \times MR$ is greater than that of $W \times MV$.

FIG. LXXVIII. 308. PROP. *If* P *and* W *act always at the ſame diſtance from the lever, in directions parallel to* FM, *and in equilibrio about* F *in the poſition* PFW, *and the lever be moved, the deſcending body will continue to preponderate.*

DEM. Let *FN*, *ps*, *wr*, be drawn parallel to the directions in which the bodies act, *pQw* be the lever, and *Fn* be the poſition of *FN*; and from ſimilar $\triangle$s $Qr : Qs :: Qw : Qp$, and therefore $Qr : Qs$ in a greater ratio than $nw : np$ or $NW : NP$ or $WM : PL$ or

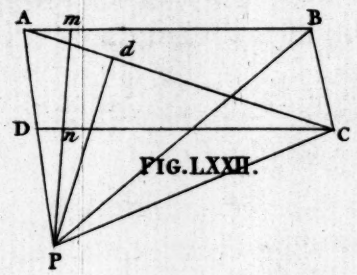


FIG. LXXII.

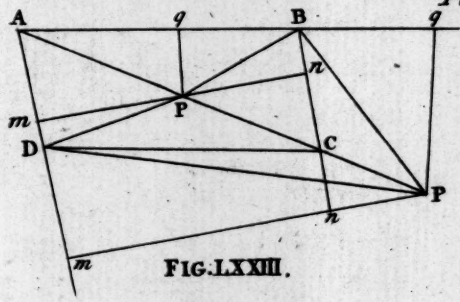


FIG. LXXIII.

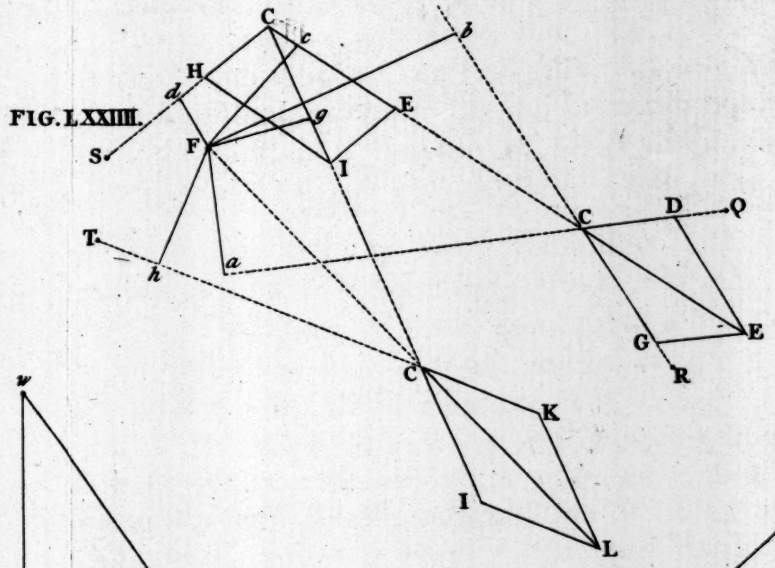


FIG. LXXIV.

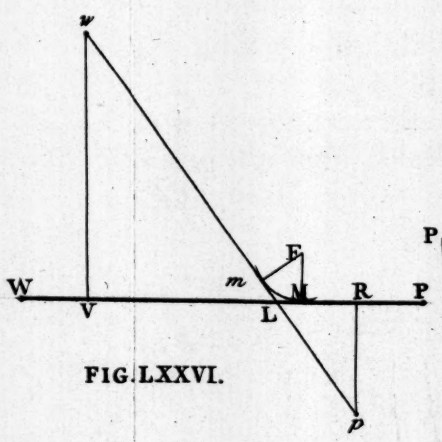


FIG. LXXVI.

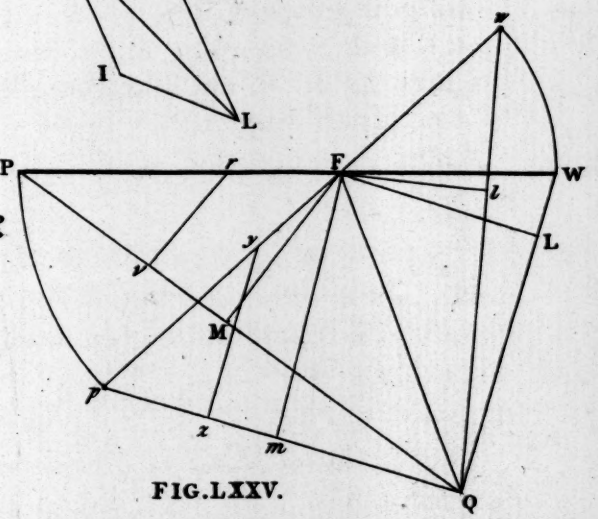


FIG. LXXV.


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PL or P to W; but $Nr : Ns$ in a greater ratio than $Qr : Qs$, and conſequently $P : W$ in a leſs ratio than $Nr : Ns$, and the body W will continue to preponderate.

309. Cor. When the bodies are placed under the lever, the deſcending body will continue to deſcend, becauſe $P : W$ in a leſs ratio than $Nr : Ns$, as eaſily appears by turning the figure.

SCHOLIUM.

310. In all communications of motion by impact, the quantities of motion loſt and gained being equal and oppoſite, the quantity of motion eſtimated in the ſame direction is invariable, and the quantities of matter vary inverſely as the velocities loſt and gained; and if two bodies A and B act upon a lever, or any other machine, they are ſo connected that A cannot deſcend without making B aſcend with the ſame quantity of motion, and their quantities of matter are therefore inverſely as each other. Theſe caſes, having ſuch marks of coincidence, are inferred to be ſimilar in every reſpect, and the cauſe of an equilibrium in the mechanical powers, is often immediately aſſigned from this equality of momenta; but they are not exactly ſimilar, becauſe when A impinges upon B, ſome part of its motion is transferred to B, and A's motion neceſſarily precedes this communication of motion; but, when they act upon any machine, the deſcending body A cannot be ſaid to communicate any part of its motion to B aſcending; becauſe, from their connection, their motion muſt neceſſarily commence and be extinguiſhed together; and beſides, the power of the lever ought to be conſidered. The ratio of the power and weight may however be aſſigned, in every machine, from their incipient momenta; for P and W, acting upon the arms of a lever, exert a preſſure and have a tendency to move; and if a and b be the velocities with which A and B ſtrike the lever, eſtimated in ſuch directions that their preſſures are ſolely employed in reſiſting each other's efforts to produce motion, no part being loſt by obliquity of direction, $A \times a \times$ into its velocity or diſtance from the centre of motion

motion is equal to $B \times b \times$ into its velocity or diſtance, when there is an equilibrium; and by ſubſtituting preſſures or P and W, for $A \times a$ and $B \times b$, the ratio of $P : W$ is found to be the ſame as was before collected from the reſolution of motion. As the velocity of any point of a lever varies as its rectilineal diſtance from the center of motion, all points deſcribing ſimilar circular arcs round it in the ſame time, if the directions of P and W be perpendicular to their rectilineal diſtances, their velocities will be the ſame as thoſe of the points where they act and wholly efficient, and when there is an equilibrium, P will be to W inverſely as their rectilineal diſtances. But if the direction of P or W be enclined to their rectilineal diſtance, their efficient velocity will not be equal to that of the point where they act; if P's direction be the line PD, it is evident that P will have two motions whilſt the lever revolves, one acceding to, or receding from, F, according as the angle DPF is leſs or greater than a right angle, and the other producing the rotation of the lever; for, deſcribing a circular arc with F as a center, and FD as radius, the power will act at every intermediate point in PE whilſt the lever deſcribes the angle PFD. The motion in the direction of the lever is inefficient, and if PD repreſent the direction and quantity of A's velocity, and be reſolved into two, PC in the direction of PF, and DC perpendicular to it, this laſt only is efficient; and if the angle at F be very ſmall, $A \times DC \times$ its perpendicular velocity or into FD, or $\frac{A \times FP \times PD \times FL}{FP}$, ſuppoſing FL to be perpendicular to the direction, is A's efficacy to turn the lever. The effective part of B's velocity being found in the ſame manner, and theſe values of a and b being ſubſtituted for them in the ſuppoſition of an equality of moments; whatever be their directions, the ratio of P to W is the ſame as that diſcovered by other principles. Thus in an equilibrium $A \times PD \times FL$ is given, and conſequently $A \times PD$ or the preſſure of P in the line PD is as $\frac{1}{FL}$. The demonſtration of this fundamental propoſition, aſcribed to Archimedes, depends upon this principle, that if a number of weights be ſuſpended upon the arms of a lever, at points equidiſtant from each other, whether on the ſame ſide of the fulcrum or not, their

FIG. LXXIX.

efficacy

efficacy to make the lever revolve is the ſame as if they were united in a point biſecting the diſtance of the points of ſuſpenſion. Mr. Huygens ſays, that many fruitleſs attempts have been made to remedy the defects of this demonſtration, and propoſes another founded on this principle, that when two equal bodies are placed upon the arms of a lever, that which is moſt remote from the fulcrum will preponderate. But this principle is not more evident than that of Archidemes, and beſides, his proceſs is prolix and tireſome. Mr. Maclaurin hath given a demonſtration of this propoſition when the arms of the lever are commenſurate, and his method might eaſily be extended to caſes in which they are incommenſurate, and be made general; but this would add to the length of a proceſs already very long. The only principle in Sir I. Newton's demonſtration that hath been controverted is this, which is taken for granted, "that the ſame power will have the ſame effect to whatever point of the direction, in which it acts, it be applied;" yet no doubts are entertained of the truth of it, though, perhaps, from its ſimplicity and intuitive evidence, it cannot be demonſtrated by more ſimple principles. If the line *PC* and the angle *PFC* be invariable, the radii *PF*, *CF* being fixed to *PC* at the points *P* and *C*, it is evident, that two equal forces *P* and *Q*, acting upon the points *P* and *C* in the directions *PC* and *CP*, will deſtroy each other, and the line *PC*, and conſequently *PF* and *CF*, will be quieſcent. If therefore *P* make a body deſcribe the arc *Ww* in any ſmall time, *Q* will make it deſcribe *wW* in the ſame time, or they would not deſtroy each other's effects. Equal forces therefore *P* and *Q*, acting at different points of the ſame direction, in the ſame time make the radii *PF*, *CF* deſcribe angles at *F*, *PFp* and *CFc* equal to *WFw*, and therefore to each other. Though this demonſtration of Newton be general, conciſe, and perfectly ſatisfactory, the great utility of the propoſition may poſſibly render other demonſtrations of it not undeſerving of attention. PLATE VII. FIG. LIX.

DEMONSTRATION OF ARCHIMEDES.

FIG. LXXX. Let AB be a homogeneous cylinder, and C the biſection of its axis, and it is evident, that if a fulcrum, or power equal and oppoſite to the preſſure upon it, be applied at C, the parts AC and CB will be in equilibrio. Let any point D be taken, and, biſecting AD in E, and DB in F, it is clear that two powers, reſpectively equal to the weights of AD and DB, applied at E and F, will ſupport them, and have the ſame effect with the fulcrum, or power, applied at C, and be in equilibrio about C: but $CE = CA - AE = \frac{AB - AD}{2} = \frac{DB}{2}$; and $CF = CB - BF = \frac{AB - DB}{2} = \frac{AD}{2}$; and conſequently $CE : CF :: DB : AD$ :: force applied at F: force applied at E. Q. E. D.

FIG. LXXXI. 311. LEMMA. *If from any point* P, *in the diagonal of a parallelogram*, ABCD, *two lines* Pm *and* Pq, *be drawn perpendicular to the ſides, the perpendiculars and ſides are inverſely as each other.*

DEM. Draw CE, CF, perpendicular to the ſides, and, from ſimilar triangles, $Pq : CF :: AP : AC :: Pm : CE$; therefore $Pq : Pm :: CF : CE :: CD\ (AB) : CB\ (AD)$ (ſim. triangles). Or, this follows from (298), where it is proved, that $AB \times Pm = AD \times Pq$, and conſequently, $Pm : Pq :: AD : AB$. Q. E. D.

FIG. LXXXII. 312. PROP. *If any three forces* W, P, Z, *whoſe magnitudes and directions are* AB, AC, AD, *act upon the lever* WP, *which is at reſt,* W *is to* P *inverſely as the perpendiculars let fall from* F *upon their directions.*

DEM.

DEM. The directions of these forces must be in the same plane, and meet in the same point, (216), and are respectively as the sides and diagonal of a parallelogram parallel to their directions; and $W : P :: AB : AC$ (196) $:: FL : FM$ (311). The effect will evidently be the same when a fulcrum is applied at F instead of the force Z. Q.E.D.

*Dr. Hamilton's demonstration of this proposition depends upon the same principle with the above. Let the forces W, P, Z, act upon the inflexible line WP at the points W, P, F, and let the directions of P and W meet in C, and the direction of a force Z, equivalent to them, must pass through C. Therefore P, W, Z, are to each other as the sides and diagonal of a parallelogram respectively parallel to their directions, FA, FB, FC, and $W : P :: AC : BC ::$ sin. $\angle AFC$ (FCB) $::$ sin. $\angle ACF :: FM : FL$. FIG. LXXXIII.

$W : Z :: AC : FC ::$ sin. $\angle FCB$: sin. $\angle FBC$ or $BCA :: P\mathcal{Q} : PN$, supposing $P\mathcal{Q}$ and PN to be perpendicular to the directions of Z and W.

$P : Z :: FA : FC ::$ sin. $\angle FCA$: sin. $\angle FAC$ or $ACB :: WG : WR$, supposing WG and WR to be perpendicular to the directions of Z and P.

The parts of W and P, which act in directions exactly opposite to those of Z, are found by resolving AC and $\mathcal{Q}A$ (BC) into two forces, one parallel to the right line joining W, P, as Am and Bn, and the other parallel to FC as $\mathcal{Q}m$, $\mathcal{Q}n$. The opposite parts Am, Bn, are equal and destroy each other, and the conspiring parts $\mathcal{Q}m + \mathcal{Q}n$ must be equal to Z; and consequently when two forces W and P are in equilibrio with a third force Z, and their directions are all parallel, FIG. LXXXIV.

$W : P :: \mathcal{Q}n : Cn :: PB : BC :: P\mathcal{Q} : \mathcal{Q}W$;
$W : Z :: Cm : \mathcal{Q}C :: CA : CW :: \mathcal{Q}P : PW$; and
$P : Z :: Cn : C\mathcal{Q} :: CB : CP :: W\mathcal{Q} : WP$.

†BALANCE.

313. DEF. *The ancient balance, commonly called the statera romana, or steelyard, is a lever of the first kind supported at the point* F, *placed near* PLATE XI. FIG. LXXXV.

* Essay on the Principles of Mechanics. † Helsham, Lect. VI. Muschenb. Ch. VIII. § CCCLXXXIII. Desaguliers, pag. 95.

near one extremity, about which the brachia FA, FN *equiponderate. On one ſide of* F *at the extremity* A, *an unknown weight,* W, *is ſuſpended, and on the other ſide a known weight,* P, *is moveable upon the arm* FN, *which is divided into parts equal to* FA, *each of which is alſo divided into* 10, 100, *&c. equal parts.*

314. Cor. If P be at x, the fourth diviſion from I, which is the nth diviſion from F, when an equilibrium obtains between it and W, and IK be divided into m equal parts, the weight of W will be equal to $P \times n + \frac{4}{m}$; for $W : P :: Fx : FA\,(274) :: FA \times n + \frac{4}{m} : FA :: n + \frac{4}{m} : 1$, and conſequently $W = P \times n + \frac{4}{m}$.

FIG. LXXXVI. 315. DEF. *The common, or modern, balance, or a pair of ſcales, is a lever of the firſt kind, as* AB, *ſupported at its biſection* F: *to the extremities* A *and* B *are ſuſpended baſins or ſcales, and the brachia and baſins, on each ſide of* F, *are ſuppoſed to equiponderate.*

316. Cor. 1. If any known weight, P, placed in one ſcale equiponderate with one unknown, W, placed in the other, the weight of W is known, being equal to that of P; for $P : W :: AF : BF\,(274) :: 1 : 1$, and conſequently $P = W$.

317. Cor. 2. A balance, whoſe arms are unequal in length, is fallacious; for if AF and BF be unequal, P and W, when in equilibrio, cannot be equal, for $W = \frac{P \times BF}{AF}$. But the relation of $BF : AF$ being known, W is alſo known.

318. Cor. 3. If a man, whoſe weight is equal to W, ſtanding in one ſcale and in equilibrio with P placed in the other, preſs the

Plate X.

FIG. LXXVII.

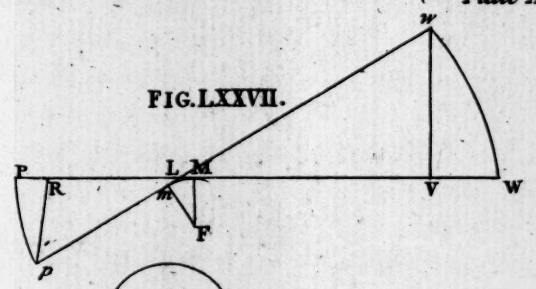


FIG. LXXVIII.

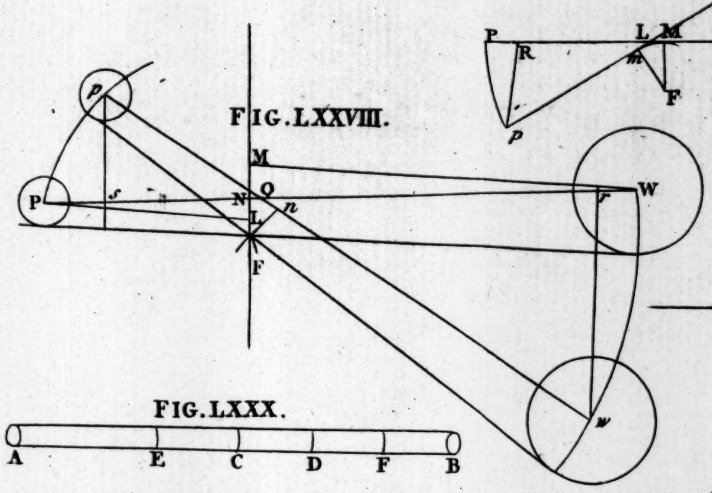


FIG. LXXIX.

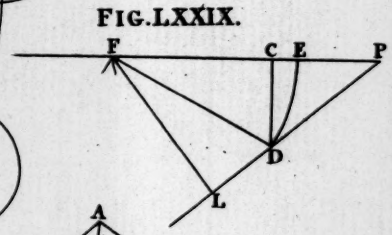


FIG. LXXX.

FIG. LXXXI.

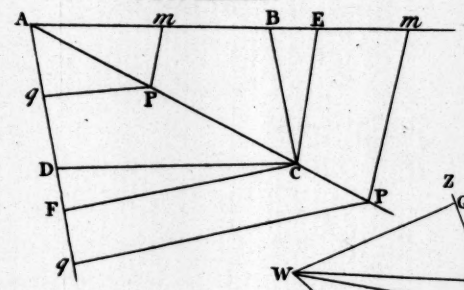


FIG. LXXXII.

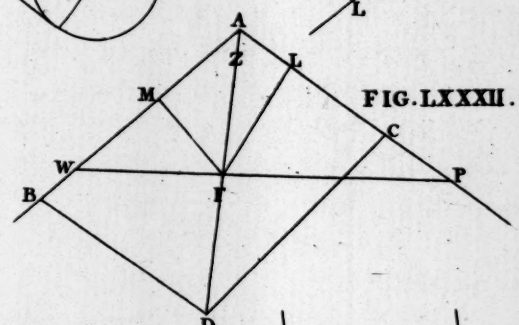


FIG. LXXXII.

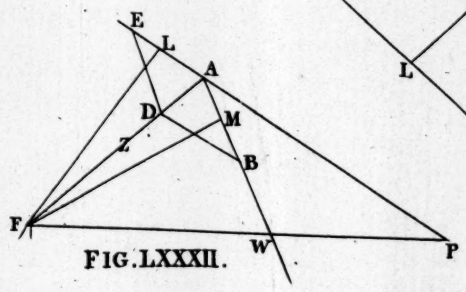


FIG. LXXXIII.

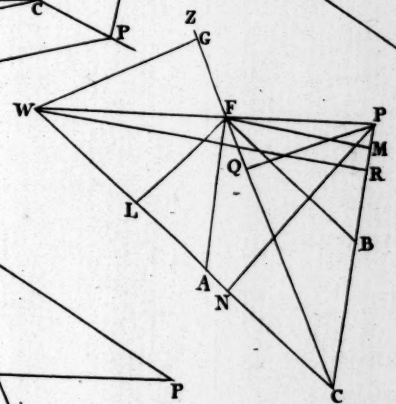
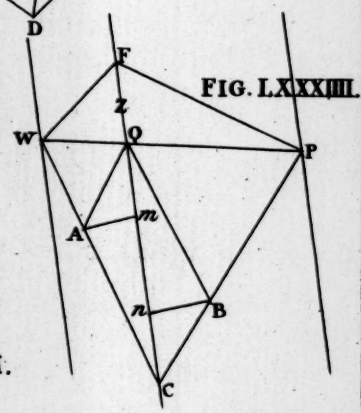


FIG. LXXXIII.




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the beam upwards in D with a force equal to Q, the diminution of W's moment is equal to $Q \times FD$; and becauſe the reaction at the ſcale is equal to Q, the encreaſe of W's moment is equal to $Q \times FA$, and conſequently W will deſcend with a force equal to $Q \times AD$. If the preſſure be upwards at E, W will deſcend with a force, reſulting from this preſſure, equal to $Q \times EF$, and, from the reaction, with a force equal to $Q \times FA$; and therefore the whole force of deſcent is equal to $Q \times EA$. When the preſſure is downwards at D, the encreaſe of W's moment is equal to $Q \times FD$, and the diminution of its moment $= Q \times FA$; and, conſequently, W will aſcend with a force equal to $W \times DA$. If the preſſure be downwards at E, the diminution of W's moment, or encreaſe of P's moment, is equal to $Q \times EF$, and a part, Q, of W's weight being transferred to E, the diminution of its moment, on that account, is equal to $Q \times FA$; and conſequently the whole diminution of W's moment, or force of P's aſcent is equal to $Q \times EA$.

319. Cor. 4. If the center of motion C be in the right line joining the centers of ſuſpenſion, the equilibrium of equal weights P and W will obtain in every poſition; the perpendiculars let fall from C upon the directions being always equal to each other. But when C is above or below WP, an equilibrium of equal weights does not obtain, unleſs WP coincide with the horizontal line AB. When WP coincides with AB, the perpendiculars let fall from C upon the directions of W and P are equal to GB and GA, CG being perpendicular to AB; but when the balance is in any other poſition WP, the perpendicular CI is greater than CH, becauſe gL, which is leſs than CI, is equal to gM, which is greater than CH. W will deſcend and continue to vibrate till its motion be deſtroyed by friction. This corollary is alſo deducible from (305). FIG. LXXXVII.

320. Cor. 5. If P and W be unequal, and C, be in the right line WP, the heavier of them will deſcend till WP be perpendicular to the horizon, or, if the center of motion be not in WP, till $P \times CH = W \times CI$.

321. Cor.

FIG. LXXXVI. 321. Cor. 6. In a balance whoſe arms are unequal, the weight of W may be ſtill aſcertained; for let W, ſuſpended at A, be in equilibrio with a known weight Q, ſuſpended at B, and $W \times FA = Q \times FB$, and, ſuſpended at B, let it be in equilibrio with a known weight R ſuſpended at A, and $W \times FB = R \times FA$; conſequently, by multiplying theſe equations together, $W^2 \times FA \times FB = Q \times R \times FA \times FB$, and $W = \sqrt[2]{Q \times R}$.

FIG. LXXXVIII. 322. *Cor. 7. If the beam of the balance be ſuppoſed to have weight and be ſimilar and homogeneous in every part, its center of gravity is in the biſection F; but if it be not homogeneous, or the center of motion be not in the biſection, let G be its center of gravity, and an equilibrium will obtain when $P \times PF + B$ (weight of the balance) $\times GF = W \times FW$.

323. Cor. 8. If from F, a ſtyle FD, perpendicular to WP, be raiſed, the equilibrium of the balance will be affected by it, except it be in an horizontal ſituation, the moment of the ſtyle being meaſured by its weight multiplied into the diſtance of its center of gravity from the line FH, perpendicular to the horizon. But the equilibrium is reſtored by continuing the ſtyle to the other ſide of F, ſo that the moments on each ſide may be equal and oppoſite.

324. Cor. 9. If the center of gravity of the balance, ſcales, and weights, be in the center of motion, F, an equilibrium obtains in every poſition of the balance; but if this center be above or below F, the balance cannot be quieſcent till the right line joining F and this center be perpendicular to the horizon. The beſt poſition therefore of the center of gravity is below F, and as little below it as poſſible, that the arcs deſcribed by it, during its vibrations, may be ſmall and ſoon deſcribed. The points of ſuſpenſion ſhould

* Theſe corollaries may be omitted till the chapter upon the center of gravity be read.

ſhould be in the ſame right line with the center of motion, which ought accurately to biſect their diſtance (317).

325. Cor. 10. The arms of the balance ſhould be as long as can be uſed conveniently; becauſe the moment of a given body varies as its diſtance from the fulcrum, and, therefore, the greater the diſtance, the more diſtinguiſhable will be the moment ariſing from any ſmall difference between *P* and *W*. And to diſtinguiſh very minute differences of weight, the friction upon the axis, in the motion of the beam and ſcales, ought to be as little as poſſible.

CHAP.

CHAP. VIII.

*WHEEL AND AXIS.

326. DEF. *A WHEEL and axis or axis in peritrochio, is a machine compoſed of a circular wheel, in whoſe center a cylindrical axis is inſerted and fixed; and the wheel revolving by the action of a power* P, *the axis, whoſe extremities are ſupported, revolves with it, and the rope, to which a body* W *is appended, is tied to the axis and wrapped round it during its motion.*

FIG. LXXXIX.

327. Cor. It is evident that every point of the axis *B*, *G*, &c. deſcribes a circle round its correſponding center *y*, *z*, &c. in the time of a revolution of the wheel; and that any points of the wheel and axis, *A* and *B* deſcribe ſimilar arcs of circles in the ſame time, and, conſequently, their velocities are as the peripheries or radii of the circle deſcribed by them.

328. PROP. *If the directions of* P *and* W *be perpendicular to the radii of the wheel and axis reſpectively, they are in equilibrio when* P : W :: *radius of the axis* : *radius of the wheel.*

DEM. The ſame power is required to ſupport *W* to whatever point of the axis it be applied, becauſe the diſtance from the correſponding center of motion is the ſame, and the wheel and axis may be reduced to a bent lever, and conſequently there will be an equilibrium when *P* : *W* :: *W*'s diſtance from the center of motion : *P*'s diſtance :: radius of the axis : radius of the wheel (277). Q. E. D.

This

* Keil's Phyſics, Lect. X. Helſham, Lect. VII. Muſchenb. Ch. VIII. Sect. CCCCXLIII. Emerſon, Prop. XXIV. Varignon, Tom. I. Sect. IV.

Otherwiſe:

This propoſition is uſually proved by the following proceſs: ſince the directions of P and W are perpendicular to their reſpective diſtances from their centers of motion, they are wholly efficient, and P's velocity is to W's velocity as the periphery of the wheel to the periphery of the axis, and conſequently, when there is an equilibrium, $P : W$ :: periphery of the axis : periphery of the wheel :: radius of the axis : radius of the wheel (272). Q. E. D.

329. Cor. 1. If the thickneſs of the rope, to which W is appended, be not inconſiderable, it ought not to be neglected; for, when one or more ſpires of the ropes are folded about the axis, the diſtance of W's direction from the center of motion is encreaſed, being equal to the ſemidiameters of the axis and ropes; and there is an equilibrium when $P : W$:: the diſtance of W's direction from the center of motion : ſemidiameter of the wheel.

330. Cor. 2. If P and W act in the ſame plane, and in the directions PD, and WD, meeting in D, and be in equilibrio, they are equivalent to a third force, or preſſure upon the axis at A, whoſe direction meets PD, and WD in D (216); and, producing PD, WD, theſe three forces are to each other, as the ſides DF, DE, and diagonal DG, of the parallelogram EF; therefore $P : W :: DF : DE$, or drawing AN, AM, perpendicular to WD and FD reſpectively, $P : W :: AN : AM$ (311). FIG. XC.

331. Cor. 3. The preſſure upon the axis at $A(Pr) : P :: DG : DF$ :: ſin. $\angle DFG$ or PDW : ſin. $\angle FGD$ or ADW; $Pr : W :: DG : DE$ :: ſin. $\angle DEG$ or PDW : ſin. $\angle DGE$ or ADP; and $P : W$ :: ſin. $\angle ADW$: ſin. $\angle ADP$. When the angle PDW is infinitely ſmall, or PD and WD are parallel, the perpendiculars AN, AM, are to each other as $AW : PA$.

FIG. XCI. 332. PROP. *In a combination of wheels* GH, FK, EL, *whoſe axes are* QM, RN, BD, *an equilibrium obtains when* P : W :: QD × RC × AB : DG × CF × AE.

DEM. Let $\mathcal{Q}$, R, W, be reſpectively in equilibrio with P, $\mathcal{Q}$, R; and $P : \mathcal{Q} :: \mathcal{Q}D : DG$, (328)

$\mathcal{Q} : R :: RC : FC$,

$R : W :: AB : AE$, and, componendo, $P : W :: \mathcal{Q}D \times RC \times AB : DG \times FC \times AE$. Q. E. D.

333. Cor. 1. If the ratios of $\mathcal{Q}D : DG$, $CR : CF$, $AB : AE$ be the ſame, and the number of wheels be n, $P : W :: \mathcal{Q}D^n : DG^n$; and if this given ratio be that of $1 : r$, $P : W :: 1 : r^n$ and $P = \frac{W}{r^n}$.

334. Cor. 2. If the peripheries $\mathcal{Q}M$, RN, were to touch the peripheries FK, EL, and operate upon them by means of teeth made in each, this analogy would ſtill obtain, and P would be to W as the product of the ſemidiameters of the axes, to the product of the ſemidiameters of the wheels. As if $X\Upsilon$ be a combination of wheels, and W be appended upon the axis ΥF, and the power P act at P; $P : W$ :: products of the ſemidiameters of the pinions B, D, F : product of the ſemidiameters of the wheels A, C, E.

FIG. XCII.

335. Cor. 3. Becauſe the number of teeth in the wheels and pinions are to each other as their peripheries, or radii, $P : W$ :: ſemidiameter of the axis to which W is appended multiplied into the number of teeth in the pinions : the length of the lever where P acts multiplied into the number of teeth in the ſeveral wheels.

336. Cor. 4. The number of revolutions of a pinion or wheel being inverſely as the time of one revolution, or inverſely as the periphery or number of teeth in it, the number of revolutions of

the wheel where *P* acts, is to the number of revolutions, in the ſame time, of the axis to which *W* is appended, as the product of the number of teeth in the wheels to the product of the number of teeth in the pinions.

337. PROP. *If a wheel with teeth* C D A *revolve about* C, *and impel the wheel* S B A *round with it, by the action of teeth* B D, b d, *&c. upon* B *and* b, *whoſe curvature is ſuch that* B, b, *deſcribe the lines* BD, bd, *whilſt they deſcribe* BA *and* bA *reſpectively, the moments of theſe wheels are equal.* FIG. XCIII.

DEM. Becauſe the points *B* and *D* come into contact at *A*, at the ſame time, every part of *BA* is applied to *AD*, and is therefore equal to it, and conſequently *B* and *D* move towards it with equal velocities; and therefore if equal weights act at *B* and *D* perpendicular to *SB*, *CD*, they muſt be in equilibrio. Q. E. D.

338. Cor. 1. If *C D n B* be a bent lever whoſe fulcrum is at *C*, equal forces, acting at *D* and *B* in directions perpendicular to *CD*, *SB*, would keep the levers *C D n B* and *S B* in equilibrio.

339. Cor. 2. Becauſe the point *B* moves over *B D* whilſt the arc *BA* rolls over *DA*, the figure of the tooth *B n D* is an epicycloidal arc; and the effect is the ſame whether *B* deſcribe the concave or convex ſide of *B n D*, and conſequently whether the wheel *C D A* impel *S B A* by the action of the convex ſide of the tooth upon *B*; or *S B A* impel *C D A* round with it by the action of *B* upon the concave ſide of the tooth *B n D*.

340. Cor. 3. If any number of epicycloidal teeth *DB*, *db* and *Aa*, be inſerted in the periphery *AD*, at equal diſtances from each other, and teeth *B*, *b*, *A*, be inſerted in the other wheel *AB*, at equal diſtances, and *Ab* = *Ad*, and *AB* = *AD*, the teeth

B, *b*, *A*, will all act together with equal moments; for because $Ab = Bb$, the velocities and consequently moments of *B* and *b* are equal, and they act together, because *b* is always found in *bd* and *B* in *BnD*.

341. Cor. 4. The effect is the same when the epicycloidal teeth are upon the periphery of the wheel *SBA*; for, whilst *B* and *D* move to *A*, *B* describes the epicycloid *BnD*, and *D* describes another epicycloid *DmB*, whose base is *BA*.

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342. The teeth should not act upon each other before they arrive at *SC*, joining their centers, and the machine is more or less complete according to the number of teeth acting together. The action of any tooth should not cease before that of the succeeding tooth begins.

PULLEY*.

FIG. XCV. 343. DEF. *A pulley is a small circular wheel, as* mEn, *revolving about an axis passing through its center, by the action of a power which is applied to a rope passing over the pulley.*

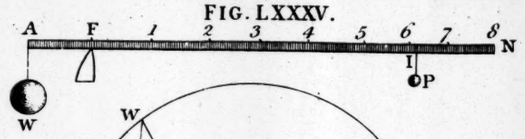
344. PROP. *In a single fixed pulley, an equilibrium obtains when the power* P, *is equal to the weight* W.

DEM. When a rope is stretched and quiescent, it is evident that the tension of every part is the same, otherwise motion would ensue; therefore the tension of *Pm* is equal to that of *Wn* and $P = W$. Q.E.D.

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* Muschenbroek, Ch. VIII. Sect. CCCCXCIV. Varignon, Tom. I. Sect. III. Keil's Physics, Lect. X. Helsham, Lect. VII. Hamilton's Principles of Mechanics, pag. 162. Emerson, Prop. XXVII. Desaguliers, pag. 99.

FIG. LXXXV.



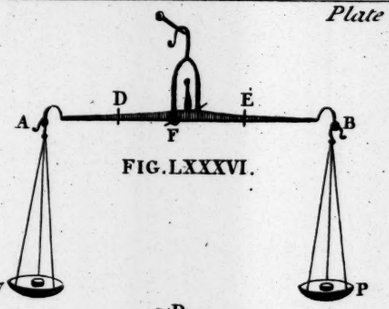


FIG. LXXXVI.

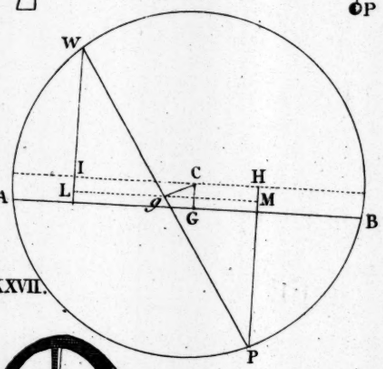


FIG. LXXXVII.

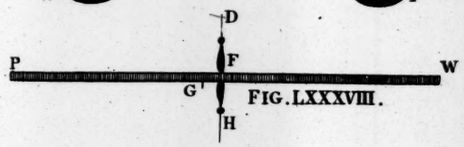


FIG. LXXXVIII.

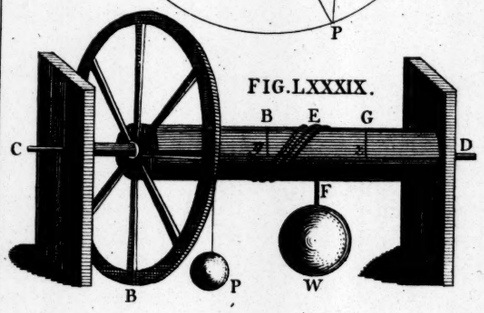


FIG. LXXXIX.

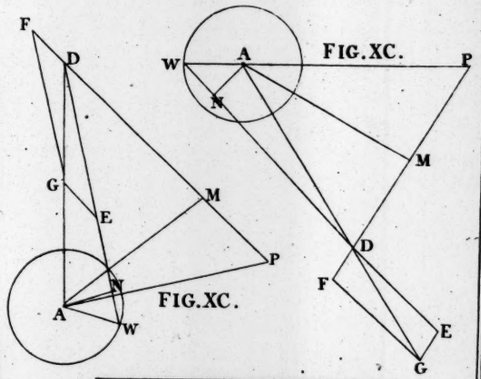


FIG. XC.

FIG. XC.

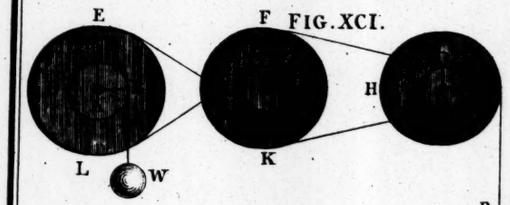


FIG. XCI.

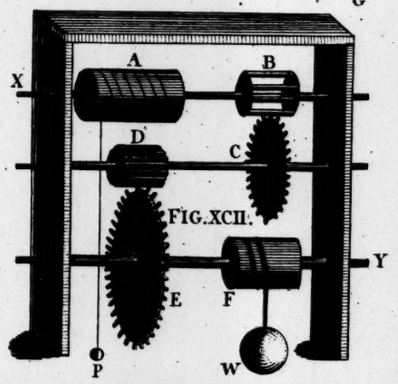


FIG. XCII.

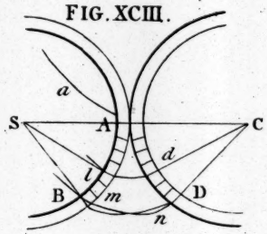


FIG. XCIII.


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345. This propoſition is ſometimes proved by conſidering the pulley as a lever; for the moments of P and W being the ſame to whatever parts of their directions they are applied, but, if applied at m and n, mEn is a ſtraight lever whoſe center of motion, E, is in its biſection, and conſequently when there is an equilibrium $P = W$ (274). And the concluſion thus deduced is certainly ſatisfactory without any other demonſtration.

346. Cor. 1. If the ſame rope paſs over any number of fixed pulleys, P is equal to W when there is an equilibrium, becauſe the tenſion of the rope is, in every part, the ſame.

347. PROP. *If the weight*, W, *be ſuſtained by a power* P, *applied to a rope paſſing over a moveable pulley* E, P : W :: 1 : 2. FIG. XCVI.

DEM. Let the tenſion of the rope $PA = m$, and that of BC, and DF, is each equal to m, and conſequently the tenſion of EW is equal to $2m$; therefore $P : W :: m : 2m :: 1 : 2$. Q. E. D.

348. Cor. 1. If W be ſupported by P, applied to a rope paſſing over any number of moveable pulleys (n), $P : W :: 1 : 2n$; for the number of ropes ſupporting the weight is equal to $2n$, each ſupports an equal part of it, and the tenſion of the rope, to which P is applied, is equal to that of one of them.

349. Cor. 2. If W be ſupported by P, applied to a rope paſſing over each pulley in two blocks, to the lower of which W is appended; $P : W ::$ 1 : number of ropes at the lower block. For all the ropes A, B, C, D, E, ſupport W, and each ſupports an equal part, becauſe their tenſion is the ſame, and the tenſion of each is equal to that of F to which P is applied. FIG. XCVII.

350. Cor.

350. Cor 3. If a moveable pulley, L, be annexed to this ſyſtem, P will ſupport a weight equal to $2 \times W$; for the tenſions of the ropes G and H are equal and they both ſuſtain the weight, which is therefore equal to $2 \times W$.

351. Cor. 4. It is evident that P acquires no moment or quantity of motion by this diſtribution of the weight amongſt a number of pulleys; for the velocity of P is to that of W as the number of ropes, ſupporting the weight, to unity. If W be elevated through any ſpace equal to s, any point in each of the ropes ſupporting it, muſt move through a ſpace equal to s, and P conſequently through a ſpace equal to s multiplied into the number of ſtrings.

FIG. XCVIII.

352. PROP. *If the body* W *be ſupported by the power* P, *in a ſyſtem compoſed of one fixed, and any number of moveable pulleys, each having a ſeparate ſtring*, P : W :: *unity* : *that power of* 2 *whoſe index is the number of moveable pulleys.*

DEM. Let the tenſion of the ſtring, to which P is applied, be equal to m, and the tenſion of A and B is each equal to m; that of C and D is each equal to $2m$; that of E and F is each equal to $4m$; and that of GH to $8m$; therefore $P : W :: m : 8m :: 1 : 8$; and, if the number of moveable pulleys be equal to n, it is evident that the tenſion of the ſtring ſupporting W is equal to the nth term of the geometric ſeries 2, 4, 8, &c. or to 2^n and $P : W :: 1 : 2^n$. Q. E. D.

353. Cor. 1. $P \times 2^n = W$ and $P = \frac{W}{2^n}$; and if the number of moveable pulleys and P or W be given, the number of moveable pulleys, or n, may be found.

354. Cor.

FIG. XCIV.

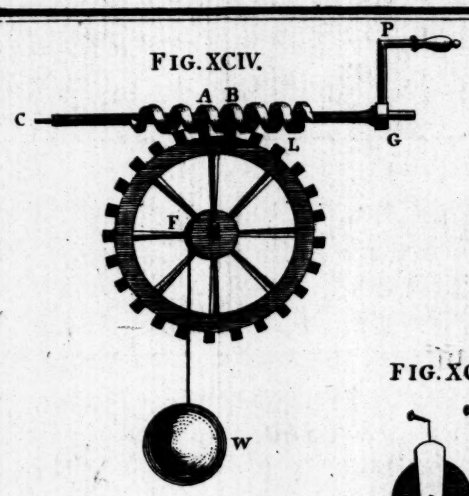


FIG. XCV.

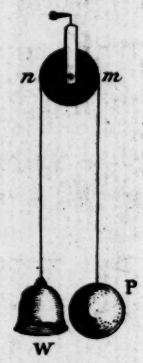


FIG. XCVI.

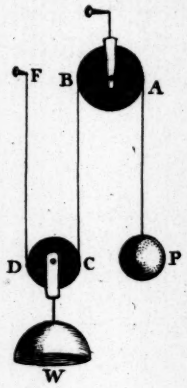
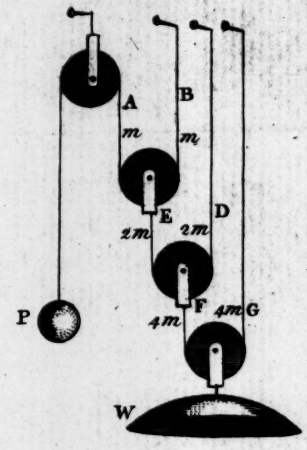


FIG. XCVII.



FIG. XCVIII.



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354. Cor. 2. The parts of W, ſuſtained by the ſeveral moveable pulleys, &c. are to each other as 2, 4, 8, 16, &c.

355. PROP. *If the body,* W, *be ſuſtained by the power,* P, *in a ſyſtem of pulleys where the rope paſſing over each pulley is immediately fixed to* W, P : W :: *unity : that power of two, diminiſhed by unity, whoſe exponent is the number of ropes, or number of moveable pulleys encreaſed by unity.* FIG. XCIX.

DEM. Let the tenſion of the rope, to which P is applied, be equal to m, and it is evident that the tenſions of the ropes G, B, D, F, are equal to m, $2m$, $4m$, $8m$, reſpectively; but theſe ropes entirely ſupport W, and conſequently $P : W :: m : \overline{1 + 2 + 4 + 8} \times m :: 1 : 15$. If the number of ropes fixed to W be equal to n, it is evident that $P : W :: 1 : 1 + 2 + 4 + 8$, &c. continued to n terms $:: 1 : 2^n - 1$. Q. E. D.

356. Cor. 1. $P \times \overline{2^n - 1} = W$ and $P = \frac{W}{2^n - 1}$; and if any two of theſe magnitudes P, W, or n, be given, the other may be found.

357. Cor. 2. m, $2m$, $4m$, $8m$, &c. expreſs the ratio of the parts of W reſpectively, ſupported by G, B, D, F, &c.

358. Cor. 3. If the rope, to which P is applied, inſtead of being fixed to W paſs over a pulley to which W is appended, $P : W :: m : 4m :: 1 : 4$; for W is ſupported by the ropes E, G, F, whoſe tenſions, compared with that to which P is applied, are reſpectively equal to m, $2m$, m, and the ſum of their tenſions $= 4m$. FIG. C.

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FIG. CI. 359. When the directions of the ropes PA, QB, to which two powers Q and P are applied, and the direction in which W acts, are parallel to each other, as is ſuppoſed in the preceding propoſitions, it is evident that W is exactly equal to $P + Q$, becauſe they juſt ſupport it, and their force is all effective, no part being loſt by obliquity of direction. This alſo appears by conſidering BCA to be a lever, for (287) $W : P + Q :: CB + CA : AB$, and conſequently $P + Q = W$ and $P = Q = \frac{W}{2}$. But if the direction in which P acts be changed to ZA, touching the pulley in D, motion will enſue, parallel to the horizon, as P acts partly in that direction, and the quantity and direction of Q muſt be changed to reſtore the equilibrium. It is evident that the pulley cannot be at reſt, till the horizontal parts of the forces P and Q be equal to each other, and the parts contributing to the ſupport of W be each equal to $\frac{W}{2}$. This is alſo obvious from (216); for the pulley BCA is acted upon by three forces, whoſe directions are not parallel, and is quieſcent; and theſe forces are therefore in the ſame plane, their directions meet in the ſame point D, and are conſequently equally inclined to the direction of W, or to the horizontal line BA.

FIG. CII. 360. PROP. *If* W *be ſupported by two powers* P *and* Q, *whoſe directions touch the pulley in* A *and* B, W : P *or* Q :: *ſin.* ∠ *contained between* PA *and* QB : *ſin. of half that angle.*

DEM. Becauſe the pulley is acted upon by three forces P, Q, W, and kept at reſt, and their directions are perpendicular to the ſides of the triangle CBA, $W : P :: BA : CA$, and $W : Q :: BA : CB$; therefore $W : P$ or Q :: ſin. ∠ BCA or its ſupplement to two right angles BDA : ſin. ∠ BDC (equal to the ∠ CBA or CAB). Q. E. D.

361. Cor.

361. Cor. 1. Becauſe $P : W :: CA : BA$ and $Q : W :: CB : BA$; therefore $P + Q : W :: CA + CB : BA$.

362. Cor. 2. If AB be an arc of 60 degrees, or the angle ADB be equal to 120 degrees, AB is equal to AC or CB, and conſequently $W = P$ or Q.

363. Cor. 3. Becauſe $P : W :: AC : AB$, $P = \frac{W \times AC}{AB}$. If therefore the arc AB vaniſh, or the angle ADB be equal to 180°, $P = \frac{W \times AC}{0}$, or P is infinitely great compared with W. As the arc AB encreaſes to a ſemicircle, P decreaſes and becomes the leaſt poſſible when it is a ſemicircle, becauſe the chord AB is then the greateſt poſſible. In this caſe PA and QB are parallel to each other and to CD, and $W = P + Q$, becauſe $AB = AC + CB$, and conſequently $P = \frac{W \times AC}{AB} = \frac{W}{2}$. As the arc BA encreaſes beyond the ſemicircle, P encreaſes and becomes infinite when it is equal to the periphery.

364. Cor. 4. If P be finite, W is either finite or evaneſcent; for $W = \frac{P \times AB}{AC} = 0$, when AB vaniſhes, and is finite when AB is finite.

365. PROP. *If* W *be ſuſtained by* P *in a ſyſtem of moveable pulleys* v, y, x, *each of which has a ſeparate rope, and the angles contained by the directions of the ropes be* FSE, DTC, BRA; W : P :: EF × DC × BA : vE × yC × xA. FIG. CIII.

DEM. Let W, T, R, P represent the tensions of the strings vW, yT, xR, PG respectively; and (360) $W : T :: EF : vE$

$T : R :: CD : yC$

$R : P :: AB : xA$; and consequently

$W : P :: EF \times CD \times AB : vE \times yC \times xA$.

Q. E. D.

366. Cor. 1. If the pulleys and the angles FSE, DTC, BRA be equal, $W : P :: EF^3 : vE^3$; and, if the number of moveable pulleys be equal to n, $W : P :: EF^n : vE^n$.

367. Cor. 2. If the directions of the ropes become parallel, $W : P :: 2Ev \times 2yC \times 2xA : Ev \times Cy \times Ax$, and, when the pulleys are equal, $W : P :: 2 \times 2 \times 2 : 1$; or as that power of two, whose exponent is the number of moveable pulleys, to unity, which coincides with (352).

368. Cor. 3. If the tension of the rope PG, to which P is applied, be equal to m, $R = \frac{m \times AB}{Ax}$, $T = \frac{m \times AB \times CD}{Ax \times Cy}$, $W = \frac{m \times AB \times CD \times EF}{Ax \times Cy \times Ev}$.

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369. The conclusions, derived from considering the tensions of the several ropes in any system of pulleys, may also be investigated by supposing the moments of P and W to be equal, the velocities involved in these moments being reduced to opposite directions. 1. In fig. 99. the ropes being all parallel to the direction in which W acts, the velocities of P and W are in opposite directions and entirely efficient; and, when an equilibrium obtains, the product of P and its velocity is equal to the product of W and its velocity: for let $Ll = Mm = Nn = Oo = v$, and if the point L be

be elevated to l, through a ſpace equal to v, the pulley EF will deſcend through a ſpace equal to v, and any point of the rope E through a ſpace equal to $2v$, and, when the point M is elevated to m, through $3v$; therefore the pulley BA will deſcend through a ſpace equal to $3v$, and when the ropes A and B are ſtretched, A muſt deſcend through a ſpace equal to $6v$, and when the point N is elevated to n, through $7v$, &c.; conſequently the velocities of W and the ſeveral ſtrings E, C, A, P, are reſpectively as v, $3v$, $7v$, $15v$, and when an equilibrium obtains $P : W :: 1 : 15$. 2. In fig. 98. let W aſcend through a ſpace equal to v, and the ropes F, E, A will evidently be elevated through ſpaces equal to $2v$, $4v$, $8v$, reſpectively, and if the number of moveable pulleys be equal to n, it is clear that the velocity of P : velocity of $W :: 2^n \times v : v :: 2^n : 1$, and when there is an equilibrium P and W are inverſely as their velocities, or as $1 : 2^n$. 3. If the ropes, ſuſtaining the pulleys be not parallel to each other; let EFV be a pulley ſuſtained by two powers E and F, whoſe directions meet in S; and if W be raiſed through a very ſmall ſpace equal to SV, E and F, acting parallel to ES and FS, will deſcribe the ſpaces SE and SF reſpectively, and the velocities of W, E, F, are to each other as SV, SE, SF. Reſolve SF and SE, each into two, Sm, mF, and Sm, mE, of which mF and mE being equal and oppoſite deſtroy each other, and the remainders are wholly efficient; therefore, in an equilibrium, $\overline{E+F} \times Sm = W \times SV$; but $\overline{E+F} \times SF : \overline{E+F} \times Sm$ (or $W \times SV$) $:: SF : Sm : FV : Fm$, and $\frac{\overline{E+F} \times SF}{2} (E \times SE) : W \times SV :: \frac{FV}{2} : Fm :: FV : FE$, or the power or tenſion at E is to the moment of W as $EV : FE$, which coincides with what is proved in (360).

FIG. CIV.

INCLINED PLANE.

PLATE XIII. FIG. CV.

370. DEF. *A plane ſurface inclined, in any angle, to an horizontal plane, is called an inclined plane. If* BA *be drawn upon the inclined plane, and* BC *upon the horizontal plane, from the ſame point* B *of their common interſection, perpendicular to it, the angle* ABC *is the angle of elevation or inclination; and if* PW *be the direction of the power, the angle* PWA *is called the angle of traction.*

371. PROP. *If a body, whoſe weight is* W, *be juſt ſupported upon the inclined plane* AB *by a power* P, *acting in the direction* PW; P : W :: PW : PC *(ſuppoſing* PC *to be perpendicular to the horizontal line* BC) :: *ſin. ∠ inclination* ABC : *coſ. ∠* AWP.

DEM. The weight *W*, acting in the direction *WL* or *PC* perpendicular to the horizon, is ſupported by the power *P*, and reaction of the plane, acting reſpectively in the directions *PW*, and *WC* perpendicular to the plane; and conſequently *P*, *W*, and preſſure upon the plane, are to each other reſpectively as the ſides *PW*, *PC* and *WC* of the triangles *PWC* (196), and *P* : *W* :: *PW* : *PC* :: ſin. ∠ *PCW* or *ABC* : ſin. ∠ *PWC* or coſ. of ∠ *PWA*. Q. E. D.

Another demonſtration:

FIG. XVI.

Let *WZ*, repreſenting the weight of *W*, or its tendency to deſcend in a direction perpendicular to the horizon, be reſolved into two, one *SZ* perpendicular to the plane, and the other *WS* parallel to it. *WS* repreſents the tendency of *W* to deſcend upon the plane, and, taking *Wx* = *WS*, and drawing *xE* perpendicular to the plane, any force *WD*, *WE*, &c. drawn from *W* and terminated by *xE*, will be in equilibrio with *W*, becauſe the only efficient

Plate XIII

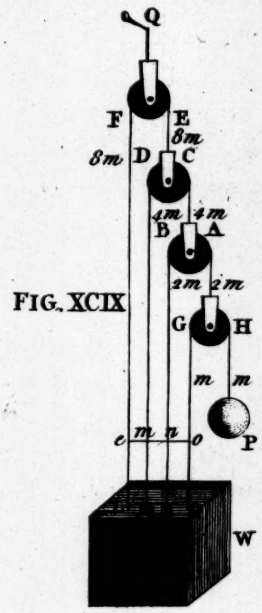


FIG. XCIX

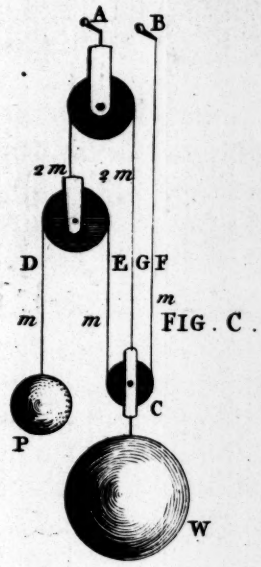


FIG. C.

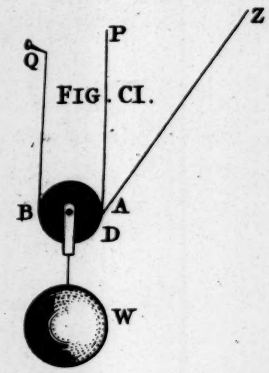


FIG. CI.

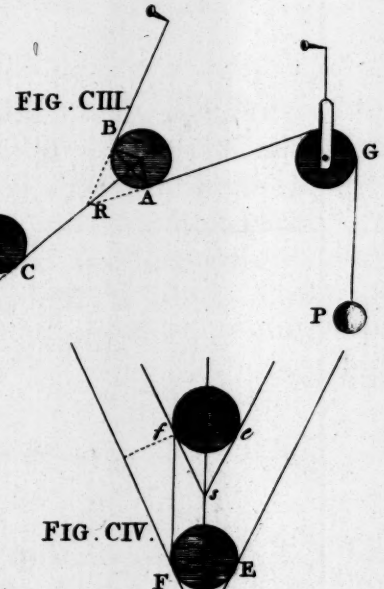


FIG. CIII.

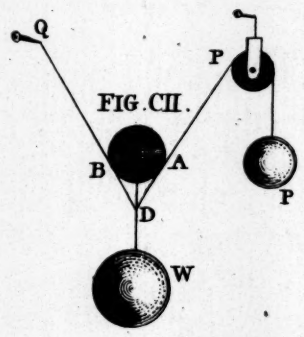


FIG. CII.

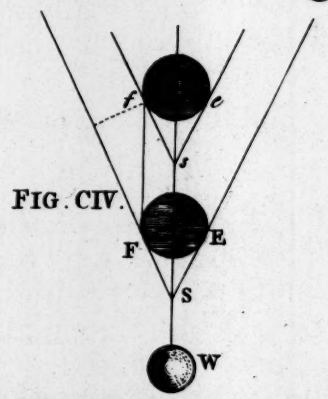


FIG. CIV.


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cient part of WE, reſulting from reſolution, is equal and oppoſite to WS; but $P : W :: WE : WZ :: \frac{WX \times \text{rad.}}{\text{cof. } \angle EWX} : \frac{WS \times \text{rad.}}{\text{ſin. } \angle SZW} ::$ ſin. $\angle SZW$ or ACB : cof. $\angle EWA$.

372. Cor. 1. The ſame force, acting in parallel directions, is required to ſupport the ſame weight upon every part of the plane; becauſe the angle of traction is the ſame, and $P = \frac{W \times \text{ſin. } \angle \text{ inclin.}}{\text{cof. } \angle \text{ of traction}}$, which is a given quantity.

373. Cor. 2. Becauſe $P = \frac{W \times \text{ſin. of } \angle \text{ inclin.}}{\text{cof. } \angle \text{ traction}}$, if W and the plane be given, P is the leaſt poſſible when the coſine of the angle AWP is the greateſt poſſible, or when PW is parallel to the plane: as the angle PWA encreaſes from hence, its coſine decreaſes, and conſequently P encreaſes, and becomes equal to W, when PW is perpendicular to the horizon, and infinitely greater than W when PW coincides with WG or WC. FIG. CV.

374. Cor. 3. When PW is parallel to the plane, $P : W ::$ ſin. $\angle$ of inclin. : radius :: $AC : AB$; when it is parallel to the baſe, $P : W ::$ ſin. $\angle$ of inclin. : ſin. $\angle BAC$ (this angle being the complement of the angle of traction) :: $AC : BC$.

375. Cor. 4. If AB be perpendicular to the horizon, and the angle $PWA = 0$, $P = W$; for the cof. of $\angle PWA =$ ſin. of $90° =$ ſin. of $\angle$ of inclination. If, in this ſuppoſition, PW be parallel to the horizon, P is infinite; for cof. of $\angle$ of traction $= 0$.

376. Cor. 5. Let Pr repreſent the preſſure upon the plane, and $Pr : W ::$ ſin. $\angle WPC$: ſin. $\angle PWC$ or cof. $\angle$ of traction, and $Pr = W \times$

$\frac{W \times \text{sin. } \angle WPC}{\text{cos. } \angle PWA} = 0$, when PW is perpendicular to the horizon, and infinite when the $\angle PWA$ is a right angle.

377. Cor. 6. Let CI be perpendicular to P's direction, and the ſides of the triangle CIB, CI, CB, and IB, are reſpectively perpendicular to the directions of P, W, and preſſure upon the plane, and conſequently P, W, and Pr, are reſpectively as CI, CB and IB.

378. Cor. 7. If WCP were an angular balance revolving upon the fulcrum C, and acted upon by two forces in the directions PW and WL, an equilibrium would obtain between them when $P : W$ inverſely as the perpendiculars let fall from C upon their directions, or $P : W :: CL : CK$ (272); but $CL : CK ::$ ſin. $\angle CWL$ or WCP : ſin. $\angle CWK$ or $CWP :: PW : PC$.

FIG. CVII. 379. Cor. 8. If P and W be in equilibrio, their perpendicular velocities are to each other inverſely as their magnitudes; for let W deſcend through an infinitely ſmall ſpace WA, and its perpendicular deſcent is WY, and P's perpendicular aſcent is equal to the difference between PA and PW, or Am, if Em be perpendicular to PA. Draw FDn perpendicular to PA, and DZ to FA, and $Am : WY ::$ P's velocity : W's velocity :: $An : DZ :: AF : DF$ (ſim. triangles) :: $W : P$ (377).

FIG. CVIII. 380. PROP. *If two bodies* W *and* V, *placed upon the inclined planes* AB, AD, *ſupport each other by means of a rope paſſing over the pulley* P, V : W :: *cof.* $\angle$ AVP $\times$ *ſin.* $\angle$ ABC : *cof.* $\angle$ PWA $\times$ *ſin.* $\angle$ ADC.

DEM. The tenſion of the rope VPW is every where the ſame, or the ſame power, P, is required to ſupport V and W; but

P :

$F : W ::$ ſin. $\angle ABC :$ coſ. $\angle PWA$; and
$V : P ::$ coſ. $\angle AVP :$ ſin. $\angle ADC$; therefore
$V : W ::$ ſin. $\angle ABC \times$ coſ. $\angle AVP :$ coſ. $\angle PWA \times$ ſin. $\angle ADC$.
Q. E. D.

381. Cor. 1. If PV and PW be parallel to AD and AB reſpectively, $P : W :: AC : AB$,
$V : P :: AD : AC$; therefore
$V : W :: AD : AB$.
This concluſion is alſo derived from the general expreſſion, the coſines of the angles PVA and PWA becoming AD and AB.

382. Cor. 2. If PV and PW be parallel to the baſes of the planes, $P : W :: AC : BC$,
$V : P :: DC : AC$, and
$V : W :: DC : BC$.
This alſo follows from the general expreſſion, the complements of the $\angle$s AVP and AWP, being, in this ſuppoſition, reſpectively equal to DAC and BAC.

WEDGE.

383. DEF. *A wedge is a hard body, generally of a triangular priſmatic figure as* AF, *which is generated by the motion of the triangle* AED *upon the right line* EF *always perpendicular to its plane.* FIG. CIX.

If AED be an iſoſceles triangle, it is called an iſoſceles wedge, if ſcalene, a ſcalene wedge. $ABCD$ is the back of the wedge, upon which a force is impreſſed uſually by percuſſion; and $ABFE$, $DEFC$, are the ſides of the wedge, upon which the reſiſtances of wood, &c. act and counterpoiſe the force of percuſſion. The angle AED is the vertical angle of the wedge.

384. PROP. *If two equal reſiſtances, acting in equal angles upon the ſides of an iſoſceles wedge* ABV, *be in equilibrio with a power,* P, *acting*

FIG. CX.

ing in a direction perpendicular to the back; P is to the resistances, as the rectangle of the sines of half the vertical angle of the wedge and the inclination of the resistance to the sides, to the square of the radius.

DEM. Let the directions and quantities of the resistances be CD and Cd; resolve each into two, DE and de, in the directions of the sides, and CE, Ce, perpendicular to them, and DE, de, are lost by their obliquity. Resolve EC, eC into two forces, EF, eF, parallel to the base, and FC, FC perpendicular to it; EF, ef, being equal and opposite, destroy each other, and the forces $2FC$, being exactly opposite to, and in equilibrio with P, are equal to it. But $FC : EC ::$ sin. $\angle FEC$ or $\angle BVC$: rad.

$EC : DC ::$ sin. $\angle EDC$: rad.; and

ex æquo $FC : DC ::$ sin. $\angle BCV \times$ sin. $\angle EDC$: $\overline{\text{rad.}}^2 :: 2FC$ or $P : 2DC$ or resistances. Q. E. D.

385. Cor. 1. If the resistances be perpendicular to the sides, the sine of the angle ADC becomes the radius; and consequently P : resistances :: sin. $\angle CVA$: radius :: $AC : AV :: AB : AV + BV$.

386. Cor. 2. If the resistances be perpendicular to the axis, P : resist. :: $BC \times CV : BV^2$; and, when the angle AVB is equal to two right angles, P : resist. :: $BC \times 0 : BC^2 :: 0 : BC$, or P is infinitely less than the resistances.

387. Cor. 3. When the resistances are perpendicular to the back AB, the angle $EDC = BVC$, and P : resist. :: sin.$^2 \angle BVC$: rad.$^2 :: BC^2 : BV^2$. If the angle BVA be equal to two right angles $BC = BV$, and $P =$ resistances; and if the angle BVA be diminished without limit, the resistances are encreased without limit.

388. Cor.

388. Cor. 4. If P be the ſame in the two ſuppoſitions of cor. 1. and 2; reſiſt. in cor. 1 : reſiſt. in cor. 2 :: $CV : BV$.

389. Cor. 5. If the reſiſtances be given in cor. 2, P is the greateſt poſſible when the angle BVA is a right angle.

390. Cor. 6. If the quantities of the reſiſtances be given, and their directions be variable; P is the greateſt poſſible in the common ſuppoſition of cor. 1.; for it varies as the ſine of the angle EDC, which is the greateſt when that angle is right.

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391. The relation, ſubſiſting between the power and reſiſtances in the wedge, has been traced through various proceſſes and modes of reaſoning, by different philoſophical writers*; and different, and frequently contradictory, concluſions have been deduced from different demonſtrations, ſome of which muſt conſequently be erroneous. As in all equilibria of forces, they muſt either obliquely, or directly, oppoſe each other, and their intenſities, eſtimated in oppoſite directions, muſt be equal; one ſource of error hath reſulted from a miſapplication of this equality of moments, which is univerſally true and will never lead to falſe concluſions, if the velocities be reduced to oppoſite directions, and the efficient parts only of the moments be ſuppoſed equal. 1. Let two equal reſiſtances R and r act perpendicularly upon the ſides of an iſoſceles wedge ABD, and be in equilibrio with a power P acting perpendicularly upon the baſe AB; and taking Dd, the continuation of PD, very ſmall, and drawing dE, de parallel, and DE, De perpendicular, to the ſides, the velocities of P, R, r, are to each other as Dd, DE, De reſpectively, theſe being deſcribed by them in the ſame time. Reſolve DE, De into two, EL and eL parallel

FIG. CXI.

* Rohault Not. ad a. 9. Maclaurin's Newt. Chap. III. Art. 21. Graves, Left. I. C. X. Deſaguliers, p. 107. Emerſon, Prop. 30. Muſchenbroek, Sect. CCCCLXIII. and CCCCCXCI.

rallel to the base, which being equal and opposite destroy each other, and DL, DL perpendicular to the base, which being opposite to P's direction, are wholly efficient; and because there is an equilibrium $\overline{R+r} \times DL = P \times Dd$, but $\overline{R+r} \times DL$ or $P \times Dd : \overline{R+r} \times DE :: DL : DE :: AP : AD :: AB : AB + AD$, the same as (385).

When R and r act in any other direction, the relation of their moments to that of P is discoverable by a similar process. If they be parallel to the base, draw Mdm parallel to their direction, and the velocities of P, R and r, are respectively as Dd, DM and dm; and resolving DM, dm into two DE, de perpendicular, and ME, me parallel, to the sides, the last not being opposed to the sides are quite inefficient; and repeating the process above $\overline{R+r} \times DL = P \times Dd$; and $P \times Dd : \overline{R+r} \times DE :: AP : AD$

$$\overline{R+r} \times DE : \overline{R+r} \times DM :: DE : DM :: PD : AD$$

and $P \times Dd : \overline{R+r} \times DM :: AP \times PD : AD^2$; which is the same as (386).

A different analogy is investigated by Rowning, from the same principle, viz. the power : the resistances :: $AP : PD$.

2. Another source of error is an improper use of this principle, "that if three forces act upon a body, which remains at rest, they are to each other as the three sides of a triangle, respectively parallel or perpendicular to their directions;" which certainly is untrue, if any parts of them be inefficient and lost by obliquity of direction; for the force FG will have exactly the same effect upon the sides of the wedge AD with FH, FI, or the perpendicular FL, supposing GL to be parallel to AD, and of the parts, resulting from a resolution of each into two forces, one perpendicular to, and the other coincident with, the sides, these last, Fx, &c. are lost. When the wedge does not fill the cleft, and the resistances act perpendicularly upon its sides, Maclaurin and Varignon have applied the above principle and deduced different conclusions, both of which cannot be true. Dr. Hamilton, Gravesands, Desaguliers, Emerson, Muschenbroek assert, that the power is to the resistances as half the back is to the height, and Keil, Whiston

and Nicholſon, aſſign this ratio to be that of the whole back to the height. The firſt analogy coincides with this general theorem, "that the power is to the reſiſtances, when in equilibrio, as a line drawn from the biſection of the baſe to one ſide, parallel to the reſiſtance upon that ſide, to the height of the wedge;" and this theorem is preſumed to be erroneous for the following reaſons.

1. If the directions of the reſiſtances be parallel to the axis, their ſum is equal to the power, according to the theorem, which cannot be true; becauſe the power acting perpendicularly is wholly efficient, but part of the reſiſtance is loſt by obliquity of direction.

2. If the vertical angle of the wedge be diminiſhed without limit, a line drawn, from the biſection of the baſe to a ſide, parallel to the reſiſtance, accedes to equality with the height when the direction of the reſiſtance is parallel to the height, and the power and reſiſtance become equal, according to the theorem; but, the power continuing the ſame, the directions of the reſiſtances accede to a parallelism with the ſides, and they become wholly inefficient.

3. When the reſiſtances act perpendicularly to the axis, and the vertical angle of the wedge accedes to two right angles; according to the theorem, the ratio of the power to the reſiſtances encreaſes without limit, which cannot be true, becauſe the inefficiency of the reſiſtances evidently encreaſes without limit, and conſequently the ratio of the reſiſtances to the power encreaſes without limit, which is directly the reverſe of the theorem.

4. If the power be the ſame in the two caſes where the directions of the reſiſtances are perpendicular to the ſides and axis, of the wedge, the reſiſtances are, according to the theorem, as the ſide and height of the wedge reſpectively; or greater reſiſtances are required to ſuſtain a given power, when their directions are perpendicular to the ſides, and therefore entirely efficient, than when they are oblique and conſequently not wholly efficient, which certainly is untrue.

It is presumed, for these reasons, that the analogy, derived from the theorem, and consequently the theorem itself, cannot be generally true; and were these reasons less decisive, it would not be difficult to point out the several oversights in the demonstration which induced a false conclusion*.

FIG. CXIII. 392. PROP. *If the power and resistances* P, R, *and* r, *act perpendicularly upon the back and sides of a scalene wedge* ABD, *and be in equilibrio,* P : R + r :: *back of the wedge : sum of its sides.*

DEM.

*THEOREM. If a power *P*, acting perpendicularly upon the back of an isosceles wedge
FIG. CXII. *ABC*, be in equilibrio with the equal resistances *E* and *F*, acting perpendicularly upon its sides, *P* : *E* + *F* as a line *PE* drawn from the bisection of the base *P* to one side, parallel to the resistance upon it, to the height of the wedge *PB*.

DEM. Let the directions of *E* and *F* meet in the bisection of the base *P*, and *PE* = *PF*, and, compleating the parallelogram, *P* : *E* + *F* :: *PN* : *PE* + *PF* (196) :: *PH* : *PE* :: *PE* : *PB*.

Cor. If *PN* be the force of *P*, *PE* and *PF* will represent the force with which *P* protrudes the resistances in directions perpendicular to the sides.

THEOREM II. If the directions of the resistances be any other lines *PD*, *PO*, equally inclined to the sides, the power is to the resistances (*E* + *F*) as *PD* to *PB*.

DEM. Resolve *PE* and *PF* into two, *PG* and *PK* in the directions of *PD* and *PO*, and *GE*, *FK* perpendicular to them; and these last forces, acting perpendicularly to the resistances, are lost. If *PN* be the force of *P*, *PE* and *PF* are its force perpendicular to the sides, and *PG* + *PK* the force in equilibrio with the resistances; therefore *P* : *E* + *F* :: *PN* : *PG* + *PK* :: *PH* : *PG* :: *PD* : *PB*, because $PE^2 = PG \times PD = PH \times PB$, and consequently *PH* : *PG* :: *PD* : *PB*. Q. E. D.

In this demonstration, *EG* and *FK*, being inclined to the sides of the wedge, are not inefficient, and, not being opposite, they do not destroy each other, and consequently ought not to be neglected. And, besides, if the power estimated in the directions of *PD* and *PO* be equal to *PG* + *PK*, these will be equal to the power estimated in the direction *PN*, or equal to *PN*. Resolve *PG* and *PK* into two, *GL*, *OL*, perpendicular to the axis, which being equal and opposite, destroy each other, and *PL* and *PL* are the only remaining parts of the force, and these are never equal to *PN*, unless *PD* and *PE* coincide. *Hamilton on the Principles of Mechanics.*

FIG. CV.

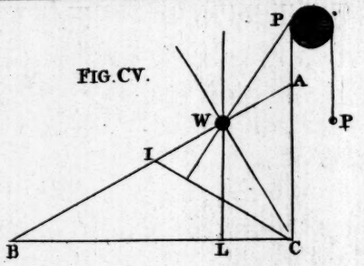


FIG. CVI.

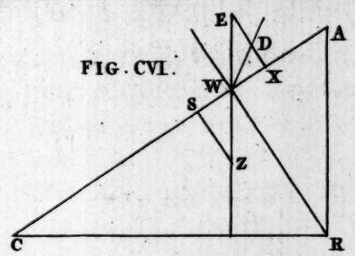


FIG. CVII.

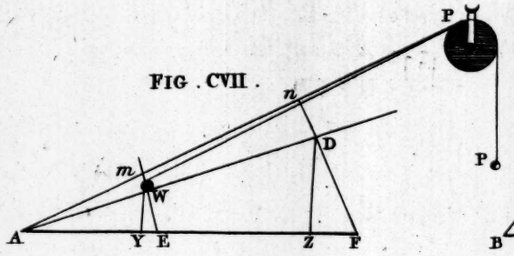


FIG. CVIII.

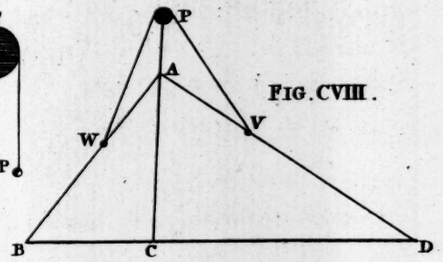


FIG. CIX.

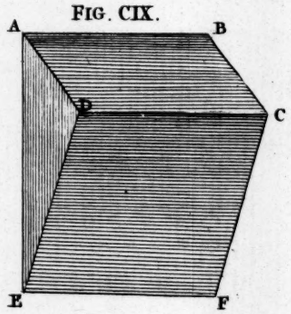


FIG. CX.

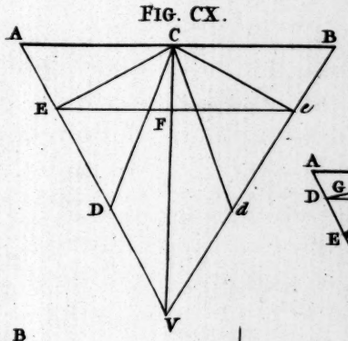


FIG. CXII.

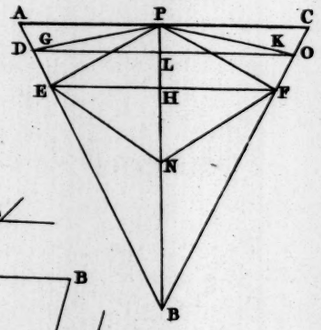
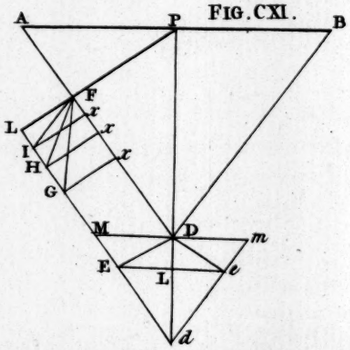


FIG. CXI.



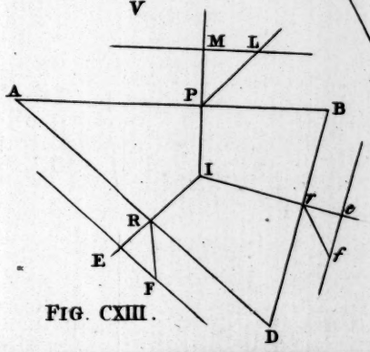


FIG. CXIII.


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DEM. Becauſe the power and reſiſtances act perpendicularly upon the back and ſides, they are wholly efficient, their directions are in the ſame plane, meet in the ſame point (216), and their magnitudes are to each other as the three ſides of a triangle parallel, or perpendicular, to their directions; therefore $P : R + r :: AB : AD + DB$. Q. E. D.

393. Cor. If PM, RE, re, be the relative magnitudes of P, R, r, in equilibrio, and lines be drawn through M, E, and e, parallel to the ſides of the wedge reſpectively, any forces, whoſe magnitudes are PL, RF, rf, drawn from P, R and r, terminated by the lines drawn parallel to the ſides, will be in equilibrio; for their perpendicular and only efficient parts PM, RE and re are in equilibrio.

SCREW.

*394. DEF. *If a right line, divided into equal parts* AC, CE, EG, &c. *repreſenting equal inclined planes, be ſo wrapped round the convex and concave ſurfaces of two cylinders with equal baſes, that* AB, CD, EF, &c. *the horizontal baſes of the planes, may be bent into the peripheries of circles parallel and equal to the baſes of the cylinders; equal ſpirals will be formed upon their ſurfaces, whoſe lengths are* AC, CE, EG, &c. *and diſtances* CB, ED, FG, &c. *the perpendicular heights of the planes. The convex cylinder is inſerted in the concave, and ſo adapted to it that the ſpirals protuberant upon one cylinder may exactly fill the excavated ſpiral or groove upon the other: the firſt is called the external, and the ſecond the internal, ſcrew.* PLATE XIV. FIG. CXIV.

Another definition:

If a cylinder move uniformly about its axis, whilſt a point moves uniformly upon its ſurface in a right line parallel to the axis, the line, deſcribed by this componnd motion, is a ſpiral, which, being raiſed upon the external ſurface of the cylinder, forms the external ſcrew; and a ſimilar ſpiral groove being cut upon the internal ſurface of a hollow cylinder,

* Maclaurin, Chap. III. XXII. Keil's Phyſics, Lect. X. Rohault, Not. ad 9. Emerſon's Mechanics, Prop. 29.

linder, of the ſame diameter, to receive the protuberant ſpiral, forms the internal ſcrew. Either of theſe ſcrews is fixed, and the other is moveable by a lever paſſing through the center, and in the plane, of its baſe.

FIG. CXV. 395. PROP. *There is an equilibrium upon the ſcrew, when the power,* P, *is to the weight,* W, *or reſiſtances acting parallel to the axis, as the diſtance between two contiguous ſpirals, to the periphery of the circle deſcribed by the power.*

DEM. It is evident that W, acting upon either of the ſcrews, in a direction parallel to its axis, will equally preſs every point of the ſpirals of the other in contact with it, in directions perpendicular to their baſes. If ABD be the baſe of one ſpiral, p, the magnitude of a power acting at B perpendicular to BC, and in equilibrio with the preſſure pr, upon one point of the ſpiral, P' a power acting at P perpendicularly to PC, and in equilibrio with p or pr; (374) $p : pr ::$ diſtance between two contiguous ſpirals $(d) : ABD$; $P' : p :: BC : PC :: ABD$: periphery of the circle deſcribed by the power, therefore $P' : pr :: d$: periphery of this circle, and the ſum of all the P's, or P is to the ſum of all the pr's, or W in the ſame ratio, that is, $P : W :: d$: periphery of the circle deſcribed by P. Q. E. D.

Another demonſtration from def. 2.

Whilſt B or P makes one revolution, W is elevated through a ſpace equal to d, therefore the velocity of P is to the velocity of W, as the periphery of the circle deſcribed by P to the diſtance between two contiguous ſpirals, and conſequently when there is an equilibrium, $P : W ::$ diſt. between two ſpirals : to the periphery deſcribed by P. Q. E. D.

396. Cor. 1. If the direction of W be inclined to the axis, and P do not act in the plane of the baſe, but in any other direction, the ratio of P to W may be found by art. 371.

397. Cor.

397. Cor. 2. The preſſure ſuſtained by any point of the ſpirals is to the part of W incumbent upon it, as the length of a ſpiral to its baſe ABD.

398. Cor. 3. In an endleſs or perpetual ſcrew, acting upon a wheel, the diſtance of whoſe teeth is equal to AB, the diſtance of two contiguous ſpirals, and in equilibrio with a body W, ſuſpended from the axis EF; $P : W :: AB \times EF$: diameter of the wheel × periphery of a circle whoſe radius is PG; for if R be the reſiſtance of a tooth to a ſpiral of the ſcrew, $P : R :: AB$: per. of the circle deſcribed by P, $R : W :: EF$: diameter of the wheel; therefore $P : W :: AB \times EF$: per. of the circle deſcribed by P × diameter of the wheel. PLATE XII. FIG. XCIV.

SCHOLIUM.

399. The ſecond demonſtration of this propoſition is generally deemed unſatisfactory, becauſe the moments of P and W are not reduced to oppoſite directions; but the ratio of P to W may be inveſtigated by finding the oppoſite and efficient parts of a power ſuſtaining a body upon an inclined plane, and combining it with the power of the lever. Let WC be an inclined plane, or ſpiral of the ſcrew, W a weight acting perpendicularly to the horizon WE, and ſupported by a power P, acting parallel to WB; and if W be elevated through a very ſmall ſpace Ww, or perpendicular height Hw, it is evident that P, acting parallel to WE, will deſcend through a ſpace equal to $WE - we$ or WH; and conſequently the velocities of W and P, eſtimated in oppoſite directions, are as Hw to HW, or as the height of the plane to its baſe, and when an equilibrium obtains, $P : W ::$ height of the plane : its baſe or ABD. But, from the nature of the lever, a power, acting at the diſtance PC, muſt be diminiſhed in the ratio of $BC : PC$, or of the circumference ABD : circumference deſcribed by the power acting at P. FIG. CXVI. FIG. CXV.

CHAP.

CHAP. IX.

CENTER OF GRAVITY.

400. DEF. *THE center of gravity of a body or ſyſtem of bodies is a point about which the parts of the body or ſyſtem are in equilibrio.*

401. PROP. *To find the center of gravity of a body.*

PLATE XIV. FIG. CXVII. Let A, B, C, D, &c. be particles of the body, and finding the centers of equilibrium p and q, of A and B, C and D reſpectively (274); let $A+B$ be placed in p, and $C+D$ in q, and their center of equilibrium, G, will be the center of gravity of the particles A, B, C, D, &c. Becauſe the force of gravity acts upon the particles in parallel directions, the efficacy of A to communicate motion to G is $A \times AG$, and that of B is $B \times BG$ (279) or $A \times \overline{Ap+pG}$ and $B \times \overline{Bp+pG}$, which are equivalent to them (181), or $\overline{A+B} \times pG$, ſince $A \times Ap$ and $B \times Bp$ are equal and oppoſite, and conſequently deſtroy each other. The ſum of the moments of C and D is found, by a ſimilar proceſs, to be the ſame as if they were placed in q; and conſequently G, which is the center of gravity of $A+B$ and $C+D$, placed in p and q reſpectively, is the center of gravity of A, B, C, D, placed at the points A, B, C, D, &c. Q. E. D.

402. Cor. 1. The particles of the body cannot be in equilibrio about any other point except G; for, if poſſible, let X be ſuch a point, and it is proved, as before, that the efforts of A and B to move $X = \overline{A+B} \times pX$, and of C and $D = \overline{C+D} \times qX$; therefore the point X is kept in equilibrio by two forces, $\overline{A+B} \times pX$ and

and $\overline{C+D} \times qX$, not acting in oppoſite directions, which is impoſſible (187).

403. Cor. 2. In every ſituation of the body compoſed of the particles A, B, C, D, &c. if the point G be ſupported, the body will be at reſt; for the force of gravity acting always in parallel directions upon the particles, their moments, or efforts to move G, will always be as $A \times AG$, $B \times BG$, &c. which by the proceſs uſed in this propoſition, will always be reduced to two forces that are equal and oppoſite.

404. Cor. 3. If $A+B+C+D$, &c. be equal to $\mathcal{Q}$, and the preſſure of each in parallel directions be equal to q, a force as $\mathcal{Q} \times q$, acting at the point G, in a direction oppoſite to that in which the particles preſs, will remove their preſſure. Or if A, B, C, &c. be deſtitute of gravity, and only reſiſt the action of a force by their inertia, a force P acting at G will communicate equal velocities to every particle; becauſe their reſiſtances, being exerted in directions oppoſite to that of P (3d law of motion) and therefore parallel to each other, vary as their diſtance from G, and conſequently the ſums of the reſiſtances on each ſide of G are equal. And $v.v$, if $\mathcal{Q}$ be moving and without gravity, a force applied, at G (the center of inertia) equal to the moment of $\mathcal{Q}$ will deſtroy all motion.

SCHOLIUM.

405. The particles which compoſe a body, being connected together by the force of coheſion, every line of particles may be conſidered as a lever, impreſſed in different points by the action of gravity, or any force which acts in parallel directions. The particle A therefore is connected with G by the coheſion of the intermediate particles, and, an infinite number of levers terminating in G will therefore be formed, upon which the particles, in any one line, act with intenſities varying as their magnitudes multiplied into their diſtances.

FIG. CXVIII. 406. PROP. *The ſum, or difference, of the products, which reſults from multiplying each particle* A, B, C, D *into its perpendicular diſtance from any plane* LN, *according as they are on the ſame or different ſides of the plane, is equal to the product of all the particles multiplied into the diſtance of their center of gravity,* G, *from that plane.*

DEM. Let P and $\mathfrak{Q}$ be the centers of gravity of A and B, C and D, and drawing right lines through P, $\mathfrak{Q}$, G, parallel to the plane, which interſect the perpendiculars drawn from thoſe points reſpectively; and $A : B :: BP : AB :: Bn$ or $Bb - Pp : Am$ or $Pp - Aa$,

and $A \times \overline{Pp - Aa} = B \times \overline{Bb - Pp}$, or

$$A \times Aa + B \times Bb = \overline{A + b} \times Pp.$$

By a ſimilar proceſs it appears, that $C \times Cc + D \times Dd = \overline{C + D} \times \mathfrak{Q}q$. But $A + B : C + D :: \mathfrak{Q}G : PG :: Gv$ or $Gg = \mathfrak{Q}q : Px$ or $Pp - Gg$, and $\overline{A + B} \times \overline{Pp - Gg} = \overline{C + D} \times \overline{Gg = \mathfrak{Q}q}$, or, by tranſpoſition and a ſubſtitution of equals, $A \times Aa + B \times Bb = C \times Cc = D \times Dd = \overline{A + B + C + D} \times Gg$, where the higher or lower ſigns are to be uſed, according as the bodies are on the ſame, or a different, ſide of the plane. Q. E. D.

FIG. CXIX. 407. Cor. 1. If the particles be placed upon the ſame right line, or Aa, Bb, Cc, Dd, Gg, become Ag, Bg, Cg, Dg, Gg, reſpectively, it is evident, that $A \times Ag + B \times Bg = C \times Cg = D \times Dg = \overline{A + B + C + D} \times Gg$; or the ſum, or difference, of the products reſulting from the multiplication of each particle into its diſtance from any point g, according as they are on the ſame, or a different, ſide of that point, is equal to the product of their ſum multiplied into the diſtance of their center of gravity from that point.

408. Cor. 2. The whole moment of a body, acting upon a lever, being equal to that of every particle, or to the ſum of the products which reſults from the multiplication of each particle into its diſtance from the center of motion (279), is equal therefore to the

the product of the whole body into the distance of the center of gravity from the center of motion, and is consequently the same as if it were collected in the center of gravity. The demonstration of this proposition obtains therefore when A, B, C, D, are collections of particles or bodies, whose centers of gravity are the points A, B, C, D. And to find the center of gravity of a system of bodies, it is evident, that, in the proposition (401), bodies, whose centers of gravity are A, B, C, D, &c. may be substituted for particles.

409. Cor. 3. If A, B, C, D, be bodies acting upon any plane LN, in parallel directions, the sum of their efforts to move it is the same as if they were collected in their center of gravity; for, if A, B, C, D, be the respective centers of gravity of each body, this sum is equal to $A \times Aa \times Bb \pm C \times Cc \pm D \times Dd = \overline{A+B+C+D} \times Gg$; or, if they be placed upon a lever, the sum of their efforts to make it revolve is the same, as if they were placed at G. When the center of gravity therefore is in the plane, or at the fulcrum of a lever, the plane and lever are quiescent. And if any point Z be taken in NL, $A \times aZ + B \times bZ + C \times cZ + D \times dZ = \overline{A+B+C+D} \times gZ$; for if a plane pass through Z, the proof is the same as that of this proposition. FIG. CXVIII.

410. Cor. 4. The distance of any plane from the common center of gravity of A, B, C, D, &c. or Gg is equal to $\frac{A \times Aa + B \times Bb \pm C \times Cc \pm D \times Dd}{A+B+C+D, \text{ \&c.}}$; and its distance from a plane passing through any point Z is equal to $\frac{A \times Za + B \times Zb \pm C \times Zc \pm D \times Zd}{A+B+C+D}$, where the lower signs are to be used for those bodies not on the same side of Z with A and B.

FIG. CXX. 411. Cor. 5. A right line drawn from A through the center of gravity G of any number of bodies A, B, C, D, &c. will pass through the center of gravity of the remainder; for $B \times Bb + D \times Dd = C \times Cc$, and consequently the center of gravity of B, C, D is in the plane passing through AG, and if this plane revolve, their center of gravity is always in the plane passing through AG, and consequently it must be in the line AG produced, which is the common intersection of the planes. If r be this center, $\overline{B+C+D} \times Gr = A \times AG$, and if the bodies be equal and n their number, $AG = \overline{n-1} \times Gr$.

FIG. CXXI. 412. Cor. 6. If a circle or sphere be described about the center of gravity G, of any number of bodies A, B, C, &c. and any point P be taken in the periphery of the circle, or surface of the sphere, $PA^2 \times A + PB^2 \times B + PC^2 \times C$, &c. is a given quantity; for, drawing GP and the perpendiculars to it Aa, Bb, Cc, $A \times Ga = B \times Gb + C \times Gc$ (409), or, by substitution of equals, $A \times \frac{\overline{GA^2 - PA^2 + GP^2}}{2GP} = B \times \frac{\overline{PB^2 - BG^2 - GP^2}}{2GP} + C \times \frac{\overline{PC^2 - GP^2 - GC^2}}{2GP}$, or, $A \times PA^2 + B \times PB^2 + C \times PC^2 = A \times \overline{GA^2 + GP^2} + B \times \overline{GB^2 + GP^2} + C \times \overline{GC^2 + GP^2}$, and this side of the equation is invariable in whatever point of the periphery or surface P be placed.

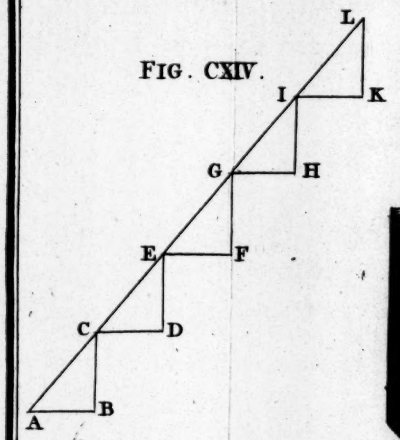
FIG. CXXII. 413. PROP. *If* A, B, C, D, *&c. be particles of a body urged by forces in parallel directions, whose magnitudes are* A a, B b, C c, *&c. the sum of their weights is equal to the weight of* A + B + C, *&c. acted upon by a force whose magnitude is* G g.

DEM. The weights of A, B, C, &c. are $A \times Aa$, $B \times Bb$, $C \times Cc$, &c. (235), and consequently the sum of their weights is equal to $\overline{A+B+C}$, &c. $\times Gg$; but this product is the weight of $\overline{A+B+C}$, &c. acted upon by the force Gg. Q. E. D.

414. Cor.

Plate XV.

FIG. CXIV.



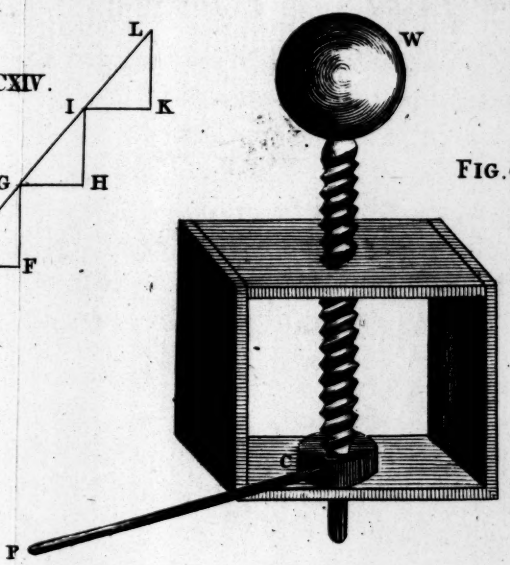


FIG. CXV.

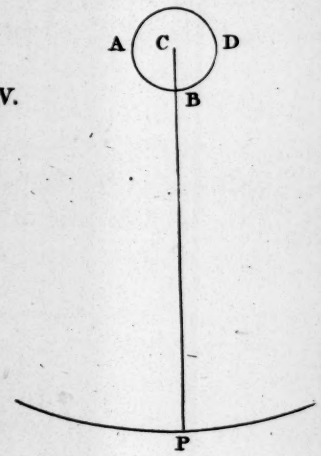


FIG. CXVI.

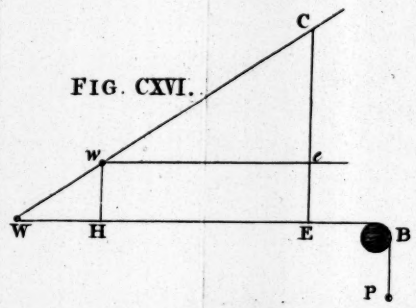


FIG. CXVII.

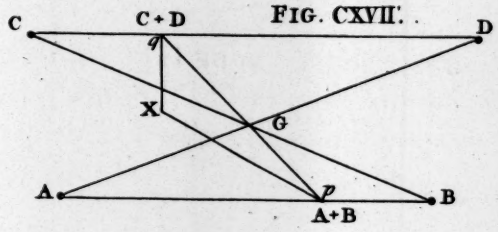


FIG. CXVIII.

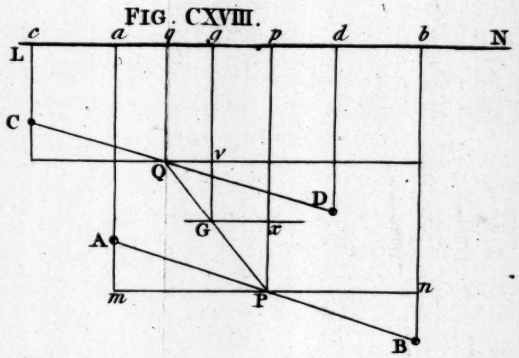
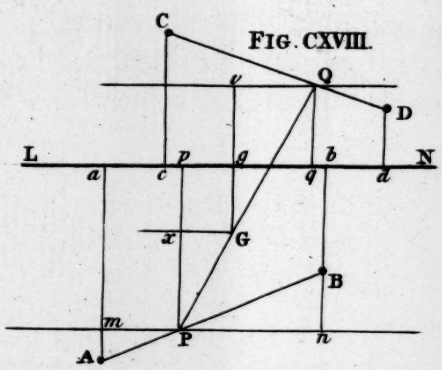


FIG. CXVIII.




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414. Cor. If the forces Aa, Bb, Cc, &c. be equal to each other, Gg is equal to one of them, or if the particles A, B, C, &c. be acted upon by the same force, their weight is the same as if they were collected in their center of gravity and acted upon by that force. The tendency therefore of a body to descend is the same as if it were collected in its center of gravity, and, consequently, if a line drawn from that center perpendicular to the horizon, fall within the base of the body, it cannot fall, and, if without the base, it cannot stand.

415. PROP. *If any number of bodies* A, B, C, *&c. move in parallel directions, with any velocities, the center of gravity will describe a right line parallel to them.* FIG. CXXIII.

DEM. Let A and B, a and b, be cotemporary positions of the bodies A and B, and G, g, their centers of gravity, and through g draw a line xy parallel, and consequently equal, to AB. From the nature of the center of gravity, $A : B :: BG : AG :: bg : ag :: yg : xg$ (sim. triangles); and the point g divides the parallel and equal lines AB, xy, in the same ratio, and Gg is a right line parallel to Aa or Bb. If H be the center of gravity of A, B, C, it is proved in the same manner, that it cuts the parallel and equal lines GC, vz, in the same ratio, and Hh is consequently a right line parallel to Gg. Q. E. D.

416. Cor. 1. If any number of bodies A, B, C, &c. ascend, or descend in parallel right lines, the sum of the products resulting from the multiplication of each body into the space described by it, is equal to the product of their sum and the space described by their center of gravity G; for, let a, b, c, g, be cotemporary positions of A, B, C, G, and, drawing any plane NL, $A \times Am + B \times Bn + C \times Cq = \overline{A + B + C} \times Gh$ (406), and $A \times am + B \times bv + C \times cq = \overline{A + B + C} \times gh$; and consequently by addition $A \times Aa + B \times Bb + C \times Cc = \overline{A + B + C} \times Gg$. FIG. CXXII.

417. Cor.

FIG. CXXIII. CXXIV. 417. Cor. 2. If any number of bodies therefore move in parallel directions with any unequal velocities, or they be placed upon the lever XY, and receive unequal impulses from any force at the same time in parallel directions, the center of gravity will, in the beginning of its motion, move uniformly in a right line parallel to them, and its velocity is equal to the products of each body into its velocity, divided by the sum of the bodies; for the spaces Aa, Bb, Cc, Gg are described in the same time, and vary as the velocities, and Gg (or velocity of G) $= \frac{A \times Aa + B \times Bb + C \times Cc}{A + B + C}$.

FIG. CXXV. 418. PROP. *The surface, or solid, described by the line or surface* AB, *moving round an axis passing through* C, *is equal to the surface or solid formed by the multiplication of* AB *into the line described by the center of gravity*, G, *of* AB.

DEM. For if AB be covered with physical points of the same thickness, AD, DE, EF, &c. $AD \times CA + DE \times CD + EF \times CE$, &c. $= \overline{AD + DE + EF}$, &c. or $AB \times CG$ (407), or because the arcs Aa, Dd, Ee, &c. are similar, $AD \times Aa + DE \times Dd + EF \times Ee$, &c. (= the sum of all the superficial, or solid, annuli Ad, De, &c.) $= AB \times Gg$.

FIG. CXXVI. 419. PROP. *The content of the solid generated by the revolution of a plane figure* ABC *round an axis* AC, *is equal to that of the solid whose base is* ABC, *and perpendicular height the periphery of the circle described by the center of gravity*, G.

DEM. For the moment of ABC or $ABC \times GD$, is equal to the sum of the moments of the elementary parts of ABC, or to the sum of all the small rectangles En multiplied into the distance of their center of gravity from AC, or to the sum of all the $y \times mn \times \frac{y}{2}$ or $\frac{y^2}{2} \times mn$; therefore $ABC \times GD$ is equal to the sum of

all

all the $\frac{y^2}{2} \times mn$, and $p \times ABC \times GD$ to the sum of all the $\frac{py^2}{2} \times mn$; but if p be the periphery of a circle whose radius is unity, $p \times GD$ is the periphery of a circle whose radius is GD; $\frac{p \times y^2}{2}$ is the area of a circle whose radius is y, and the sum of all the $\frac{py^2}{2} \times mn$, is therefore equal to the solid described by ABC.

420. DEF. *The center of gravity of a line or surface, is the center of gravity of equal physical points, which are supposed to compose that line or surface.*

421. Cor. 1. The center of gravity of a right line is evidently in its bisection, and the center of gravity of a parallelogram, prism, cylinder, are in the bisection of the diameter or axis, because the number of physical lines in a parallelogram, or of physical laminæ in the prism and cylinder, on each side of this point, are equal.

422. Cor. 2. The center of gravity of any number of lines AB, CD, EF, &c. is found by joining the centers of gravity m, n, of any two AB, CD, and taking a point p, so that $pm : pn :: CD : AB$, and joining p, and q the center of gravity of EF, and taking a point r so that $pr : rq :: EF : AB + CD$, &c. (401). The center of gravity of the sides of a triangle ABC may be found as above, or it may be found by bisecting the sides in m, n, p, joining the points of bisection, and bisecting the angles mpn, and nmp, by the lines pr and mq, whose intersection, G, is the center of gravity; for $pq : qn :: pm : mn :: AC : BC$, or the center of gravity is in qm; and $mr : rn :: mp : pn :: AC : AB$, or this center is in pr, and therefore must be in their intersection G.

FIG. CXXVII.

FIG. CXXVIII.

423. PROP.

FIG. CXXIX. 423. PROP. *If a regular polygon* ABCDE *be inſcribed in the ſegment of a circle whoſe center is* F, *half the ſum of its ſides is to the ſine of half the arc :: perpendicular let fall from* F *upon a ſide : the diſtance of* G *from* F.

DEM. For let m, n, p, q, be reſpectively the centers of gravity of the ſides, and joining vs, the centers of AB, BC, and CD, DE, it is clear that the center of gravity of all the ſides will be in vs, and it will be alſo in FC, and conſequently at G the interſection of FC and vs. But from the ſimilar triangles sFq, EID, $ED : EI (:: ED + DC : EC) :: Fq : Fs$, and from the ſimilar triangles GsF, CEH, $CE : HE :: Fs : FG$, and, by adding theſe analogies together, $ED + DC : EH :: Fq : FG$. Q. E. D.

424. Cor. 1. If a regular polygon be inſcribed in a circle, its center of gravity is the center of the circle; for $FG = \frac{\text{ſine of half the periphery} \times Fq}{\frac{1}{2} \text{ the ſum of the ſides}} = \frac{0 \times Fq}{\frac{1}{2} \text{ the ſum of the ſides}}$.

425. Cor. 2. As the arc EC : ſine of that arc :: FE : diſtance of the center of gravity of twice EC from the center F; for, let the ſides of the polygon inſcribed, be diminiſhed without limit, ſo as to coincide with the circular arc and be equal to it, and Fq is then equal to the radius. If the arc be a ſemicircle, the quadrant $CK : FC :: FC : FG$; and the center of gravity of the whole periphery is in F, becauſe $FG = \frac{0 \times FC}{KCL} = 0$.

426. PROP. *The diſtance of the center of gravity of a triangle from any of its angles, is equal to two third parts of the line drawn from that angle to the biſection of the ſide oppoſite to it.*

DEM.

Plate XVI.

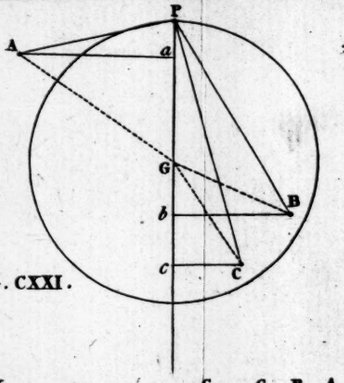


FIG. CXXI.

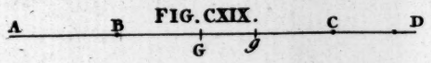


FIG. CXIX.

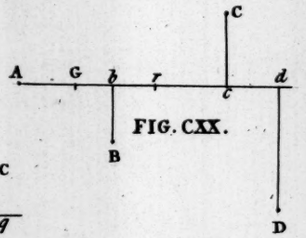


FIG. CXX.

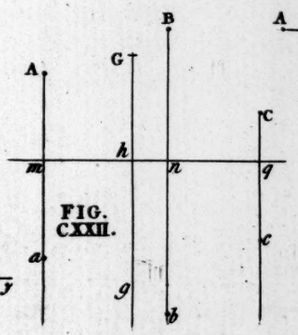


FIG. CXXII.

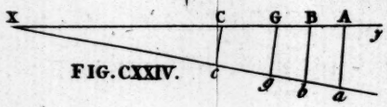


FIG. CXXIV.

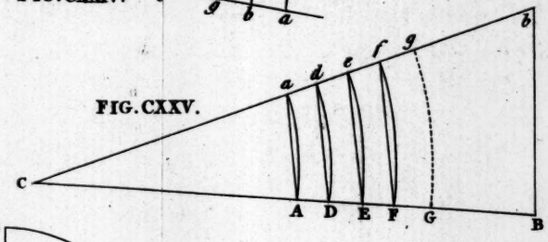


FIG. CXXV.

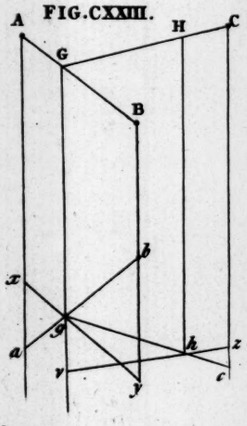


FIG. CXXIII.

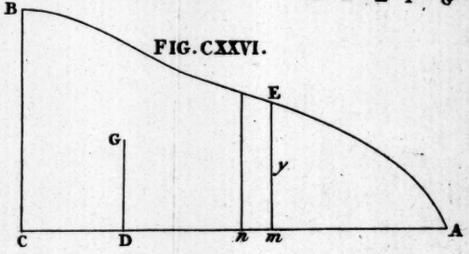


FIG. CXXVI.

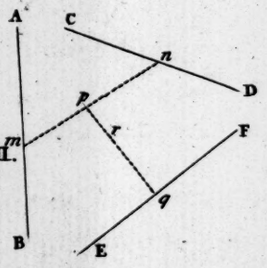


FIG. CXXVII.

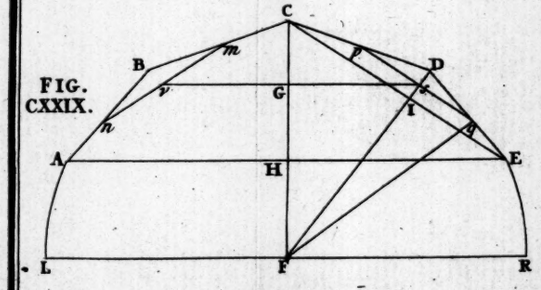


FIG. CXXIX.

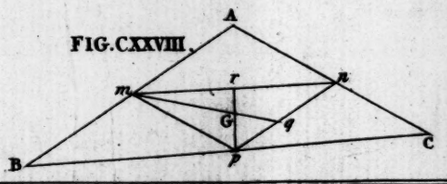


FIG. CXXVIII.


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DEM. Let Ap biſect the oppoſite ſide BC, and drawing ab, cd, ef, &c. infinitely near and parallel to BC, each will be biſected, and the center of gravity of each, and conſequently of all, which make up the triangular ſurface, will be in Ap; and, for the ſame reaſon, it is in Bq biſecting AC, and therefore muſt be in their interſection G; but, from ſimilar triangles, $AG : Gp :: AB : pq :: BC : pC :: 2 ; 1$. Q. E. D. FIG. CXXX.

427. Cor. 1. The center of gravity of any right lined figure is found by dividing it into triangles, and conſidering them as bodies: let m be the center of gravity of ABC, and n of BDC and joining mn, and taking a point G, ſo that $mG : nG :: DBC : BAC$, it will be the center of gravity of the trapezium $ABDC$ (401). The proceſs is ſimilar when the figure has more ſides. FIG. CXXXI.

428. Cor. 2. The center of gravity of a regular polygon, inſcribed in a circle, is the center of that circle; for drawing a right line BH, from any of the angles through the center, the figures on each ſide are ſimilar and equal, and the center of gravity is in BH; and, for the ſame reaſon, it is in DI, and therefore muſt be at C, and conſequently the center of gravity of a circle is its center. FIG. CXXXII.

429. PROP. *The diſtance of the center of gravity of a regular polygon, inſcribed in a ſector of a circle* FEAC, *from its center, is equal to two thirds of diſtance of the center of gravity of the periphery* EDABC. FIG. CXXXIII.

DEM. Find the center of gravity, H, of the periphery (423), and, taking $IE = \frac{1}{3} FE$, deſcribe a polygon ſimilar to $EDABC$. The centers of gravity of the triangles FED, FDA, FAB, FBC, are at L, S, T, V, the biſections of IK, KN, NP, PQ; and the centers of gravity of $FEDA$ and $FABC$, are at R, and X, the biſections of SL, VT, and the center of the whole figure is at G, the biſection of RX; but $FE : EI :: FM : LM$, therefore $LM = \frac{1}{3} FM$, and, for the ſame reaſon, $RO = \frac{1}{3} FO$ and $GH = \frac{1}{3} FH$. Q. E. D.

430. Cor. 1. The distance of the center of gravity of a sector of a circle from the center, is therefore equal to two thirds of the distance of the center of gravity of the circular arc upon which it stands.

431. Cor. 2. As the arc AE : sine of AE :: radius : distance of the center of gravity of twice AE from F, or FH (425); but $FG = \frac{2}{3} \times FH$; therefore AE : sine of AE (:: $EABC$: its subtense EC) :: $\frac{2}{3}$ radius : FG.

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432. The distance, from the vertex, of the center of gravity of a parabolic space is equal to three fifths, and of a paraboloid to two thirds, of the axis; but these, and the most difficult propositions upon the center of gravity, are best investigated by a fluxional process.

FIG. CXXXIV. 433. PROP. *If one or more of the bodies* A, B, C, *&c. move uniformly in the same right line, with velocities equal to* a, b, c, *&c. their common center of gravity will move uniformly.*

DEM. Let A and B move uniformly in the same, or an opposite, direction, P be their center of gravity, and D their distance: and because the motions of A and B are uniform, D either continues the same, or encreases and decreases uniformly; but $AP = \frac{D \times B}{A+B}$, and consequently varies as D, and P moves uniformly. If another body C move uniformly in the same right line, and R be the center of gravity of A, B, C; the distance CP either continues the same, or encreases and decreases uniformly, because C and P move uniformly; but $PR = \frac{CP \times C}{A+B+C}$, and consequently varies as CP, or R moves uniformly. Q. E. D.

434. Cor.

434. Cor. 1. The velocity of the center of gravity is equal to $\frac{Aa \pm Bb \pm Cc}{A+B+C}$; for, let p, r, a, b, c, be cotemporary positions of P, R, and the bodies, and (407) $A \times Ap$ or $\times \overline{Aa+ap} - B \times Bp$ or $\times \overline{bp \pm Bb} = \overline{A+B} \times Pp$, and $Pp = \frac{A \times Aa \pm B \times Bb}{A+B}$, because $A \times ap - B \times bp = 0$. And, placing $A+B$ in P and repeating the above process, it appears that $Rr =$ the velocity of R (106) $= \frac{A \times Aa \pm B \times Bb \pm C \times Cc}{A+B+C}$. It is from hence again collected, that the velocity of R is uniform; because Aa, Bb, Cc are constant, and consequently their sum, or difference, multiplied into the same given quantities, or the velocity of R, is always the same.

435. Cor. 2. Because $\overline{A+B+C} \times Rr = A \times Aa \pm B \times Bb \pm C \times Cc$, the velocity of the center of gravity is such as would be communicated to the sum of the bodies acted upon by a force equal to $A \times Aa \pm B \times Bb \pm G \times Cc$.

436. *PROP. *If one or more bodies* A, B, C, *&c. move uniformly in right lines, either in the same or different planes, their common center of gravity* S *will move uniformly in a right line.* FIG. CXXXV.

DEM.

* This proposition is proved very neatly, though, perhaps, less intelligibly by the following process.

CASE I. Let two bodies move, in the same plane, in the directions DE, AB; and let D and A, E and B be cotemporary positions, and H, K, the centers of gravity in those positions, respectively; and taking $BP = AD$, joining EP and drawing DL parallel to HK, $DE : AB$ in the given ratio of the motions of the bodies; and, because the angle EDP is given, all the angles of the triangle EDP are given, and DP is to PE in a given ratio: but $PE : PL :: BE : BK$, which is a given ratio, therefore $DP : PL$ in a given ratio; and, because all the angles of the triangle DPL are given, the angle PDL is given, and L is always in DL. By the nature of the center of gravity, $DA : DH :: EB : EK :: PB$ or $DA : LK$; therefore $DH = LK$, and $DHKL$ is a parallelogram, HK is parallel to DL, and the angle BHK is given, and the center of gravity K is always in the right line HK given in position. And, because all the angles of the triangles DPL and DLE are given, the lines DP, DE, DL, FIG. CXXXVI.

DEM. 1. Let B deſcribe Bb uniformly in the time T, and P, Q, be the centers of gravity of A and B; and $A+B : B :: AB : AP :: Ab : AQ :: Bb : PQ$ (EUC. B. 6. P. 5.), and PQ is parallel to Bb, and equal to $\frac{Bb \times B}{A+B}$, and varies therefore as Bb or uniformly. Let A deſcribe Aa uniformly in the time T, either in the ſame plane with Bb, or not, and R be the center of gravity of A, and B placed at b; and QR, the path of the center of gravity, will appear, by the ſame proceſs with the above, to be parallel to Aa, and equal to $\frac{Aa \times A}{A+B}$, and conſequently it varies as Aa, or encreaſes uniformly. When both bodies move at the ſame time, the point P will have two motions PQ and QR, and will conſequently deſcribe the diagonal PR uniformly in the time T (185).

2. Let a third body C be added, and the common center of gravity be S, and CS produced will paſs through the center of gravity of A and B (411). From the nature of the center of gravity, $A+B+C : A+B :: CP : CS :: CQ : CT :: QP : ST$, and $ST = \frac{}{A+B}$

DL, that is, AB, DE, HK, are in a given ratio, and conſequently the point K moves uniformly in HK. The demonſtration is the ſame if one of the bodies moves from B towards A.

CASE II. Let the paths of the bodies AB and DE be in different planes; and through AB draw a plane Bde parallel to DE, and through DE draw the plane $DdeE$ perpendicular to Bde; produce BA to d, and let Dd, Ee, be perpendicular to de, and the planes DdA, EeB, will be perpendicular to the plane edB. Let A and D, B and E be cotemporary poſitions of the bodies. If the body at D were to move in de, the center of gravity would move uniformly in ſome line HK (caſe 1.); through HK erect the plane $HKkb$ perpendicular to HBK. From ſimilar triangles, and the nature of the center of gravity, $Ab : bD :: AH : Hd :: BK : Ke :: Bk : kE$; therefore bk is the path of the center of gravity of the bodies moving in AB, DE. And, becauſe $Dd : Hb :: Ad : AH :: Ee : BK :: Ee$ or $Dd : Kk$, $Hb = Kk$ and kb is equal and parallel to HK; therefore the center of gravity of the bodies, moving uniformly in AB, DE, moves uniformly in bk.

CASE III. The common center of gravity of two bodies and a third body, is either at reſt, or moves uniformly in a right line; for two may be placed in their common center of gravity, which was proved to move uniformly, and the center of gravity of the three or more bodies is proved, by the ſame proceſs as before, to move uniformly.

$\frac{A+B \times QP}{A+B+C}$, and varies as QP or uniformly; and for the ſame reaſon TV, the motion of T ariſing from A's motion, is equal to $\frac{QR \times A+B}{A+B+C}$, and therefore varies as QR or uniformly (caſe 1.). When A and B move together, the motions ST, TV, will be combined into one, SV; and if C deſcribe Cc uniformly in the time T, the common center of gravity will deſcribe VY, and this new motion, combined with SV, will make it deſcribe SY uniformly in the time T. Q. E. D.

437. Cor. 1. It is evident, that the paths of the center of gravity, ariſing from the motion of any one body, is always parallel to that of the moving body: PQ and ST are parallel to Bb; QR and TV are parallel to Aa and VY to Cc.

438. Cor. 2. The centers of gravity of two, three, &c. bodies will deſcribe polygons or curves ſimilar to that of the moving body to which their motion is owing; and if the velocity of the body be variable, the velocity of each center will be variable according to the ſame law.

439. Cor. 3. The velocity of the center of gravity of two, three, &c. bodies is the ſame as if they were placed in it, and acted upon by forces, equal to the moments of the moving bodies, in their reſpective planes and directions; for $B \times Bb = \overline{A+B} \times PQ$ and $A \times Aa = \overline{A+B} \times QR$; and if $A+B$ were placed at P, and acted upon by forces equal to $B \times Bb$ and $A \times Aa$ in the planes and directions of Bb and Aa, they would deſcribe the diagonal PR (185).

440. Prop. *The common center of gravity of two or more bodies is not affected by any action of the bodies upon each other.*

Dem.

FIG. CXXXVIII.

DEM. Let A and B be two bodies in a ſyſtem, acting upon each other, G their common center of gravity, and Aa, Bb, the velocities loſt by A and gained by B reſpectively in oppoſite directions, and $A \times Aa = B \times Bb$ (3d law of motion), or $A : B :: Bb : Aa :: BG : AG :: bg : aG$, or $A : B ::$ B's diſtance from the center of gravity : A's diſtance from it; and conſequently the ſame point G is ſtill the center of gravity of A and B, or it has been immoveable. What is proved of theſe two, is true of every two bodies, and therefore of all. Q. E. D.

441. Cor. If two parts of a ſyſtem A and B, attract or repel each other, or moving with unequal rectilineal motions, diſturb each others motion by the force of their inertia, the center of gravity will not be affected by their mutual action.

CHAP.

FIG. CXXX.

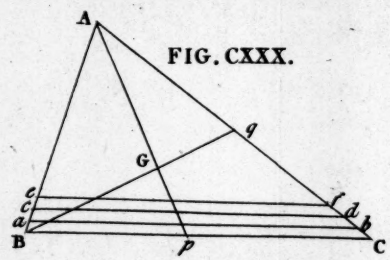
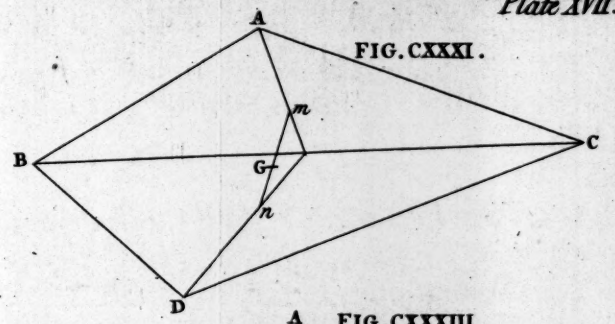


FIG. CXXXI.



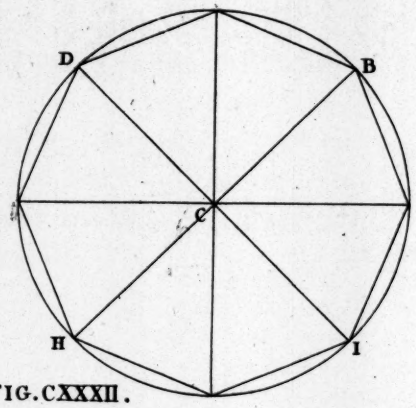


FIG. CXXXII.

FIG. CXXXIII

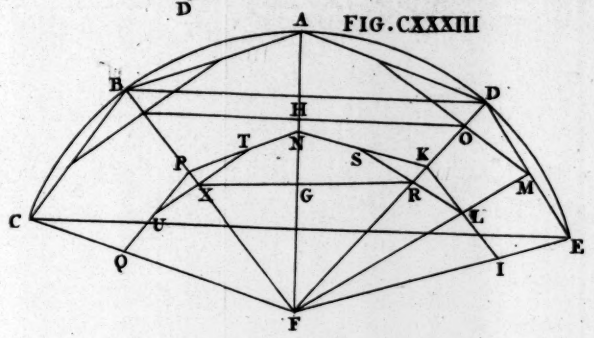
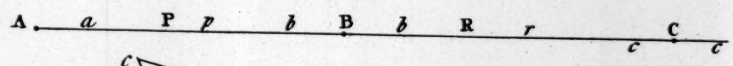


FIG. CXXXIV.



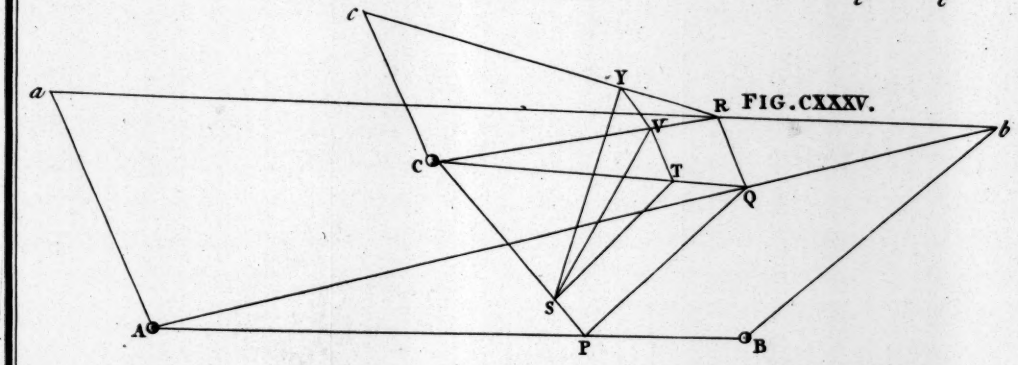
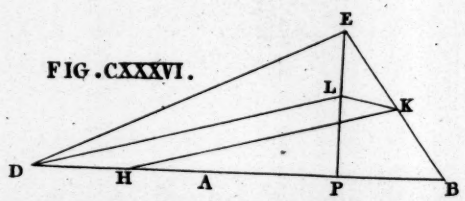


FIG. CXXXV.

FIG. CXXXVI.




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CHAP. X.

COMMUNICATION OF MOTION BY DIRECT IMPACT.

442. DEF. *TWO bodies are said to impinge directly, when the right line, in which their centers of gravity move, passes through the point of contact.*

443. DEF. *The center of gravity of a body, or system of bodies, supposed to be without gravity, is the same point with that center when its influence obtains.*

LEMMA. If two bodies X and Y, consisting of any number of particles without gravity A, B, C, and D, E, &c. be immoveably connected to the right line SR, passing through their centers of gravity G and H, which is perpendicularly impressed at F by a force F, the moments communicated to these bodies are inversely as the distances of their centers of gravity from the point where E acts. FIG. CXXXIX

DEM. The sum of the resistances of D and E to the action of F is $D \times Fd + E \times Fe$ (279) $= \overline{D+E} \times FH$ (407), and consequently if d, e, b, be the respective velocities of D, E, and H, and $D+E=Y$ be placed in H, $D \times d + E \times e = Y \times b$; and for the same reason $A \times a + B \times b + C \times c$, &c. $= X \times g$, supposing a, b, c, g, to be the respective velocities of A, B, C, and of X collected at G. Let a fulcrum be placed at G, and because F is in equilibrio with

with $D \times d + E \times e$ or $\Upsilon \times b$, $F : \Upsilon \times b :: GH : FG$, and if a fulcrum be placed at H, $X \times g : F :: FH : GH$, and ex æquo $X \times g : \Upsilon \times b :: FH : FG$. Q. E. D.

444. Cor. Becauſe $g : b :: FH \times \Upsilon : FG \times X :: D \times Fd + E \times Fe : A \times Fa + B \times Fb + C \times Fc$, the velocities of G and H, and of the points A, B, C, &c. are the ſame, whether the particles be placed in thoſe points, or collected in their centers of gravity G and H; and the effect is evidently the ſame, whether G and H be in the right line SR, or in PQ, making an invariable angle with SR, and at the ſame perpendicular diſtances from F's direction.

445. PROP. *If a material ſurface* SUR, *compoſed of the particles* A, B, C, D, *&c. be impreſſed by a force equal to* F, *acting in the plane of the ſurface at the center of gravity* F, *and perpendicularly to a right line* SFR *paſſing through it, the particles will move with equal velocities in directions parallel to that of* F.

DEM. Let the particles, on one ſide of F's direction, $A + B + C$, &c. $= X$ be placed at their center of gravity G, and $D + E$, &c. on the other ſide at their center of gravity H, and GFH is a right line (411). But $g : b :: \Upsilon \times FH : X \times FG :: 1 : 1$ and $g = b$; and the incipient motions of X and Υ are parallel to F's direction (2d law of motion). Since X and Υ begin to move with equal velocities, in parallel directions, they are relatively at reſt, and conſequently will not diſturb each others motions; and becauſe the relative ſituations of the points A, B, C, &c. are always the ſame, they will move with velocities equal to thoſe of G and H, and in parallel directions. The effects are the ſame whether A, B, C, &c. be in their real places, or collected in their centers of gravity (444); and the initial motions of G and H are not diſturbed by the mutual actions of the particles (441). Q. E. D.

446. Cor.

446. Cor. 1. If any particles B, C, &c. be not in the ſuperficies SR, but on different ſides of it, their center of gravity will be in q (411), and their efforts, reſulting from their inertia, to diſturb the incipient motion of the particles, are meaſured by the products of each particle into its perpendicular diſtance from the plane SR(279), and theſe being equal and oppoſite, and conſequently deſtroying each other, the particles will continue to move with their initial velocities. If a body therefore be impelled by any power F, in a direction paſſing through its center of gravity, its conſtituent particles will move with equal velocities, in directions parallel to that of F; and if the particles be equal, and their number n, the velocity of each, and of the center of gravity, is the ſame whether that center be impelled by the force F, or each particle by a force equal to $\frac{F}{n}$, in the direction of F. FIG. CXXXIX.

447. Cor. 2. In the impact of one body A upon another body B, either moving or quieſcent, when the right line joining their centers of gravity, and the lines in which they move, are in the ſame right line, the forces of impact and reſiſtance produce a change of velocity, in each body, only in this line, or their impact is direct.

448. PROP.* *Let a body* A, *moving with a velocity equal to* a, *impinge directly upon* B, *moving in the ſame, or an oppoſite, direction, with a velocity equal to* b, *to find their common velocity.*

If A and B be inelaſtic, (3d law of motion) $Aa \pm Bb$ is the ſame before and after impact (where the higher ſign is to be uſed when the bodies move in the ſame direction, and the lower when they move in oppoſite directions,) and, becauſe they move together, if v be their common velocity, $Aa \pm Bb = \overline{A+B} \times v$ and $v = \frac{Aa \pm Bb}{A+B}$. Q. E. I.

449. Cor.

* Newt. Cor. 3. ad leges motus. Maclaurin's Newt. Chap. IV. Rohault. Not. Part I. Ch. II. Art. 6. Helſham. Lect. V. and Appendix. Nov. Com. Petropol. Tom. XVII. pag. 315.

449. Cor. 1. The velocity, which A loſes by impact, is equal to the difference between its velocities before and after impact, or to $a - v$; and the velocity which B gains by impact, in the direction of A's motion, is equal to the difference between its velocities after and before impact, or to $v \mp b$. Subſtituting the value of v, $a - v = a - \frac{Aa \mp Bb}{A+B} = \frac{Aa + Ba - Aa \mp Bb}{A+B} = \frac{B \times \overline{a \mp b}}{A+B} = l$, and, reſolving this equation into a proportion, $A+B : B :: a \mp b : l$. The velocity gained by $B = g = v \mp b = \frac{Aa \pm Bb}{A+B} \mp b = \frac{Aa \pm Bb \mp Ab \mp Bb}{A+B} = \frac{A \times \overline{a \mp b}}{A+B}$, and $A+B : A :: a \mp b : g$. From hence we have the following rules.

1.* The ſum of the bodies is to the ſtruck body as the difference or ſum (according as they move in the ſame, or an oppoſite, direction) of the velocities, before impact, to the velocity loſt by the ſtriking body.

2. The ſum of the bodies is to the ſtriking body as the difference or ſum of the velocities, before impact, to the velocity gained by the ſtruck body.

450. Cor. 2. If A and B move in oppoſite directions with velocities that are always inverſely as the bodies, their common center of gravity will not be affected by their motion; but, l and g, repreſenting the velocities loſt by A and gained by B, are made in oppoſite

* Theſe rules are inveſtigated differently by the following proceſs:

1. $A \times l = B \times g$ (3d law of motion) and $g = \frac{A \times l}{B}$; but $a - l$ (A's velocity after impact) $= g \pm b$ (B's velocity after impact) $= \frac{A \times l}{B} \pm b$ and $B \times a - B \times l = A \times l \pm B \times b$ and $l \times \overline{A+B} = B \times \overline{a \mp b}$, and $\overline{A+B} : B :: \overline{a \mp b} : l$.

2. $l = \frac{B \times g}{A}$ and $a - l = a - \frac{B \times g}{A} = g \pm b$, (becauſe the bodies move on together) and $Aa - B \times g = Ag \pm A \times b$ and $\overline{A+B} \times g = A \times \overline{a \mp b}$, and $A+B : A :: a \mp b : g$.

oppoſite directions, and $A \times l = B \times g$ or $A : B :: g : l$, or g and l are inverſely as the bodies, and conſequently the velocity of the center of gravity of A and B is not affected by impact, and is, therefore, both before and after impact, equal to their common velocity, v.

451. PROP. *If* A, *moving with a velocity equal to* a, *impinge upon* B, *moving in the ſame, or an oppoſite, direction with a velocity equal to* b, *and* A *and* B *be perfectly elaſtic, or one of them be perfectly hard, and the other perfectly elaſtic, to find the velocity loſt by* A (L), *and that gained by* B (G).

When A and B are inelaſtic, $A + B : B :: a \mp b : l$, and $A + B : A :: a \mp b : g$; but $L = 2l$, and $G = 2g$ (257); therefore $A + B : 2B :: a \mp b : L$, and $A + B : 2A :: a \mp b : G$; and we have theſe rules.

1. The ſum of the bodies is to twice the ſtruck body as the difference or ſum of the velocities, before impact, to the velocity loſt by the ſtriking body.

2. The ſum of the bodies is to twice the ſtriking body as the difference or ſum of the velocities, before impact, to the velocity gained by the ſtruck body. Q. E. I.

452. Cor. 1. $L : G :: 2l : 2g :: B : A$ and $A \times L = B \times G$; therefore in all perfectly elaſtic bodies $L = \frac{B \times G}{A}$ and $G = \frac{A \times L}{B}$.

453. Cor. 2. A's velocity after impact is equal to the difference between its velocity before impact and the velocity loſt, or to $a - \frac{2B \times a \mp b}{A + B} = \frac{Aa + Ba - 2Ba \pm 2Bb}{A + B} = \frac{A - B \times a \pm 2Bb}{A + B}$.

B's velocity after impact is equal to the ſum of its velocity before,

and the velocity gained, or to $\mp b + \frac{2A \times \overline{a \mp b}}{A+B} = \frac{2Aa \mp Bb \mp Ab}{A+B}$.

If $A = B$, $\frac{\overline{A-B} \times a \mp 2Bb}{A+B} = \mp b$, and $\frac{2Aa \mp Bb \mp Ab}{A+B} = a$, or the bodies move with interchanged velocities.

454. Cor. 3. If A be greater or less than B, L is less or greater than $a \mp b$, and G greater or less than $a \mp b$. If A be to B, as m to n, $L = \overline{a \mp b} \times \frac{2n}{m+n}$, and $G = \overline{a \mp b} \times \frac{2m}{m+n}$.

455. Cor. 4. If there be any number of bodies A, B, C, D, &c. in geometric progression, encreasing or decreasing, and A, moving with a velocity equal to a, communicate motion to B at rest, and B, moving with the velocity gained, communicate motion to C at rest, &c. the velocities communicated to B, C, D, &c. will be in geometric progression, decreasing or encreasing. For (451)

$A+B : 2A :: a :$ velocity of B
$B+C : 2B ::$ vel. of $B :$ vel. of C
$C+D : 2C ::$ vel. of $C :$ vel. of D
$D+E : 2D ::$ vel. of $D :$ vel. of E. And, because A, B, C, &c. are in geometric progression, $A+B : 2A :: B+C : 2B :: C+D : 2C :: D+E : 2D$; and consequently $a :$ vel. of $B ::$ vel of $B :$ vel of $C ::$ vel of $C :$ vel. of D, &c. And if the bodies encrease or decrease in magnitude, a is greater or less than the velocity of B, or the velocities communicated decrease or encrease.

456. Cor. 5. If the number of bodies in geometric progression be equal to n, $a :$ velocity of the last body $:: \overline{A+B}^{n-1} : \overline{2A}^{n-1}$; for $a :$ vel. of the last body in a ratio compounded of the ratios of $a :$ vel. of B, vel. of $B :$ vel. of C, vel. of $C :$ vel. of D, &c. each of which is equal to the ratio of $A+B : 2A$, and the number of ratios is equal to $n-1$. The velocity of A, or a, is also to the velocity of the last body $:: \overline{A+B \times B+C \times C+D}$, &c. $: A \times B \times C$, &c. (the product of all except the last) $\times 2^{n-1}$.

457. Cor.

457. Cor. 6. The velocities loſt by A, B, C, &c. are in geometric progreſſion encreaſing or decreaſing, according as the bodies decreaſe or encreaſe; for,

$A+B : 2B ::$ a : vel. loſt by A

$B+C : 2C ::$ vel. of B : vel. loſt by B

$C+D : 2D ::$ vel. of C : vel. loſt by C, &c.

But A, B, C, &c. being in geometric progreſſion, $A+B : 2B :: B+C : 2C :: C+D : 2D$, &c. and conſequently a : vel. loſt by A :: vel. of B : vel. loſt by B :: vel. of C : loſt by C, &c.; therefore the ratios of the velocities loſt and gained are the ſame, and the laſt being in geometric progreſſion (456), the firſt are ſo too.

458. PROP. *If there be three perfectly elaſtic bodies,* A, X, Q, *and* A, *moving with a velocity equal to* a, *impinge upon* X *at reſt, and* X *impinge upon* Q *at reſt,* Q's *velocity is the greateſt poſſible when* X *is a mean proportional between* A *and* Q.

DEM. Let q be the velocity of $\mathcal{Q}$, and $a : q :: \overline{A+X} \times \overline{X+\mathcal{Q}}$ $4AX$ (456) $:: A+X+Y+\mathcal{Q} : 4A$ (ſuppoſing A to be to X as Y to $\mathcal{Q}$, and conſequently $A+X : X :: Y+\mathcal{Q} : \mathcal{Q}$, and for $\overline{A+X} \times \mathcal{Q}$, ſubſtituting its equal $\overline{Y+\mathcal{Q}} \times X$); and $q = \frac{4Aa}{A+X+Y+\mathcal{Q}}$, and, $4Aa$ being given, q varies $\frac{1}{A+X+Y+\mathcal{Q}}$, and is greateſt when their ſum is leaſt, or when $X = Y$; for $A \times \mathcal{Q}$ being given, $X \times Y$ is given, and the ſum of two factors containing a given product is leaſt * when they are equal. Q. E. D.

459. Cor. 1. The velocity, communicated to $\mathcal{Q}$, will be encreaſed, by encreaſing the number of bodies in geometric progreſſion, between it and A, and arrive at its limit when that number

* Let the given product be repreſented by the given rectangle AB, contained by two unequal lines AL, LB. Make the ſquare LE equal to the rectangle AB and $AL + LB = AD = 2CM$ is always greater than $2LM$ or $LM + LF$. FIG. CXL.

number is infinitely great, and then $a : q :: \sqrt{Q} : \sqrt{A}$. Let the ſucceſſive differences of the bodies be x, y, z, &c. or $B = A + x$, $C = B + y$, $D = C + z$, &c. and let their reſpective velocities be a, b, c, c, d, &c.; becauſe $A + B : 2A :: a : b$ and $B = A + x$, by ſubſtitution, $A + A + x : 2A$ or $A + \frac{x}{2} : A :: a : b :: \sqrt{B} : \sqrt{A}$ (31); $B + C : 2B$ or $2B + y : 2B$ or $B + \frac{y}{2} : B :: b : c :: \sqrt{C} : \sqrt{B}$; and, for the ſame reaſon, $C + \frac{z}{2} : C :: c : d :: \sqrt{D} : \sqrt{C}$, &c. but $a : d$ in a ratio compounded of the ratios of $a : b$, $b : c$, $c : d$, &c. or their equals $\sqrt{B} : \sqrt{A}$, $\sqrt{C} : \sqrt{B}$, $\sqrt{D} : \sqrt{C}$, &c. or of the ſquare root of the laſt body to the ſquare root of the firſt (41); and conſequently $a : q :: \sqrt{Q} : \sqrt{A}$.

460. Cor. 2. Becauſe $A \times L = B \times G$, $A : B :: G : L$; but G and L are made in oppoſite directions, and are to each other as the ſpaces deſcribed by the bodies in the ſame time, and being inverſely as the diſtances from the center of gravity, this center will evidently not be affected by impact, but will continue to move with a velocity equal to v, their common velocity after impact when inelaſtic. This alſo appears from article 440.

461. PROP. *If two perfectly elaſtic bodies* A *and* B *impinge directly, the ſums of the products, reſulting from multiplying each body into the ſquare of its velocity, is the ſame before and after impact.*

DEM. Retaining the ſame notation, $a - v = l$, and $2a - 2v = L$, and $2v \mp 2b = G$; and conſequently A's velocity after impact is equal to $a - 2a + 2v = 2v - a$, and B's velocity $= \pm b + G = 2v \mp b$; but $A \times \overline{2v - a}^2 + B \times \overline{2v \mp b}^2 = A \times 4v \times \overline{v - a} + a^2 + B \times 4v \times \overline{v \mp b} + b^2 = A \times a^2 + B \times b^2$, becauſe $A \times a + B \times \pm b = \overline{A + B} \times v$, and conſequently $A \times \overline{v - a} +$

$B \times$

$B \times \overline{v \mp b} = 0$, or, multiplying this equation by $4v$, $A \times 4v \times \overline{v - a} + B\,4v \times \overline{v \mp b} = 0$*. Q.E.D.

462. Cor. 1. Whatever therefore be the number of elaſtic bodies A, B, C, &c. which impinge ſucceſſively, beginning with A, the ſums of the products, reſulting from the multiplication of each body into the ſquare of its velocity, are the ſame before and after impact.

463. Cor. 2. The relative velocities of A and B are the ſame before and after impact; for, before impact, the relative velocity is equal to the velocity of A diminiſhed or encreaſed by that of B, according as they move in the ſame, or an oppoſite, direction, or equal to $a \mp b$; and, after impact, it is equal to B's velocity diminiſhed or encreaſed by that of A, or equal to $2v \mp b - 2v + a = a \mp b$, the ſame as before impact.

464. Cor. 3. In this propoſition the force of elaſticity is equal to that of compreſſion, and this obtains very nearly in ſome kinds of bodies, as glaſs, ivory, tempered ſteel, &c. and, in theſe, the conſervatio vis vivæ is preſerved, or the products of the bodies into the

* This propoſition may be proved a little differently by the following proceſs: let p and q be equal to the velocities of A and B reſpectively, after impact, and $A \times a \pm B \times b = B \times q \mp Ap$ (3d law of motion) and $Aa \mp Ap = Bq \mp Bb$. But the relative velocities of A and B are the ſame before and after impact, or $a \mp b = q \mp p$, and $a \pm p = q \pm b$, and, multiplying $Aa \mp Ap$ into $a \pm p$, and $Bq \mp Bb$ into $q \pm b$, the reſults are

$$A \times a^2 \mp Apa \pm Apa - Ap^2 = B \times q^2 \mp Bqb \pm Bqb - Bb^2, \text{ or}$$
$$A \times a^2 + B \times b^2 = B \times q^2 + A \times p^2.$$

EXAMP. Let the ratios of $A : B$, and $a : b$, be reſpectively as $1 : 9$, and $6 : 1$, and, before impact, $A \times a^2 + B \times b^2 = 1 \times 36 + 9 \times 1 = 45$, and, after impact, $A \times p^2 = A \times \overline{a - \frac{2B \times \overline{a-b}}{A+B}}^2$, and $B \times q^2 = B \times \overline{b + \frac{2A \times \overline{a-b}}{A+B}}^2$; the firſt $= \overline{6 - \frac{18 \times 5}{10}}^2 = \overline{\frac{60-90}{10}}^2 = -\overline{3}^2$ or 9, and the ſecond $= 9 \times \overline{1 + \frac{2 \times 5}{10}}^2 = 9 \times 4 = 36$, and $A \times p^2 + B \times q^2 = 45$.

the ſquare of their velocities, are the ſame before and after impact. But in all bodies, endued with imperfect degrees of elaſticity, the equality of theſe products is not preſerved.

465. PROP. *When either* A *or* B *is perfectly hard, and the other imperfectly elaſtic, or when they both are imperfectly elaſtic, and the whole force of reſtitution is to that of compreſſion, as any number* n : 1, *to find the velocities of* A *and* B *after impact.*

DEM. When A and B are inelaſtic, let the velocities loſt by A and gained by B, reſpectively, by the force of impact, be l and g; and $1 : n :: l : nl =$ the velocity which A loſes by the force of reſtitution, and $nl + l = A$'s whole loſs of velocity. Let $m = n + 1$ and $a - ml = A$'s velocity after impact, $= \left(\text{becauſe } l = \frac{B \times g}{A}\right)$ $\frac{Aa - mgB}{A}$. B's velocity after impact $= \mp b + g + ng = \mp b + mg$. Q. E. I.

466. Cor. 1. Becauſe $l = \frac{B \times \overline{a \mp b}}{A + B}$, $\overline{n + 1} \times l$, or A's whole loſs of velocity, $= \overline{n + 1} \times \frac{B \times \overline{a \mp b}}{A + B}$, and conſequently $A + B : \overline{n + 1} \times B :: a \mp b$: velocity loſt by A. And becauſe $\overline{n + 1} \times g$, or B's whole gain of velocity, $= \overline{n + 1} \times \frac{A \times \overline{a \mp b}}{A + B}$, $A + B : \overline{n + 1} \times A :: a \mp b$: velocity gained by B.

467. Cor. 2. It is proved in the ſame manner, as when the bodies were perfectly elaſtic, that the velocities loſt and gained, by a ſeries of imperfectly elaſtic bodies in geometric progreſſion, which are all at reſt, except the firſt, are in geometric progreſſion; for let the

the velocities of A, B, C, D, &c. be a, b, c, &c. and

$$A+B : \overline{n+1} \times A :: a : b$$
$$B+C : \overline{n+1} \times B :: b : c$$
$$C+D : \overline{n+1} \times C :: c : d,$$

and becauſe $A : B :: B : C :: C : D$, &c.

$A+B : \overline{n+1} \times A :: B+C : \overline{n+1} \times B :: C+D : \overline{n+1} \times C$, &c.;

or $a : b :: b : c :: c : d$, &c.

468. Cor. 3. If elaſticity be equal to the reſiſtance made to compreſſion, $n = 1$, and, ſubſtituting p and q for the velocities of A and B after impact, $p = a - \frac{mBg}{A}$, and $q = mg \mp b$. Let elaſticity be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of the reſiſtance, or $n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, &c.; and $p = a - \frac{\frac{3}{2}Bg}{A}$, $a - \frac{\frac{4}{3}Bg}{A}$, $a - \frac{\frac{5}{4}Bg}{A}$, &c. and $q = \frac{3}{2}g \mp b$, $\frac{4}{3}g \mp b$, $\frac{5}{4}g \mp b$, &c.: if therefore A, B, g and n be given, the ratio of the relative velocities, or of $a \mp b : q \mp p$ may be found; and $v : v$, if the ratio of the relative velocities, and A, B, a, g be known, the degree of elaſticity, or n, may be found: for $p = a - \frac{mBg}{A}$ and $q = \mp b + mg$, and the ratio of $q - p \left(= b - a + g + \frac{Bg}{A} \times m\right) : a - b$ is a given ratio, and conſequently $m(n+1)$ is known.

469. Cor. 4. The converſe of prop. (461) is true, and if the products of A and B into the ſquares of their velocities, be the ſame before and after impact, the force of reſtitution is equal to that of compreſſion; and generally if the products of A and B into that power of the velocity, whoſe exponent is any number r, be ſuppoſed equal before and after impact, the relation between the forces of elaſticity and reſiſtance may be found: for $a - \frac{mBg}{A} = p$ and $\mp b + mg = q$, and by ſolving the equation $A \times a^r +$

$B \times b' = A \times a - \frac{mBg}{A}\Big|^r + B \times \pm\overline{b + mg}^r$, m, and consequently n may be found.

FIG. CXLI. 470. LEMMA. *If two ſperical bodies* A *and* B *move from* A *and* B *at the ſame time, with velocities which are to each other as* a : b, *or* AC : BD, *to find a plane that ſhall touch both bodies at the point of impact.*

Join AB, and deſcribe the parallogram AH, whoſe ſides are AC and AB; join DH, and from C, as a center, deſcribe a circular arc, with a radius equal to the ſemidiaters of the bodies, cutting DH in L; draw LE parallel to AC, and EF parallel to the line joining C and L, and the bodies will at the ſame time be in E and F, and a plane perpendicular to EF will touch both bodies at the point of impact; for $DE : EL$ or $FC :: DB : BH$ or AC, and

$DB : DB \pm DE$ or $BE :: AC : AC \pm FC$ or AF,

and $DB : AC :: BE : AF :: b : a$, and BE, AF are therefore deſcribed in the ſame time by B and A, and becauſe $EF = CL =$ the ſum of the ſemidiameters of the bodies, they will be in contact at P, with a plane perpendicular to EF. Q. E. I.

FIG. CXLII. 471. LEMMA. *If two ſperical bodies* A *and* B *inpinge obliquely, to determine the velocities loſt by* A *and gained by* B, *and the velocity of each after impact.*

Caſe 1. Suppoſe A and B to be inelaſtic, and to meet in D, and Ll to be a plane in contact with both the bodies, and the velocities to be FD and GD. Reſolve each velocity into two, FL, and GM, perpendicular to the plane, and DL, DM, parallel to it: DL and DM are not affected by impact, and FL, GM, are directly oppoſite to each other. Taking therefore $DH = \frac{A \times FL - B \times GM}{A + B}$

(448),

Plate XVIII.

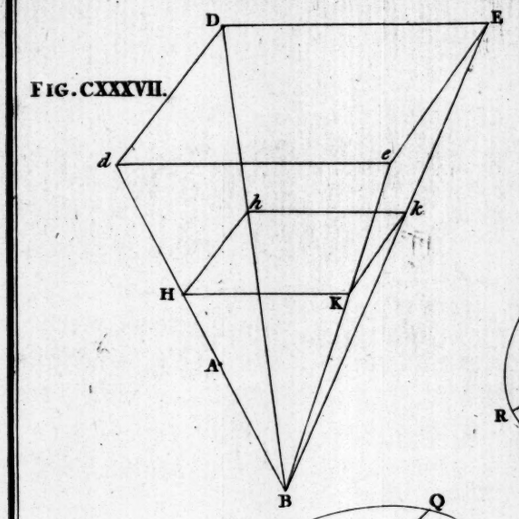


FIG. CXXXVII.

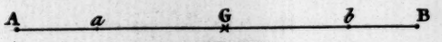


FIG. CXXXVIII.

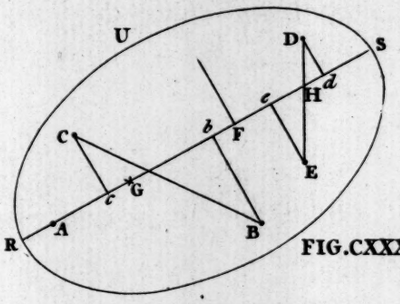


FIG. CXXXIX.

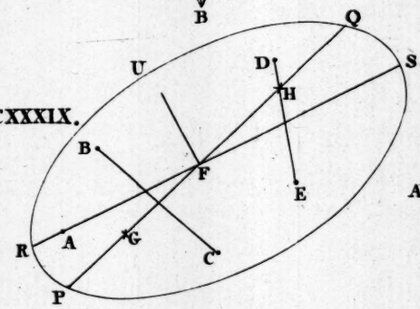


FIG. CXXXIX.

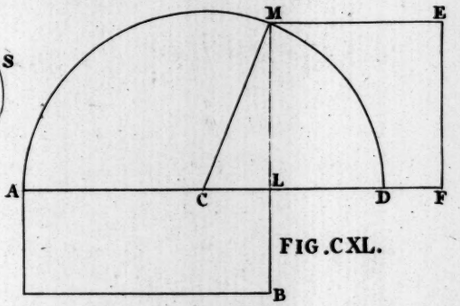


FIG. CXL.

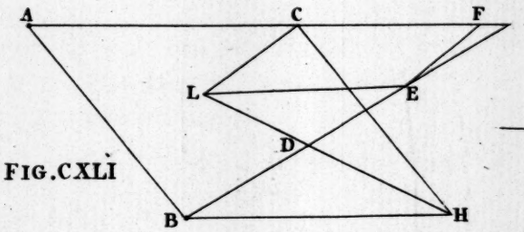


FIG. CXLI

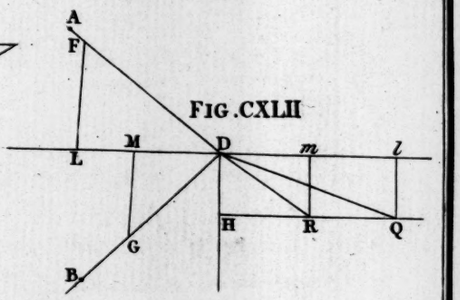


FIG. CXLII


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(448), and Dm, Dl respectively equal to DM, DL, A and B will describe the lines DQ, DR respectively.

Case 2. If A and B be either perfectly or imperfectly elastic, they will be reflected, and the velocities DE, DH be equal respectively to $FL - \frac{2B \times \overline{FL+GM}}{A+B}$ and $-GM + \frac{2A \times \overline{FL+GM}}{A+B}$ (453): taking therefore $Dm = DM$ and $Dl = DL$, A will be reflected along DK, and B along DP; for the velocities DM, DL remain after impact, and these, compounded with DH and DE respectively, will make A and B describe the diagonals of parallelograms, whose sides are DE, Dl, and DH, Dm. Q. E. I. FIG. CXLIII.

472. *LEMMA. *To find the spontaneous center of conversion of the particles* P *and* Q, *without gravity, or a point about which the right line* SR, *adhering to those particles, begins to revolve, when impressed perpendicularly by a force* F, *acting at a point* F *which is not the center of gravity* G. FIG. CXLIV.

Let pqS be a new position of SR, infinitely near to the former, and the point S is evidently stationary whilst P and Q describe the small spaces Pp and Qq: but $Q \times FQ : P \times FP :: Pp : Qq$ (443) $:: SP$ or $SF + FP : SQ$ or $SF - FQ$, and $Q \times FQ \times SF - Q \times FQ^2 = P \times FP^2 + P \times FP \times SF$, and $SF = \frac{P \times FP^2 + Q \times FQ^2}{Q \times FQ - P \times FP} = \frac{P \times FP^2 + Q \times FQ^2}{P + Q \times FG}$. Q. E. I.

473. Cor. 1. Whilst F acts at the same point, the point S is always the same whatever be the magnitude of F, because $\frac{P \times FP^2 + Q \times FQ^2}{P + Q \times F}$ is a given quantity.

474. Cor.

* Philos. Trans. for 1780, pag. 550, where this subject is treated with great perspicuity by Mr. Vince.

474. Cor. 2. To whatever point of QP the ſame force F be applied, the incipient motions of Q and P or $Q \times Qq + P \times Pp$, are invariably the ſame (2d law of motion); and conſequently the velocity of the center of gravity, or Gg, is always the ſame as if both particles were placed at G and acted upon by the ſame force F; for $Gg = \frac{P \times Pp + Q \times Qq}{P + Q}$ (406).

FIG. CXLV. 475. Cor. 3. If SUR be a material ſurface compoſed of the particles A, B, C, &c. and a force F act in the line PF placed in the ſurface, and perpendicular to a line SG drawn through the center of gravity G, each particle will receive an impulſe in a direction parallel to PF (2d law of motion); and becauſe G is not affected by the action of the particles upon each other, it will move in a right line (440), and the center of ſpontaneous converſion will be in ſome point of the line SG perpendicular to PF. Let S be that center, and the whole plane will begin to revolve about S; but if a force Q equal to F were to act in an oppoſite direction, all motion would evidently be deſtroyed; and conſequently the combined force of every particle in the ſurface (revolving round S, which reſults from the action of F), to make it revolve about F and deſtroy the action of Q would be equal to nothing. But the force of any particle $A = A \times SA$ multiplied into the perpendicular diſtance of its direction from $F = A \times SA \times Fm$, or, becauſe (letting fall Sa perpendicular to SF) $Fm : Fn :: Sa : SA$, and $Fm = \frac{Fn \times Sa}{SA}$ and $A \times SA \times Fm = A \times Fn \times Sa = A \times Sa \times \overline{SF - Sn}$ $= A \times Sa \times SF - A \times SA^2$: and by a ſimilar proceſs $B \times Sb \times SF - B \times SB^2 = B$'s force: conſequently $SF = \frac{A \times SA^2 + B \times SB^2 \&c.}{A \times Sa + B \times Sb \&c.}$ $= \frac{A \times SA^2 + B \times SB^2 \&c.}{\overline{A + B \&c.} \times SG}$. If S be the center of ſuſpenſion, F will therefore be the center of oſcillation (art. 512).

476. Cor.

476. Cor. 4. If pqS be the next position of SR, and xz be drawn parallel to it, P has been progressive through xp, and Q regressive through zq, subtending an angle at g equal to that at S. The initial motion of the center of gravity is not affected by the unequal motions of P and Q, because their actions upon it, or $P \times pg$ and $Q \times qg$ are equal and opposite, and the motion of G is consequently rectilineal and uniform (1st law of motion); and because the incipient angular motions of P and Q about g continue to be uniform, it is evident that when the angle GSg would become equal to four right angles, or P and Q have made one revolution round g, Gg, the line described by the center of gravity, would be equal to the periphery of a circle whose radius is SG. FIG. CXLIV.

477. PROP. *Let a body* B, *be impelled by a force* F, *acting in the direction* F D *not passing through its center of gravity* G, *to define its motion.* FIG. CXLVI.

Through G draw a right line SR perpendicular to FD produced, and, if S be the spontaneous center of conversion of the plane surface SUR, every particle will receive an impulse, and begin to move in a direction parallel to this surface (2d law of motion), and consequently to revolve round an axis passing through S perpendicular to it. And if the parts of every section of the body, perpendicular to DF, on each side of the plane SUR, be similar and similarly situated, the plane of conversion SUR will not be affected by either the progressive or rotatory motion of the particles: for their effects upon SUR, arising from their progressive motion, are as the sums of the products resulting from the multiplication of each particle into its distance from this plane, which are equal and opposite; and if p and q be two equal particles, similarly situated in any plane perpendicular to DF, their effects, arising from their rotation, being measured by the product of each particle into the distance from the axis, will be in that plane, and equal and opposite to each other. Consequently the incipient plane of rotation remains unaltered, and if a right line be drawn through G, perpendicular to the plane SUR, every particle

ticle will move round this line as an axis, whilſt G moves uniformly in a right line.

478. Cor. 1. In a body, B, revolving by the action of a force impelling it in a direction not paſſing through the center of gravity, the diſtance of the ſpontaneous center of converſion from the interſection of F's direction with a right line paſſing through the center of gravity, that is, SD is equal to the ſum of the particles multiplied into the ſquare of the diſtance of each, divided by B multiplied into SG.

FIG. CXLV.

479. Cor. 2. $A \times SA^2 = A \times \overline{SG^2 + GA^2 + 2SG \times Ga}$ (Euc. B. 2. P. 12)

$$B \times SB^2 = B \times \overline{SG^2 + GB^2 + 2SG \times Gb}$$

$$C \times SC^2 = C \times \overline{SG^2 + GC^2 + 2SG \times Gc}$$

$$D \times SD^2 = D \times \overline{SG^2 + GD^2 - 2SG \times Gd}, \text{ \&c.}$$

and, becauſe $A \times 2SG \times Ga + B \times 2SG \times Gb + C \times 2SG \times Gc - D \times 2SG \times GD$, \&c. $= 0$, $A \times SA^2 + B \times SB^2 + C \times SC^2 + D \times SD^2 = A \times \overline{SG^2 \times GA^2} + B \times \overline{SG^2 + GB^2} + C \times \overline{SG^2 + GC^2}$, \&c.; therefore $SF = SG + GF = \frac{A \times \overline{SG^2 + GA^2} + B \times \overline{SG^2 + GB^2}}{A + B + C, \text{ \&c.} \times SG}$, \&c. \&c.; and $FG = \frac{A \times GA^2 + B \times GB^2 + C \times GC^2}{A + B + C, \text{ \&c.} \times SG}$, \&c., $SG \times GF$ is equal to a given quantity, or SG and GF vary inverſely as each other.

FIG. CXLVI.

480. Cor. 3. Becauſe the velocity of G is the ſame as if the particles were concentrated in G, and acted upon by the force F, the time of deſcribing the angle GSg or qgz is the ſame to whatever point F be applied: but, Gg being given, the angle GSg varies inverſely as SG, or directly as GF (laſt cor.). If different forces act at the ſame point, Gg, or the angle of rotation, will evidently be as the magnitude of the force; and, conſequently, the angular velocity will vary generally as $F \times GF$, or as the magnitude of the force multiplied into the perpendicular diſtance of the center of gravity from its direction.

481. PROP.

481. PROP. *Let a body, whoſe quantity of matter is* Q, *moving with a velocity equal to* V, *impinge upon the body* B, *in the direction* FC *paſſing through the center of gravity of* Q, *to find the velocity of the center of gravity,* G, *of the body* B. FIG. CXLVI.

Let g be the velocity of G, and S the ſpontaneous center of converſion of B; and $SG : SD :: g :$ velocity of D, which is therefore equal to $\frac{SD \times g}{SG}$. $V - \frac{SD \times g}{SG}$ is therefore equal to the velocity loſt by Q in the direction FC, and (3d law of motion) $Q \times V - \frac{SD \times g}{SG} = B \times g$, and $\frac{Q \times V \times SG - SD \times g}{SG} = B \times g$, and $g = \frac{Q \times V \times SG}{B \times SG + Q \times SD}$. Q. E. I.

482. Cor. 1. If the bodies be perfectly elaſtic, the velocity of G, after impact, is equal to $\frac{2Q \times V \times SG}{B \times SG + Q \times SD}$.

483. Cor. 2. Becauſe the velocity of Q, after impact, is equal to that of D, or to $\frac{SD \times g}{SG}$, or, ſubſtituting the velocity of g, equal to $\frac{Q \times V \times SD}{B \times SG + Q \times SD}$; $V - \frac{Q \times V \times SD}{B \times SG + Q \times SD}$ is Q's loſs of velocity, when the bodies are inelaſtic, which is equal to $\frac{B \times V \times SG}{B \times SG + Q \times SD}$ = Q's loſs of velocity, and conſequently Q's velocity after impact, when the bodies are perfectly elaſtic, is equal to $V - \frac{2B \times V \times SG}{B \times SG + Q \times SD}$ = $\frac{V \times Q \times SD - B \times SG}{B \times SG + Q \times SD}$.

484. Cor.

484. Cor. 3. If FC pafs through the center of gravity G, of the body B, or the impact become direct, $SG = SD$, and G's velocity $= \frac{2\mathcal{Q} \times V \times SG}{B \times SG + \mathcal{Q} \times SG} = \frac{2\mathcal{Q} \times V}{B + \mathcal{Q}}$. $\mathcal{Q}$'s velocity $= \frac{V \times \overline{\mathcal{Q} \times SD - B \times SD}}{B \times SG + \mathcal{Q} \times SD}$ $= V \times \frac{\overline{\mathcal{Q} - B}}{B + \mathcal{Q}}$; and rules are deduced from thefe expreffions, which are the fame as in direct impact before inveftigated.

485. Cor. 4. Becaufe $\mathcal{Q} \times V = \overline{\mathcal{Q} + B} \times g$ (3d law of motion) $g = \frac{\mathcal{Q} \times V}{\mathcal{Q} + B}$, or the progreffive motion of B is the fame upon whatever point of B the body $\mathcal{Q}$ impinges.

486. Cor. 5. If Υ be the magnitude of a body placed at D, which receives the fame velocity from the impact of $\mathcal{Q}$, that was communicated to the point D, and, confequently, the velocity of Υ, after impact, will be equal to $\frac{\mathcal{Q} \times V}{\mathcal{Q} + \Upsilon} = \frac{\mathcal{Q} \times V \times SD}{B \times SG + \mathcal{Q} \times SD}$, and $\Upsilon = \frac{B \times SG}{SD}$. Hence if the direction does not pafs through the center of gravity of $\mathcal{Q}$, find the diftance of the fpontaneous center of converfion of $\mathcal{Q}$, s, and its center of gravity I, and a body whofe magnitude is equal to $\mathcal{Q} \times \frac{sI}{sd}$ impinging directly, would have the fame effect upon B, whofe velocity confequently may be eftimated as above.

CHAP.

CHAP. XI.

*CENTERS OF PERCUSSION, GYRATION AND OSCILLATION.

487. DEF. *THE center of percuſſion of a body, or ſyſtem of bodies, is a point, which being ſtopped by an immoveable obſtacle, the body or ſyſtem is perfectly quieſcent.*

If *A*, *B*, *C*, &c. be particles of a body, or bodies, whoſe centers of gravity are the points *A*, *B*, *C*, &c. connected by inflexible lines, and moving with equal velocities, in directions parallel to any right line *DN*, an immoveable obſtacle oppoſed to the center of gravity *G* in the direction *ND* will deſtroy all motion. For, when *G* is ſtopped, each particle will endeavour, by its inertia, to proceed in the line in which it was moving, with a moment equal to the product of its quantity of matter and velocity; and, if any plane be drawn through *DGN*, the effort of each particle to communicate motion to it varies as its moment multiplied into its perpendicular diſtance (279). But, the velocities of *A*, *B*, *C*, &c. being equal, the ſums of theſe efforts to move the plane, on each ſide of it, are equal (409), and being oppoſite they conſequently deſtroy each other, and *A*, *B*, *C*, &c. are perfectly quieſcent. If different bodies, or particles of the ſame body *A*, *B*, *C*, &c., whoſe relative ſituation is unchangeable, revolve about an axis paſſing through any point *S*, and a plane be drawn through their center of gravity *G* perpendicular to the axis, each particle will deſcribe a plane parallel to this plane, their angular velocities will be equal, and their lineal velocities will be as their perpendicular diſtances from the axis of ſuſpenſion.

FIG. CXLVII.

FIG. CXLVIII.

* Simpſon's Fluxions, pag. 210. Lyons's Fluxions, pag. 244. Emerſon's Mechanics, Sect. VI.

ſion. And if, as before, a plane be drawn through G, parallel to the axis, the ſums of the moments of A, B, C, &c. multiplied into their perpendicular diſtances, on each ſide of this plane, are not equal, becauſe the velocities are not equal; and conſequently if G be ſtopped, A, B, C, &c. will not be quieſcent. But the lateral efforts to move the plane paſſing through SG perpendicularly to the axis, being ſuppoſed equal, it is evident that the center of percuſſion will be in SG.

FIG. CXLVIII.

488. PROP. *Let* A, B, C, *&c. be particles of a body, revolving about an axis paſſing through the point* S, *it is required to find the center of percuſſion.*

Let A, B, C, &c. be the places of the particles reduced to the plane paſſing through SG, perpendicular to the axis, by letting fall perpendiculars from them upon it; and let A's moment or $A \times SA$, becauſe its velocity is as SA, be repreſented in quantity and direction by Ap perpendicular to SA, and reſolved into two forces Aq perpendicular, and pq parallel to SG. The whole moment of A: its force perpendicular to $SG :: Ap (A \times SA) : Aq :: SA : Sa$ (ſim. triangles), and $Aq = \frac{A \times SA \times Sa}{SA} = A \times Sa$; and, ſuppoſing P to be the center of percuſſion, the efficacy of A to communicate motion to P is equal to its perpendicular moment multiplied into the diſtance at which it acts $= A \times Sa \times Pm = A \times Sa \times \overline{SP - Sm} = A \times Sa \times SP - \frac{SA^2}{Sa} = A \times Sa \times SP - A \times SA^2$. By a ſimilar proceſs it appears, that the efficacy of B, C, &c. to communicate motion to P, is $B \times Sb \times SP - B \times SB^2$, $C \times SC^2 - C \times Sc \times SP$, &c.; and, becauſe P is quieſcent, theſe forces are equal on each ſide of it, or $A \times Sa \times SP - A \times SA^2 + B \times Sb \times SP - B \times SB^2 = C \times SC^2 - C \times Sc \times SP$, and conſequently $SP = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A \times Sa + B \times Sb + C \times Sc}$, &c. Q. E. I.

489. Cor.

489. Cor. 1. Becauſe $A \times Sa + B \times Sb + C \times Sc$, &c. $= \overline{A+B+C}$, &c. $\times SG$ (410), $SP = \dfrac{A \times SA^2 + B \times SB^2 + C \times SC^2}{\overline{A+B+C}, \text{ &c.} \times SG}$, &c.

490. Cor. 2. The diſtance of the center of percuſſion from the center of gravity, or GP, is equal to $\dfrac{A \times GA^2 + B \times GB^2 + C \times GC^2}{\overline{A+B+C}, \text{ &c.} \times SG}$, &c. (479).

491. Cor. 3. In the ſame body, or ſyſtem of bodies, A, B, C, &c. $\overline{A+B+C}$, &c. $\times SG \times GP = A \times GA^2 + B \times GB^2 + C \times GC^2$, &c. $SG \times GP$ is alſo a given quantity, wherever the point of ſuſpenſion be placed, becauſe $\dfrac{A \times GA^2 + B \times GB^2}{\overline{A+B}, \text{ &c.}}$, &c. is given; and if SG be infinitely great, or the velocities of A, B, C, &c. be equal to each other, GP is evaneſcent.

492. Cor. 4. If a circle be deſcribed from G as a center, with a radius equal to SG, and, the plane of motion remaining as before, the points of ſuſpenſion be in different points of the periphery of this circle, the diſtance between the centers of gravity and percuſſion will be invariable; for SG being given, GP is given (laſt cor.)

493. Cor. 5. If particles, whoſe magnitudes are $\dfrac{A \times SA^2}{SP^2}$, $\dfrac{B \times SB^2}{SP^2}$, $\dfrac{C \times SC^2}{SP^2}$, &c. be concentrated in P, the ſame angular velocity will be generated, in theſe particles, by a force F acting for a given time, and in A, B, C, &c. at their reſpective diſtances SA, SB, SC, &c. For, let X, Y, Z, &c. be reſpectively in equilibrio with A, B, C, &c. and their moments are inverſely as their perpendicular diſtances (272), or $A \times SA : X \times SP :: SP : SA$, and $A \times SA^2 = X \times SP^2$,

$\times SP^2$, and $X = \frac{A \times SA^2}{SP^2}$. By a ſimilar proceſs it is proved that $Y = \frac{B \times SB^2}{SP^2}$, and $Z = \frac{C \times SC^2}{SP^2}$.

494. Cor. 6. A pendulous body moving with a given angular velocity, will have the greateſt effect upon another body againſt which it impinges, when the point of impact is the center of percuſſion; for in this caſe all its motion is communicated. But when the direction of impact does not paſs through this center, it will have a lateral motion, or endeavour to continue its rotation.

495. Cor. 7. If the points A, B, C, &c. and G be reduced to any plane perpendicular to the axis, and the center of percuſſion be ſuppoſed to be in this plane, its diſtance from the axis of ſuſpenſion is found, by ſubſtituting the diſtances of their places in this plane from the axis, inſtead of SA, SB, &c. in the expreſſion $\frac{A \times SA^2 + B \times SB^2}{A + B, \text{ &c.} \times GS}$, &c.

496. LEMMA. *The velocity* V, *generated in a body whoſe quantity of matter is* Q, *by the action of a conſtant force* F, *for a given time, will vary as the force directly and quantity of matter inverſely.*

DEM. Let V, Q, F, be variable, and it is evident that V will be encreaſed and diminiſhed in the ſame ratio with F directly, and in the ſame ratio with which the inertia or (pag. 63.) Q is diminiſhed and encreaſed; or V varies as $\frac{F}{Q}$. Q. E. D.

497. Cor. Becauſe the velocity is equal to the moment of a body divided by its quantity of matter, if m repreſent the quantity of motion generated in a given time by F, or the numeral product of

of the velocity and quantity of matter in a given body, V will always be equal to $\frac{m}{2}$.

498. PROP. *If any constant force* F *act, for a given time, perpendicularly upon the line* SV, *at the point* V, *and* SV *and the particles* A, B, C, &c. *whose center of gravity is* G, *are connected to an axis passing through a fixed point* S, *the velocity communicated to the point* V *will be as* $\frac{F \times SV^2}{A \times SA^2 + B \times SB^2 + C \times SC^2}$ &c. FIG. CXLVIII.

DEM. Suppose particles, whose magnitudes are $\frac{A \times SA^2}{SV^2}$, $\frac{B \times SB^2}{SV^2}$, $\frac{C \times SC^2}{SV^2}$, &c. to be concentrated in V, and they would be in equilibrio with A, B, C, &c. respectively (493), and consequently equally resist the action of F. But the velocity generated in these particles, collected in V, is as $\frac{F}{2}$ (496), or as $\frac{F}{\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SV^2}}$, &c. or as $\frac{F \times SV^2}{A \times SA^2 + B \times SB^2 + C \times SC^2}$. Q. E. D.

499. Cor. 1. If m, as before (497), represent the absolute quantity of motion generated by F in a given time, then the velocity of $V = \frac{m \times SV^2}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c.

500. Cor. 2. Because all points describe equal angles at the axis, in equal times, the velocity of V is to the velocity of $A :: SV : SA$, and the velocity of $A =$ velocity of $V \times SA = \frac{m \times SV \times SA}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c.; and by a similar process, the velocity

velocity of $B = \frac{m \times SV \times SB}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c.; and the velocity of $C = \frac{m \times SV}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c.

501. Cor. 3. The angular velocity of a body, being as its lineal velocity directly and distance inversely, the angular velocity of A, or B, &c. $= \frac{m \times SV}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c. And the moment of a body being measured by the quantity of matter and velocity, the sum of the moments communicated to A, B, C, &c. will be equal to $\frac{\overline{A \times SA + B \times SB + C \times SC, \text{ \&c.}} \times m \times SV}{A \times SA^2 + B \times SB^2 + C \times SC^2}$ (last cor.), &c.

FIG. CXLIX. 502. Cor. 4. If A, B, C, &c. be collected in any point L, the quantity of motion generated in them, in a given time, by the action of the constant force F at V is equal to $\frac{\overline{A+B+C, \text{\&c.}} \times SL \times m \times SV}{\overline{A+B+C, \text{\&c.}} \times SL^2}$ (500) $= \frac{A \times SA + B \times SB + C \times SC, \text{ \&c.} \times m \times SV}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c. and consequently $SL = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \text{ \&c.}}{A \times SA + B \times SB + C + SC, \text{ \&c.}}$: but $A \times SA + B \times SB + C \times SC = \overline{A + B + C, \text{ \&c.}} \times SG$, and $SL = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{\overline{A + B + C, \text{ \&c.}} \times SG}$, &c., and L is the center of percussion. If therefore any system of particles be concentrated in the center of percussion, the quantities of motion generated in them, in a given time, placed in that point and at their respective distances, SA, SB, &c., by the action of F at any point V, are the same.

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503. In the preceding proposition and corollaries A, B, C, &c. are supposed to be unaffected by gravity, and motion to be communicated

FIG. CXLIII.

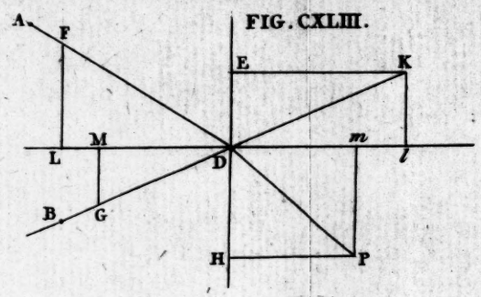


FIG. CXLIV.

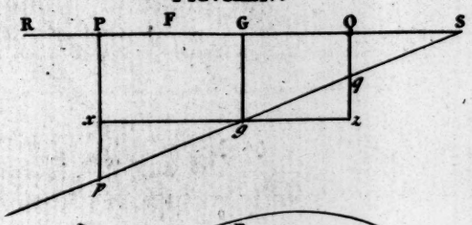
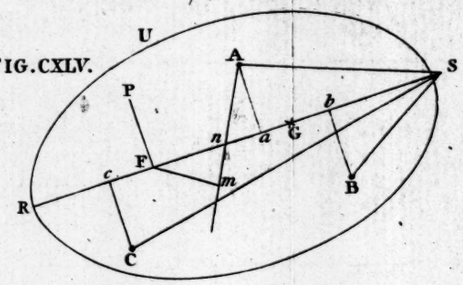


FIG. CXLV.



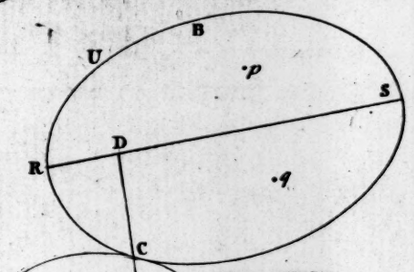


FIG. CXLVI.

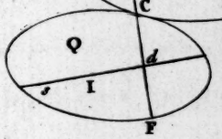
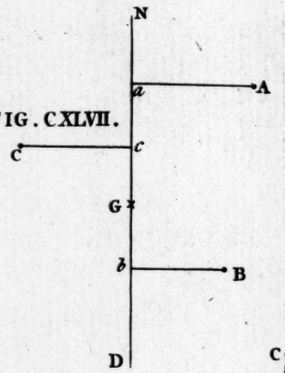


FIG. CXLVII.



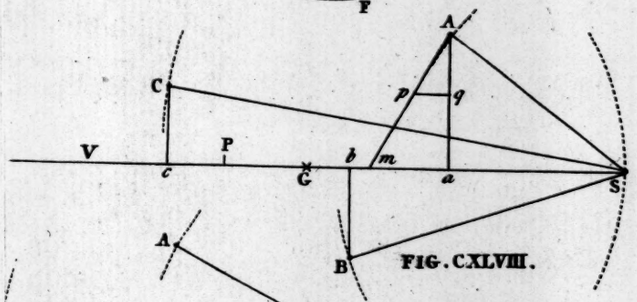


FIG. CXLVIII.

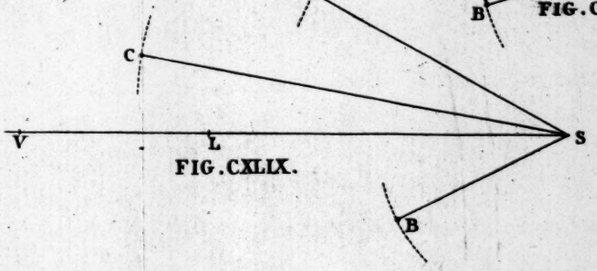


FIG. CXLIX.


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municated by ſome conſtant external force F, which is only reſiſted by their inertia; but the rules inveſtigated are applicable when F is the force of gravity, or that force acts in conjunction with F: for during an infinitely ſmall time the force of gravity may be conſidered as conſtant. If a ſyſtem of bodies A, B, C, &c. whoſe center of gravity is G, be ſuſpended from an horizontal axis paſſing through S, the ſum of their efforts to deſcend is the ſame as if they were collected in G, and they will therefore deſcend till SG be perpendicular to the horizon. In any other poſition of SG, let GL repreſent the force of deſcent in a direction perpendicular to the horizon, and be reſolved into two forces, GE touching the arc, and EL in the direction of SG. EL is the tendency from S, and GE is that part of the whole force of gravity which produces a rotation about S, and may be deemed conſtant whilſt G deſcribes an infinitely ſmall arc, and is to be added to, or ſubtracted from, F, according as they conſpire with, or oppoſe, each other. FIG. CL.

CENTER OF GYRATION.

504. DEF. *The center of gyration of a body, or ſyſtem of bodies,* A, B, C, *&c. is a point* Y, *in which, if* A + B + C, *&c. be collected, the ſame angular velocity will be generated, in a given time, by any conſtant force* F, *acting at any point* V *in a given direction, whether* A + B + C, *&c. be collected in* Y, *or be at their original diſtances* SA, SB, SC, *&c.* FIG. CLI.

505. PROP. *To find the center of gyration of the particles* A, B, C, *&c. which are connected to an axis paſſing through the point* S, *and preſerve their relative ſituation.*

Let SA, SB, SC, &c. be the perpendicular diſtances of A, B, C, &c. from the axis; and, ſuppoſing the quantity of motion generated by F, in a given time, to be m, their angular velocity is equal to

to $\frac{m \times SV}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c. (501); and, when $A + B + C$, &c. are concentrated in Y, their angular velocity $= \frac{m \times SV}{\overline{A + B + C, \&c.} \times SY^2}$. But, from the definition of the center of gyration, $\frac{m \times SV}{A \times SA^2 + B \times SB^2, \&c.} = \frac{m \times SV}{A + B + C \times SY^2}$, and $SY^2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \&c.}{A + B + C, \&c.}$, and $SY =$ the distance of the center of gyration from $S = \sqrt{\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A + B + C, \&c.}}$, &c.

506. Cor. 1. If therefore $p =$ a particle of a material surface or body, B, $d =$ its perpendicular distance from the axis of suspension passing through S, the distance of the center of gyration from the axis is equal to the sum of all the $\frac{d^2 \times p}{B}$.

507. Cor. 2. If any part of a system of particles A, B, &c. be collected in their center of gyration, the center of gyration of the whole system will continue the same; for the same force will communicate an equal angular velocity to A and B and $A + B$ placed in their center of gyration. A, B, C, &c. may consequently be considered as bodies of any magnitude, whose centers of gyration are the points A, B, C, &c. respectively.

508. Cor. 3. Because $\overline{A + B + C, \&c.} \times SY^2 = A \times SA^2 + B \times SB^2 + C \times SC^2$, &c. $= \overline{A + B + C} \times SG \times SP$ (489), therefore $SY^2 = SG \times SP$, or the distance of the center of gyration from the axis of suspension is a mean proportional between the distances of the center of gravity and the center of percussion from that axis.

509. Cor.

509. Cor. 4. If a ſyſtem of bodies A, B, C, &c. whoſe ſum $= \mathfrak{Q}$, connected as in this propoſition, be ſtruck by a given moment at the point V, in a direction perpendicular to a line SV, drawn from V through the center of gravity and perpendicular to an axis paſſing through S, their angular velocity may be found: for, let Υ be the center of gyration, G the center of gravity, and the angular velocity of A, B, C, &c. is the ſame, whether $\mathfrak{Q}$ be collected at Υ, or a body equal to $\frac{\mathfrak{Q} \times S\Upsilon^2}{SV^2}$ (489), or $\frac{\mathfrak{Q} \times SG \times SP}{SV^2}$ be directly oppoſed to the impinging body at the point V. But $A + B + C$, &c. or $\mathfrak{Q}$ being given, and alſo the quantity of matter and velocity of the ſtriking body, the velocity of $\mathfrak{Q}$ may be found by the common rules in the direct impact of bodies (449). And the arc deſcribed in a given time by $\mathfrak{Q}$ and the diſtance SV, being known, the angle which it ſubtends may be found. The converſe of this, or the velocity of the impinging body, may be found, if its quantity of matter, and $\mathfrak{Q}$ and its velocity, be given. FIG. CLI.

510. Cor. 5. In a given ſyſtem of bodies A, B, C, &c. whoſe center of gravity is G, if a circle be deſcribed from G as a center with any radius, and the center of ſuſpenſion S be in its periphery, the diſtance of the center of gyration from S is always the ſame; for $S\Upsilon^2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2, \text{ \&c.}}{A + B + C, \text{ \&c.}}$, which is a given quantity (412).

511. Cor. 6. If the periphery of a circle, compoſed of the particles A, B, C, &c. were to revolve about an axis perpendicular to its plane and paſſing through its center, the center of gyration will be in the periphery; for $S\Upsilon = \sqrt{\frac{A + B + C, \text{ \&c.} \times \text{rad.}^2}{A + B + C}} =$ radius. The angular velocity is therefore the ſame as if all the matter were collected in any one point of the periphery; and if all the matter in a gyrating body is to be placed in the center of gyration Υ, it may be placed in any point of the periphery of a

circle whose radius is SY, or collected into two equal portions, and placed in two points diametrically opposite to each other, and whose distances from $S = SY$; for then the center of gravity will be in S, and there will be no lateral motion.

CENTER OF OSCILLATION.

FIG. CLII.

512. DEF. *The center of oscillation of a body, or system of bodies,* A, B, C, *&c. connected to an axis passing through* S, *and moving by the action of gravity, is a point, in which, if the body or system be collected, it, and* A, B, C, *&c. placed at their original distances* SA, SB, SC, *&c. will describe equal angles at the axis in the same times.*

513. PROP. *Suppose* A, B, C, *&c. to be particles of a body connected to an horizontal axis passing through* S, *and acted upon by the force of gravity in a direction parallel to the vertical line* SV, *to find their center of oscillation.*

Let SGO be drawn in the plane described by the center of gravity G, and O be the center of oscillation, and the lines SO, SA, SB, &c. will describe equal angles at the axis in equal times. If the weight of A, which is as A (234), acted perpendicularly to SA, its moment would be $A \times SA$: let this be represented by Ap, perpendicular to the horizon, and resolved into two, Aq perpendicular to SA, and qp parallel to it, and this last being lost, Aq is the only efficient part of A's moment. But $Ap\ (A \times SA) : Aq :: SA : Aa$ (sim. triangles) and $Aq = \frac{A \times SA \times Aa}{SA} = A \times Aa$*; and the angular velocity of A generated in a given time is (501) $\frac{A \times Aa}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, &c. By a similar process the angular velocities of B, C, &c. generated in the same time, are $-B \times b$

* This follows immediately from (279), for Aa is equal to the perpendicular let fall from the center of motion upon the direction of A's weight.

$\frac{-B\times Bb}{A\times SA^2+B\times SB^2+C\times SC^2}$, &c., $\frac{-C\times Cc}{A\times SA^2+B\times SB^2+C\times SC^2}$, &c.

When $A+B+C$, &c. are concentrated in O, the angular velocity of O generated in the same time, would be as $\frac{\overline{A+B+C,\&c.}\times Oo}{\overline{A+B+C,\&c.}\times SO^2}$ $=\frac{Oo}{SO^2}=\frac{Gg}{SG\times SO}$ (sim.trian.): therefore $\frac{A\times Aa-B\times Bb-C\times Cc}{A\times SA^2+B\times SB^2+C\times SC^2}$ $=\frac{Gg}{SG\times SO}$; but $A\times Aa-B\times Bb-C\times Cc=\overline{A+B+C,\&c.}\times Gg$, and consequently by substitution, $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{\overline{A+B+C,\&c.}\times SG}$.

514. Cor. 1. The distances of the centers of oscillation and percussion from the axis are equal, each being equal to $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{\overline{A+B+C}\times SG}$, &c., and consequently $SG\times GO$ equal is to $\frac{A\times GA^2+B\times GB^2}{\overline{A+B+C,\&c.}}$, &c. (490), and SG and GO are inversely as each other. If SG be infinitely great, or A, B, C, &c. move with equal velocities in parallel lines, GO is evanescent, or the centers of gravity and oscillation coincide.

515. Cor. 2. If any number of particles be connected to an axis and revolve round it, retaining their relative situation, by the action of a force whose direction is in the plane of motion and perpendicular to the axis, the time of describing any given angle is the same as if they were concentrated in O and urged in the same direction by the same force; and the points where the forces act are transferred from A, B, C, &c. to O. But in the center of gyration Y, the same force is supposed to act at the same point, when A, B, C, &c. are placed at their respective distances SA, SB, SC, &c. and concentrated in Y.

516. Cor. 3. If A, B, C, &c. be bodies compoſed of any number of particles, let l, m, n be the reſpective diſtances of their centers of gravity, and p, q, r, the diſtances of their centers of oſcillation from the axis, and SO will be equal to $\frac{l \times p \times A + m \times q \times B + n \times r \times C}{A + B + C, \&c. \times SG}$: for let a, b, c, be particles of A, B, C, and x, y, z, their reſpective diſtances from the axis, and, by this propoſition, $SO =$ $\frac{\text{ſum of all the } a \times x^2 + \text{ſum of } b \times y^2 + \text{ſum of } c \times z^2}{SG \times A + B + C}$; but the ſums of all the $\frac{a \times x^2}{l \times A}$, $\frac{b \times y^2}{m \times B}$, $\frac{c \times z^2}{n \times C}$ are reſpectively equal to p, q, r, and conſequently, the ſums of $a \times x^2 + b \times y^2 + c \times z^2 = p \times l \times A + q \times m \times B + r \times n \times C$, and $SO = \frac{l \times p \times A + q \times m \times B + r \times n \times C}{A + B + C, \&c.}$.

FIG. CXLVIII. 517. Cor. 4. If A, B, C, conſiſting of a number of particles, were urged by a conſtant force F at the point V, as in (498): let q, r, s, &c. be particles in A, t, u, v, &c. particles in B, and x, y, z, &c. particles in C, and $A = q + r + s$, &c., $B = t + u + v$, &c., $C = x + y + z$, &c.; and the velocity communicated to V in a given time is as $\frac{F \times SV^2}{q \times Sq^2 + r \times Sr^2 + s \times Ss^2, \&c. + t \times St^2 + u \times Su^2, \&c. + x \times Ss^2 + y \times Sy^2, \&c.}$; but if P, p, π be the centers of percuſſion, and G, g, γ the centers of gravity of A, B, C, &c. reſpectively, then (489)

$$q \times Sq^2 + r \times Sr^2 + s \times Ss^2, \&c. = SP \times SG \times A$$
$$t \times St^2 + u \times Su^2 + v \times Sv^2, \&c. = Sp \times Sg \times B$$
$$x \times Sx^2 + y \times Sy^2 + z \times Sz^2, \&c. = S\pi \times S\gamma \times C;$$

and the velocity of V is as $\frac{F \times SV^2}{SP \times SG \times A + Sp \times Sg + B + S\pi \times S\gamma \times C}$.

FIG. CLIII. 518. PROP. *If* O *be the center of oſcillation when the axis of ſuſpenſion paſſes through* S, *the point* S *will be the center of oſcillation when the axis of ſuſpenſion paſſes through* O, *the plane of motion being ſuppoſed to be unaltered.*

DEM.

DEM. When S is the point of ſuſpenſion, the diſtance of the center of oſcillation from the center of gravity G, or $GO = \frac{A \times AG^2 + B \times BG^2 + C \times CG^2}{A + B + C, \&c. \times SG}$, &c. (514); and when the axis paſſes through O, the diſtance of the center of oſcillation from $G = D = \frac{A \times AG^2 + B \times BG^2}{A + B + C \times OG}$, &c.; and conſequently $SG \times GO = D \times GO$ and $SG = D$, and the diſtance of the center of oſcillation from $O = SO$. Q. E. D.

519. Cor. 1. If two circles be deſcribed from G as a center with the radii GS and GO, the diſtances of the centers of oſcillation from the points of ſuſpenſion would be the ſame in whatever parts of the peripheries they were placed; for SG, or GO being given, the other is given (491), and conſequently the times of deſcribing equal angles at the axis will be equal, if the points of ſuſpenſion be any where in theſe peripheries.

520. Cor. 2. If the axis of rotation paſſed through the center of gravity G, and Υ were the center of gyration, $G\Upsilon^2$ would be equal to $\frac{A \times GA^2 + B \times GB^2 + C \times GC^2}{A + B + C}$ (505) $= SG \times GO$, and conſequently GO is a third proportional to SG and $G\Upsilon$.

521. Cor. 3. Becauſe $SG \times GO$ is a given quantity, $SG + GO$ is the leaſt poſſible when the peripheries of the circle, whoſe radii are SG and GO, meet in Υ, becauſe in that ſuppoſition $SG = GO$ (note to 458). And becauſe in this caſe SO is the leaſt poſſible, the time of deſcribing a given angle at the axis is the leaſt poſſible. But becauſe GO encreaſes as GS decreaſes, the time of deſcribing a given

a given angle at the axis decreaſes and arrives at its limit when V and S coincide in the center of gyration Y, and no point of ſuſpenſion can be aſſumed, where the time would not be greater than when the axis paſſes through Y. As GS encreaſes from GY to infinity, GO is diminiſhed without limit, and the time of deſcribing a given angle, whether the axis paſſes through S or O, is perpetually encreaſed.

CHAP.

CHAP. XII.

RECTILINEAL MOTION OF BODIES.

522. LEMMA. *IF a ſide* AB *of a right-angled triangle* ABC *be divided into very ſmall equal parts,* A b, b c, c d, d e, *&c. and rectangular parallelograms be deſcribed upon them about the triangle, the ſum of the rectangles is equal to the area of the triangle, when* A b, b c, *&c. are diminiſhed without limit.* FIG. CLIV.

For the difference between the triangle and the ſum of the rectangles, is the ſum of the triangles $Alm+mnp+pqr$,&c. which is equal to $\frac{Ab \times BC}{2} = 0$ when Ab vaniſhes. Q. E. D.

523. PROP.* *If a quieſcent body be acted upon by a conſtant force in the ſame right line, and any right line* AB *repreſent the time, and* LM, *perpendicular to it, the velocity communicated in the time* AL, *the ſpace deſcribed in the time* AL *will vary as the triangle* ALM.

DEM. Let AB be divided into very ſmall equal parts Ab, bc, cd, &c. and let the forces act only at the beginning of each part, and the velocity conſequently during each will be uniform. If the velocity communicated at A be repreſented by bm, the increments communicated at b, c, d, &c. will be each equal to bm, and conſequently LM encreaſes in the ſame ratio with AL, and AM is a right line. But the ſpaces deſcribed in the particles of time Ab,

* Keil's Phyſics, Lect. XI. Graves, Lect. I. Ch. XIV. Muſchenbroek, Ch. VI. CLVIII. Cotes de Deſcenſu Gravium. Morgan's Notes to Rohault. Maclaurin's Phil. Diſc. B. II. Ch. V. Emerſon, p. 5, &c.

Ab, bc, cd, ef, &c. are as the rectangles Am, bp, cr, ds, &c. (107), and the whole ſpace deſcribed in the time AL, is as the ſum of theſe rectangles, which, when Ab, bc, cd, &c. vaniſh and the force acts inceſſantly, are equal to the triangular area (ALM). Q. E. D.

524. Cor. 1. If the velocity and time be repreſented by V and T, and be divided into very ſmall increments V' and T', whoſe number is n, $V = n \times V'$ and $T = n \times T'$.

525. Cor. 2. If V, T and v, t repreſent correſponding velocities and times, $V : v :: T : t$ and $\frac{V}{v} = \frac{T}{t}$; and, if for v a number of feet uniformly deſcribed in t ſeconds be ſubſtituted, $V = \frac{v \times T''}{t''}$ will be the number of feet deſcribed in t'' by the velocity V.

526. Cor. 3. Whatever be the magnitude of the conſtant force, the ſpace deſcribed, S, will be repreſented by a triangular area, and conſequently S, or the ſpaces deſcribed by the action of different conſtant forces, are generally as $V \times T$, and, when numbers are ſubſtituted for V and T, S is equal to the product of $\frac{V \times T}{2}$*. In the ſame conſtant force, the triangular areas ALM and ABC are always ſimilar, and V and T are directly as each other, and conſequently S is as V^2 or T^2. If therefore the time be divided into equal parts, the ſpaces deſcribed in theſe times are as the odd numbers 1, 3, 5, 7, &c.; for the ſpaces deſcribed in 1, 2, 3, 4, &c. parts of time, are as 1, 4, 9, 16, &c. and conſequently the ſpaces deſcribed in the 1ſt, 2d, 3d, &c. alone are as 1, 3, 5, 7, &c.

527. Cor.

* This equation may be deduced from finding the ſum of an arithmetic progreſſion: let V' and T' be increments of V and T, and $S = \overline{1 + 2 + 3 + 4 \ldots n} \times V'T' = \frac{n^2}{2} \times V'T$ (becauſe n is infinitely great, and conſequently vaniſhes compared with n^2) $= \frac{nV \times nT}{2} = \frac{V \times T}{2}$.

527. Cor. 4. The ſpace which is deſcribed in any time T from a ſtate of reſt, is half of that deſcribed, in the ſame time, with the laſt velocity, V, continued uniform: for, in the firſt caſe, the ſpace $= \frac{V \times T}{2}$ (526), and, in the ſecond, the ſpace $= V \times T$ (107).

528. Cor. 5. If a body, projected with any velocity, be acted upon by a conſtant force, in a direction oppoſite to its motion, it will be uniformly retarded: for the force will evidently deſtroy equal parts of velocity in equal times. A body therefore projected with the laſt acquired velocity, will aſcend to the point from whence it fell, and the velocities in the aſcent and deſcent are the ſame at the ſame point. And, if bodies be projected with different velocities and reſiſted by the ſame conſtant force, the time that elapſes, T, till the velocity be deſtroyed, will vary as the velocity of projection V; the ſpace will vary as the ſquare of the time T^2, or ſquare of velocity V^2; the ſpaces deſcribed in equal portions of time, will be as the odd numbers in a retrograde order 7, 5, 3, 1; and, whether theſe retarding forces be the ſame or different, the ſpace will vary as $V \times T$, and be equal to $\frac{V \times T}{2}$.

SCHOLIUM.

529. At the ſurface of the earth the force of gravity is conſtant, for a body deſcends through $16\frac{1}{12}$ feet nearly in the firſt ſecond, and conſequently acquires a velocity that would make it deſcribe $32\frac{2}{12}$ feet uniformly in 1″; and this is found to be the velocity communicated in every ſucceeding ſecond. Any of theſe quantities, ſpace deſcribed S, velocity acquired, or time T, being known, the others may be diſcovered.

1. The proper meaſure of velocity is the ſpace uniformly deſcribed by it in any given time: if therefore a body fall by the force of gravity for t'', and acquire a velocity which would make

it deſcribe V feet uniformly in 1″, V and 32 are the proper meaſures of the velocities communicated in t'' and 1″ (106), and $V : 32 :: t'' : 1''$ (525), and $V = 32 \times t''$. If $t = 2, 3, 4$, &c. ſeconds, $V = 2 \times 32$, 3×32, 4×32, &c. feet reſpectively.

2. If V, or the number of feet uniformly deſcribed in 1″, by the velocity acquired in falling for an unknown time, be given, this time may be found; for it is equal $\frac{V}{32}$ ſeconds. If $V = 192$ feet, the time of falling $= \frac{192''}{32} = 6''$.

3. If Y be the ſpace deſcribed in falling t'', $16 : Y :: 1^2 : t''^2$ (526), and $Y = 16 \times t''^2$ feet. If $t = 2, 3, 4$, &c. ſeconds, $Y = 16 \times 2^2$, 16×3^2, 16×4^2, &c. feet reſpectively. Or Y may be found by knowing the velocity (V) at the end of the deſcent; for $Y : 16 :: V^2 : 32^2$, and $Y = \frac{16 \times V^2}{32^2} = \frac{16 \times V^2}{4 \times 16^2} = \frac{V^2}{64}$ feet. If $V = 192$ feet, $Y = \frac{192^2}{64} = 576$ feet.

4. If Y, or the number of feet deſcribed in falling, be known, the time of deſcent, and velocity acquired, may be found; for $Y = 16 \times t''^2$, and conſequently $t'' = \frac{\sqrt{Y}}{4}$. If $Y = 576$ feet, $t = \frac{\sqrt{576}}{4} = \frac{24}{4} = 6$ ſeconds. And becauſe $Y = \frac{V^2}{64}$, $V = \sqrt{Y} \times 8$. If $Y = 576$ feet, $V = \sqrt{576} \times 8 = 24 \times 8 = 192$ feet.

530. PROP. *If bodies be acted upon by different conſtant forces, the velocities communicated will vary in a ratio compounded of the forces and times.*

Let

Let S, V, T, repreſent force, velocity and time, and be ſuppoſed variable, and it is evident that the velocity will be encreaſed and diminiſhed in the ſame ratio with both the force and time, and, theſe being independent of each other, V will be as $F \times T$. Q.E.D.

531. Cor. 1. V therefore is as $F \times T$, and if F be compared with the force of gravity f, or any other known force, capable of generating a velocity equal to v in the time t, then $V : v :: F \times T : f \times t$, and $\frac{V}{v} = \frac{F \times T}{f \times t}$.

532. Cor. 2. Becauſe, in all conſtant forces, the ſpace varies generally in a ratio compounded of the velocity and time, and the velocity varies generally as the force and time; the following analogies are general:

S is as $V \times T$, or as $F \times T^2$, or as $\frac{V^2}{F}$;

T varies as $\frac{S}{V}$, or as $\sqrt{\frac{S}{F}}$;

V varies as $\frac{S}{T}$, or as $\sqrt{S \times F}$; and

F varies as $\frac{V}{T}$, or as $\frac{S}{T^2}$, or as $\frac{V^2}{S}$.

If numbers be ſubſtituted for V, F, and T; or the number of feet deſcribed in a given time 1, by the action of any conſtant force F, be ſubſtituted for the force; the number of feet which would be uniformly deſcribed in this given time 1, by the velocity acquired in the time T, $= V$; and $T =$ the number of parts of time each equal to 1, contained in the whole time in which the velocity is communicated: then (106) V and $2F$ are proper meaſures of the velocity acquired in the times T and 1, and $V : 2F :: T : 1$ (525) and $V = 2F \times T$; and ſubſtituting this value of V in the general equation $\left(S = \frac{V \times T}{2}\right)$, the following general equations are deduced:

$S = \frac{V \times T}{2} = F \times T^2 = \frac{V^2}{4F}$ feet;

$T = \frac{2S}{V} = \sqrt{\frac{S}{F}}$, equal portions of time, each of which is equal to 1; and

$V = \frac{2S}{T} = \sqrt{S \times 4F}$ feet.

533. Cor. 3. These equations and analogies are applicable to all accelerating and retarding forces, such as gravity, and to all resistances that are constant and produce equal decrements of velocity in equal times. If the motion of a ball shot into a bank of earth or piece of wood, be uniformly resisted, and the magnitude of this resistance compared with gravity, and the velocity of impact, be known; the depth, or space through which it moves before all motion is destroyed, may be discovered; and vice versâ *.

FIG. CLV. 534. PROP. *If a body descend down an inclined plane* AB *by the action of a constant force* F, *whose direction is perpendicular to any right line* BC; F *will be to that part of* F *which makes the body descend, as the radius to the sine of the plane's inclination to* BC.

DEM. Let any given line *LM*, perpendicular to *BC*, represent the constant force *F*, in quantity and direction, and be resolved into two forces, *LN* parallel, and *MN* perpendicular, to the plane; *MN* is destroyed by the resistance of the plane, and *LN* is the only part of *F* that communicates motion to the body; therefore *F*:

* EXAMPLE. Suppose a resistance were equal to the resistance of gravity *F*, when a body is projected perpendicularly upwards, multiplied into any number *n*, and all motion were destroyed in *t''*, then *S* (being equal to the force multiplied into the square of the time) $= n \times 16 \times t^2$. If the velocity of projection were such as would make a body describe *f* feet in a second uniformly, then *S* (being equal to $\frac{\text{vel.}^2}{4 \times \text{force}}$) $= \frac{f^2}{4 \times n \times 16}$ $= \frac{f^2}{n \times 64}$. If the space described and force be known, then the time is equal to $\sqrt{\frac{S}{n \times 16}}''$, and the velocity $= \sqrt{S \times n \times 64}$ feet.

F: that part of F which makes the body move $(A) :: LM : LN :: AB : AC$ (sim. triangles) :: rad. : sin. of $\angle ABC$. Q. E. D.

535. Cor. 1. F is to that part of F which is destroyed by the reaction of the plane as $LM : MN :: AB : BC$ (sim. triangles) :: radius : cos. $\angle ABC$; and consequently the part of F, destroyed by the resistance of the plane, $= \frac{F \times \text{cos. } \angle ABC}{\text{radius}}$, which is constant upon the same, or parallel, planes, and upon different planes, not parallel, varies as the $\frac{\text{base}}{\text{length}}$, if F be given.

536. Cor. 2. Because the accelerating force $LN = A = \frac{F \times AC}{AB}$, and F is given, A is constant upon the same, or parallel planes, and, upon different planes, inclined in unequal angles to BC, it varies as the height divided by the length, or as $\frac{H}{L}$, calling H the perpendicular height and L the length.

537. Cor. 3. Because the force upon the same plane is constant, the analogies and equations in (526) are applicable to the motion of a body upon the same plane; and the analogies and equations in (532) obtain when bodies move upon different planes. If BC be horizontal, and F be the force of gravity, then the space described upon the plane in $T'' = S = A \times T''^2 = F \times T^2 \times \frac{H}{L}$ (536) $= \frac{16 T^2 \times H}{L}$; $V = \frac{2F \times T'' \times H}{L} = \frac{32 T'' \times H}{L} = 8 \times \sqrt{H}$ feet; and $T'' = \frac{V \times L}{32 \times H} = \frac{L}{4 \times \sqrt{H}}$.

538. Because the space described is generally as the force multiplied into the square of the time (532), the spaces that are to each

each other as the forces must be described in the same time, or when S is as F, T is given. If therefore two bodies descend at the same instant, one down the plane AB, and the other in AC perpendicular to BC, and CP be drawn perpendicular to AB, P and C are cotemporary positions of the bodies; for $AP : AC :: LN : LM :: A : F$, that is, the space is as the force, and T^2 and T are given. If the diameter AD of a circle be perpendicular to the horizon, a body will descend through it, and any chord AP drawn from its extremity A, in the same time, because the angle APD is a right one; and the times of descent through all the chords drawn from A are consequently equal to each other.

FIG. CLVI.

539. Cor. 5. The velocities acquired in falling down different inclined planes, are as the square roots of their perpendicular heights; for V^2 is always as the space multiplied into the force, or as $L \times A$, or as $\frac{L \times H}{L}$ (536), or as H, and V is as $\sqrt{H}$; this also follows from art. 537. The velocities therefore acquired in falling down the perpendicular AC, and any planes AB, AE, AG, &c. drawn from A and terminated by the base CB, are equal to each other. And because V is always equal to $2A \times T$, and in this cor. V is given, the times of falling are as the forces inversely, or as the lengths of the planes. Or because S is as $V \times T$, and V is given, T is as S, that is, as the length.

FIG. CLV.

540. Cor. 6. The times of descending through different planes are as the lengths directly and square roots of the heights inversely; for the time is generally as the space described directly and last velocity inversely (532), or T is as $\frac{L}{V}$, or as $\frac{L}{\sqrt{H}}$ (last cor.); this also follows from art. 537. The times therefore of describing different planes, equally inclined to the direction of F, are as the square roots of their lengths; for H is as L (sim. triangles), and consequently $\frac{L}{\sqrt{H}}$ is as $\frac{L}{\sqrt{L}}$, or as $\sqrt{L}$ or T.

541. PROP.

541. PROP. *The velocity acquired in falling down any number of contiguous planes, ſuppoſing none to be loſt in paſſing from one plane to another, is equal to the velocity acquired in falling down the ſame perpendicular altitude.* FIG. CLVII.

DEM. The velocities acquired in falling down AB and EB, $AB + BC$ and EC or PC, $AB + BC + CD$ and PD, or down the perpendicular PR, are equal (539). Q. E. D.

542. Cor. The velocities, acquired in falling down any ſyſtems of planes, are therefore as the ſquare roots of their perpendicular heights; and if the body be projected from D with the velocity acquired, it will aſcend through this or any other ſyſtem of planes to the ſame perpendicular altitude.

543. PROP. *The velocity loſt in paſſing from any plane to the next, is to the whole velocity as the ſine of the angle of their inclination to the radius.* FIG. CLVIII.

DEM. Let CB repreſent the velocity acquired in falling down AC, and be reſolved into two, CP coincident with the plane CD, and BP perpendicular to it; and this laſt is evidently deſtroyed by the reſiſtance of the plane: but $BP : CB ::$ ſin. of $\angle PCB$ or $\angle PCA$: radius. Q. E. D.

544. Cor. 1. $CB : CP$ (velocity upon the plane CD) :: radius : coſine of $\angle PCB$ or PCA. If the angle ACD become equal to two right angles, or ACD be any circular arc, CP and CB coincide, or the velocity loſt is equal to nothing; and conſequently a body moving in any circular arc ſuſtains no loſs of velocity by changing the direction of its motion; and a body, projected with the velocity acquired, up any curvilineal arc, will riſe to the ſame height from whence it fell.

545. Cor.

545. Cor. 2. If ACD be any circular arc, the velocity acquired in falling from A to D is conſequently equal to that acquired in falling through the ſame perpendicular height; and the velocities acquired or deſtroyed in deſcending or aſcending through two curve lines are as the ſquare roots of their perpendicular altitudes: for every curve may be conſidered as conſiſting of an indefinite number of inclined planes.

FIG. CLVII. 546. PROP. *The times of deſcribing two ſyſtems of inclined planes* ABCD *and* abcd, *whoſe number, inclinations, and ratios of their lengths are the ſame, are to each other as the ſquare roots of the lengths of the planes.*

DEM. Becauſe the planes are equally inclined to the direction of the force, the time of falling down AB is as $\sqrt{AB}$ (540), or as $\sqrt{EB}$, or $\sqrt{EC}$ (hypoth.), or as the time down EB, or EC; and, dividendo, the time of falling down BC, after having fallen down AB or EB, is as $\sqrt{AB}$; conſequently the time of falling down $AB + BC$ is as $\sqrt{AB}$, or as $\sqrt{AB + BC}$, &c.: for from the ſuppoſition $\sqrt{AB} : \sqrt{ab} :: \sqrt{AB+BC} : \sqrt{ab+bc}$, &c. Q. E. D.

547. Cor. 1. The times of deſcribing ſimilar parts of ſimilar curves, equally inclined, in ſimilar parts, to the direction of the force, are as the ſquare roots of their lengths.

548. Cor. 2. If two bodies vibrate in ſimilar circular arcs, the times of performing theſe vibrations are as the ſquare roots of the lengths of theſe arcs, or as the ſquare roots of their radii.

SCHOLIUM.

549. In the preceding propoſitions and corollaries, inclined planes and bodies, deſcending or aſcending upon them, are ſuppoſed to be without any aſperities upon their ſurfaces, and the bodies are ſuppoſed to move by the action of a force in a given direction,

Plate XI.

FIG.CL.

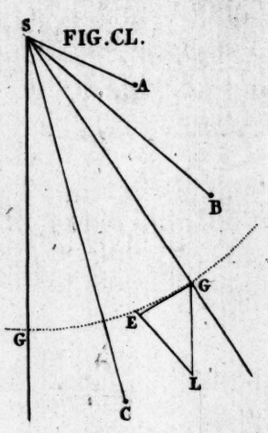
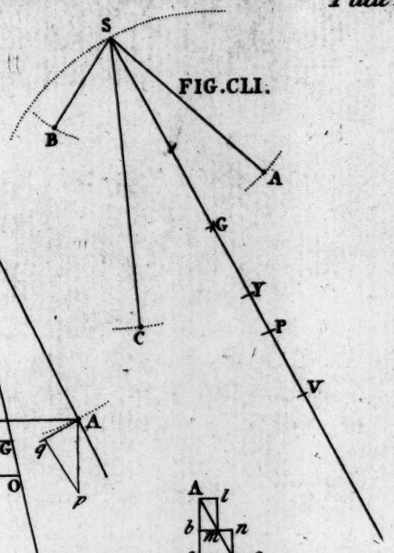


FIG.CLI.



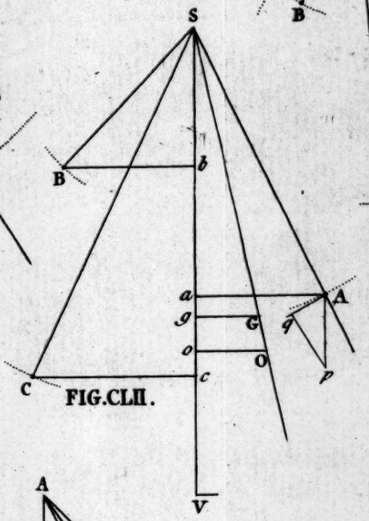


FIG.CLII.

FIG.CLIII.

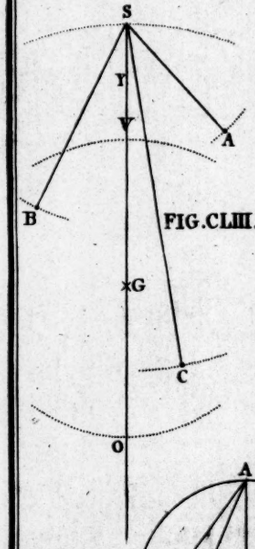


FIG.CLIV.

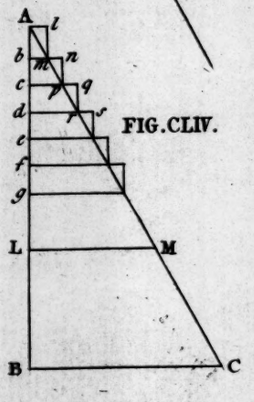


FIG.CLV.

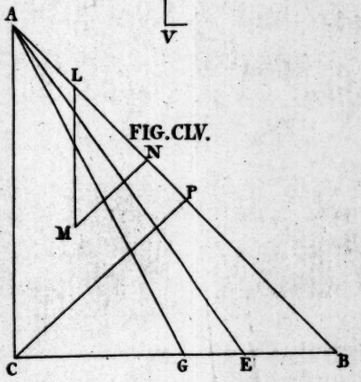


FIG.CLVI.

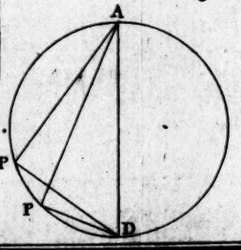
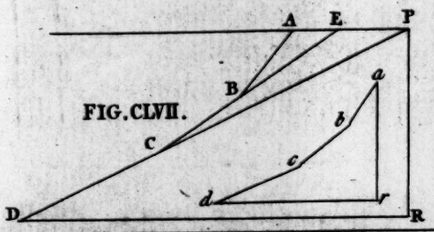


FIG.CLVII.




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direction, without any retardation from friction, or rotation about an axis. In this ſuppoſition, the tendency of each particle of a body to deſcend, when impelled by the force of gravity, is equal to the particle multiplied into $\frac{H}{L}$, and the tendency of the body W is equal to $W \times \frac{H}{L}$, and is the ſame as if it were collected in its center of gravity. But if the parts of W and the inclined plane adhere together, the force of this adheſion muſt be eſtimated and allowed for in practice. If W be a ſpherical body, and the parts of its ſurface in contact with the plane adhere by the preſſure of W upon it, their tendency to deſcend will be diminiſhed by this adheſion, and conſequently they will be regreſſive with regard to the center of gravity G, and revolve round it. The abſolute velocities of different particles in W, and conſequently their accelerating forces, are different; and the tendency to deſcend of the center of gravity is diminiſhed by this motion of rotation round it. The angular velocity of every particle round G is the ſame, and the ſame as if all the particles were concentrated in their center of gyration Υ, or a body whoſe magnitude is $\frac{W \times G\Upsilon^2}{GA^2}$ were collected in A (498).

550. PROP. *Suppoſe the ſame ſpherical body to ſlide and roll down the ſame inclined plane, to find the ratio of the forces acting upon it.* FIG. CLIX.

From the nature of the circle, the initial regreſſive velocity of the point A is equal to the progreſſive velocity of the center of the ſphere, or center of gravity G; but the regreſſive velocity of a body whoſe magnitude is $\frac{W \times G\Upsilon^2}{GA^2}$ placed at A (laſt ſcholium), would be deſtroyed by the action of a force in a contrary direction equal to $\frac{H}{L}$; and in this caſe G would not be impeded by the regreſſive motion of A, but G and A would ſlide down with equal velocities; conſequently

the force acting upon W when it ſlides: the force when it rolls down the plane $:: W + \frac{W \times GY^2}{GA^2} \times \frac{H}{L} : W \times \frac{H}{L}$. Q. E. I.

551. Cor. If the center of gyration be found, and GY, and GA be expreſſed in numbers, the ratio of theſe forces will be expreſſed in terms of the weight.

SCHOLIUM.

552. In the communication of motion by the external application of forces ſuch as impact, protruſion, &c. their magnitude, or capacity to communicate motion, is not always meaſurable by their cotemporary effects; but in this chapter every particle of matter is ſuppoſed to be impelled by a force, ſimilar to that of gravity, with the ſame intenſity in parallel directions, which will conſequently communicate the ſame velocity, or change of velocity, to bodies of different magnitudes, whether quieſcent or moving. The magnitude of theſe forces at any inſtant, or the magnitude of their accumulated action, is meaſurable by their cotemporary effects, or by the velocity generated or deſtroyed in equal times; for the increments or decrements of velocity, produced in equal times, by the ſame conſtant force, being equal, the whole velocity is encreaſed or diminiſhed uniformly; and when the forces are unequal, the changes of velocity being as the forces, it is evident that the velocities are proper meaſures of the intenſity of forces, and will vary as the forces multiplied into the times in which they are generated or deſtroyed. And the magnitudes of variable forces, whoſe intenſities and cotemporary effects are perpetually encreaſed or diminiſhed, are meaſured at any inſtant of time by their conſtant action at that inſtant, or by the velocities which they would produce in the ſame time were their intenſity conſtant during that time; and the magnitude of their accumulated actions for any finite time is meaſured by the addition of theſe cotemporary effects.

553. PROP.

453. *PROP. *If* F, V, T, *repreſent any finite variable force, velocity and time, reſpectively, and* $\dot{V}$, $\dot{T}$, *be any ſmall changes of* V *and* T, $\dot{V}$ *will vary as* $F \times \dot{T}$.

DEM. The force F being always finite, and being ſuppoſed to encreaſe or decreaſe according to the ſame law, an encreaſe or diminution of it in any finite time muſt be finite, and conſequently, in an infinitely ſmall time, evaneſcent compared with the whole force F, which therefore may be conſidered as conſtant during the time $\dot{T}$ in which $\dot{V}$ is generated or deſtroyed; therefore (530) $\dot{V}$ varies as $F \times \dot{T}$. Q. E. D.

554. Cor. 1. If $\dot{V}$ be the fluxion of the velocity, or the change of velocity generated by F at any inſtant, ſuppoſed to act conſtantly for any time $\dot{T}$, $\dot{V}$ will therefore vary as $F \times \dot{T}$.

555. Cor. 2. $\dot{T}$ is therefore as $\frac{\dot{V}}{F}$ and F as $\frac{\dot{V}}{\dot{T}}$, and the relation of any two being known, the other may be found; and becauſe $\dot{T}$ is as $\frac{\dot{S}}{V}$, $V\dot{V}$ is as $F \times \dot{S}$.

556. PROP. *If any right line* AS *repreſent the time, and* OR *be as the force at any point* O, *the change of velocity at* O *will be as the area* AR. FIG. CLX.

DEM. The increment of velocity $\dot{V}$ is as $F \times \dot{T}$, or as $AB \times Ap$ or as Aq; and the next increment of velocity is as ps, and the ſum of the increments, or the whole change of velocity, is as the ſum of theſe areas; conſequently the velocity communicated in the time AO is as AR. Q. E. D.

557. Cor. 1. If the time in acquiring any given velocity be known, and the areas AR can be ſquared, the velocities communicated in any other time AO may be found; and vice verſâ.

558. Cor.

* Newt. Prin. Tom. I. Sect. VII. Euler's Mechan. Ch. III.

558. Cor. 2. If the force F be finite, and the time Ap infinitely ſmall, the increment of velocity, $\dot{V}$, is evaneſcent; becauſe $AB \times Ap =$ a finite quantity multiplied into one that is evaneſcent $= 0$; and conſequently the increment of velocity, or $\dot{V}$, is infinitely ſmall, and no finite change of velocity can be generated by F in an inſtant.

559. Cor. 3. If the body aſcend or deſcend, and the velocity be as the time in which it is acquired or deſtroyed, the force is invariable; for, from this propoſition, the change of velocity is as AR, or as AO (hypoth.), and conſequently AR is a rectangular parallelogram, or OR is conſtant.

FIG. CLXI. 560. PROP. *If* PO *repreſent the ſpace deſcribed by the action of a force tending to* S, *and the ordinate* OR *be the relative magnitude of the force at the point* O, *the velocity*, V, *acquired at* O *will vary in a ſubduplicate ratio of the correſponding area* PR.

DEM. Let V, F, S, be the velocity, force, and ſpace deſcribed, reſpectively, and becauſe $\dot{V}$ is as $F \times \dot{T}$ (554), and $\dot{T}$ is as $\frac{\dot{S}}{V}$; $V\dot{V}$ is as $F \times \dot{S}$, or as $PB \times Pp$, and the fluent of $V\dot{V}$, or $\frac{V^2}{2}$ is as the fluent of $PB \times Pp$ or PR, and V is as $\sqrt{PR}$. Q. E. D.

561. Cor. 1. If x be the diſtance of the moving body from the center of force, S, and the force be as any power of the diſtance, whoſe exponent is $n - 1$; $V\dot{V}$ is as $F \times -\dot{x}$, or as $-x^{n-1}\dot{x}$, and $\frac{V^2}{2}$ is as $\frac{-x^n}{n}$ + correction: but when $V = 0$ at P, $x = SP = p$ and $\frac{x^n}{n} = \frac{p^n}{n}$; therefore the correction $= \frac{p^n}{n}$, and V is as $\sqrt{p^n - x^n}$.

562. Cor.

562. Cor. 2. If the force be conſtant, or $n = 1$, then V being as $\sqrt{p^n - x^n}$ is as $\sqrt{p - x}$, or in a ſubduplicate ratio of the ſpace deſcribed, either in acceding to S from a ſtate of reſt, or receding from S till the velocity be deſtroyed. The fluxion of the time $\dot{T}$ is as the fluxion of the ſpace directly and velocity inverſely; or as $\frac{-\dot{x}}{\sqrt{p-x}} = -\dot{x} \times \overline{p-x}^{-\frac{1}{2}}$, and T is as $\frac{\overline{p-x}^{\frac{1}{2}}}{\frac{1}{2}}$, and when $T = 0$, $x = p$, and $\frac{\overline{p-x}^{1\frac{1}{2}}}{\frac{1}{2}} = 0$; therefore T is as the ſquare root of the ſpace deſcribed from reſt, which coincides with art. 526.

563. Cor. 3. If the force vary directly as the diſtance, or $n = 2$, then V varies as $\sqrt{p^2 - x^2}$, or as the right ſine of a circular arc, deſcribed from S as a center, and radius equal to SP or p, whoſe verſed ſine is the ſpace deſcribed. And deſcribing this circle, the fluxion of the time $\dot{T}$ is as $\frac{\dot{S}}{V}$, or as $\frac{Oo}{OR}$, or as $\frac{Rn}{Sn}$ (ſim. triang.), and the fluent T is as the fraction $\frac{PRD}{SD}$, which is a conſtant quantity. If the force therefore vary directly as the diſtance, the times of deſcent to S from a ſtate of reſt are equal wherever P be taken. FIG. CLXII.

564. Cor. 4. If the force vary inverſely as any power of the diſtance, or n be negative, or leſs than 1, let this power be expreſſed by the number $-m - 1$; and V is as $\sqrt{\frac{1}{m \times x^m} - \frac{1}{m \times p^m}}$, or as $\sqrt{\frac{p^m - x^m}{m \times p^m x^m}}$. If the force be inverſely as the diſtance, or $m = 0$, this expreſſion does not ſhew the variation of velocity. If the force be inverſely as the ſquare of the diſtance, or $m = 1$, V is as $\frac{\sqrt{p-x}}{\sqrt{m \times px}}$, or the velocity is as the ſquare root of the ſpace deſcribed directly, and inverſely as the ſquare root of the diſtance from S.

565. PROP.

FIG. CLXIII. 565. PROP. *If the ordinates* PB, AL, OR, *be always proportional to the forces at thoſe points, and the force at* A, *continued conſtant through the ſpace* AM, *communicate a velocity,* V, *equal to that acquired by the body at* O, *deſcending by the variable force, the area* PR *will be equal to the rectangle* AN, *and the body muſt have fallen from* P.

DEM. Let $\dot{v}$ and $\dot{V}$ be the fluxions of the velocities communicated, by the variable and conſtant forces reſpectively, in any periods of their deſcent; and $\dot{v} : \dot{V} :: \frac{PB \times P\dot{O}}{v} : \frac{AL \times A\dot{M}}{V}$ (555),

and $v\dot{v} : V\dot{V} :: PB \times P\dot{O} : AL \times A\dot{M}$, and, taking the fluents, $v^2 : V^2 :: PR : AN$. If therefore $v = V$, the areas PR and AN are equal, and the body muſt have fallen from P. Q. E. D.

566. Cor. 1. If the areas PR and AN be equal, the velocities, acquired by the action of the conſtant and variable forces, whilſt the bodies deſcribe, from reſt, the ſpaces AM and PO, will alſo be equal.

567. Cor. 2. Becauſe the areas AN and PR are always equal, when the velocities at M and O are equal, their increments Or and Mn muſt be equal; and if F be the given force at A, and $y =$ the ſpace deſcribed AM, $F \times \dot{y} = Or = OR \times Oo =$ the force at O multiplied into the fluxion of the ſpace.

568. PROP. *If a body be attracted towards the point* S, *by forces which always vary as that power of the diſtance whoſe exponent is* n—1, *and begin to move at any given diſtance* SP, *it is required to aſſign the velocity,* V, *acquired in deſcribing any ſpace* PO, *ſuppoſing the magnitude of the force at any given diſtance* SA *to be known.*

Let

Let the magnitude of the force at A be to the force of gravity as $F : 1$, and let

$SA = a$,

$SP = p$,

$SO = x$; and, from the ſuppoſition, F: force at any point $O :: a^{n-1} : x^{n-1}$, and the force at $O = \frac{F \times x^{n-1}}{a^{n-1}}$. But, if y be the ſpace through which a body muſt fall, when acted upon by a conſtant force equal to F, to acquire a velocity equal to that at O, from (567) $F \times \dot{y} = \frac{F \times \overline{x^{n-1} - \dot{x}}}{a^{n-1}}$, and the fluents are equal, or $F \times y = \frac{-Fx^n}{n \times a^{n-1}}$ + correction; but when $F \times y$ vaniſhes, $x = p$, and conſequently the fluent corrected $= \frac{F \times \overline{p^n - x^n}}{n \times a^{n-1}}$. But if s be the ſpace deſcribed by the force of gravity in 1″, or any other given time 1, and the force of gravity be expreſſed by 1, $V =$ *$\sqrt{4sFy} = \sqrt{\frac{4sF \times \overline{p^n - x^n}}{n \times a^{n-1}}}$ feet in 1″. Q. E. I.

569. Cor. 1. If the force be as ſome negative power of the diſtance, or $n - 1 = -m$, then F: the force at $O :: a^{-m} : x^{-m}$, and the force at $O = \frac{F \times x^{-m}\dot{x}}{a^{-m}}$; the fluxion of $F \times y = \frac{F \times x^{-m}\dot{x}}{a^{-m}}$ and its fluent $= \frac{F \times x^{1-m}}{1 - m \times a^{-m}}$ + cor. $= \frac{F \times a^m}{1 - m \times x^{m-1}} - \frac{Fa^m}{1 - m \times p^{m-1}}$ $= \frac{F \times a^m \times \overline{p^{m-1} - x^{m-1}}}{1 - m \times p^{m-1}x^{m-1}}$, and $V = \sqrt{\frac{4sF \times a^m \times \overline{p^{m-1} - x^{m-1}}}{1 - m \times p^{m-1}x^{m-1}}}$.

570. Cor.

* Let $f =$ the force of gravity,
$s =$ the ſpace deſcribed by the conſtant action of f in 1″ = 16 feet nearly, and
$v =$ the velocity acquired in falling 1″; and

$V^2 : v^2 :: F \times y : f \times s$ and $V^2 = \frac{v^2 \times F \times y}{f \times s}$; but if $f = 1$, $v = 2s$ and $V^2 = 4sFy$ and $V = \sqrt{4sFy}$.

570. Cor. 2. Becauſe $4Fs$ and a^{n-1} are given quantities, V varies as $\sqrt{p^n - x^n}$.

571. Cor. 3. If the body deſcend to the center, or $x = 0$, and the force vary according to any direct law of the diſtance, or inverſe law leſs than the ſimple, that is, if n be any affirmative, whole number, or fraction leſs than unity, $V = \sqrt{\frac{4sF \times p^n}{n \times a^{n-1}}}$ is a real finite quantity. Let the force be as that power of x, whoſe exponent is $-\frac{1}{2}$, 1, 2, 3, &c. or $n = \frac{1}{2}$, 2, 3, 4, &c. and $V = \sqrt{\frac{4sF \times p^{\frac{1}{2}}}{\frac{1}{2} \times a^{-\frac{1}{2}}}}$, $\sqrt{\frac{4sF \times p^2}{2 \times a}}$, $\sqrt{\frac{4sF \times p^3}{3 \times a^2}}$, $\sqrt{\frac{4sF \times p^4}{4 \times a^3}}$, &c.

572. Cor. 4. If the force be inverſely as the diſtance, or $n = 0$, and the body deſcend to the center, as before, $V = \sqrt{\frac{4sF \times p^0}{0 \times a^{-1}}}$ is infinitely great. If the force be inverſely as the ſquare of the diſtance, or $n = -1$, $F \times y = -\frac{Fx^{-2}\dot{x}}{a^{-2}}$ and $Fy = \frac{Fa^2}{x} +$ cor. $= \frac{Fa^2}{x} - \frac{Fa^2}{p} = \frac{Fa^2 \times \overline{p-x}}{px}$, and $V = \sqrt{4sFa^2 \times \frac{\overline{p-x}}{px}}$. From hence it appears, as before (564), that the velocity varies as $\sqrt{\frac{PO}{OS}}$.

573. PROP. *Suppoſe a body to be impelled by a force varying as that power of the diſtance from the center of force* S, *whoſe exponent is* n — 1, *it is required to aſſign the time of deſcribing any ſpace* PO, *ſuppoſing it to fall from a ſtate of reſt.*

The fluxion of the ſpace $\dot{S} = V \times \dot{T}$, and conſequently $\dot{T}$ is equal to the fluxion of the ſpace divided by the velocity $= -\frac{\dot{x}}{V} =$

—

$-\frac{\dot{x}\times\sqrt{na^{n-1}}}{\sqrt{4sF\times p^n-x^n}}$; and the time is equal to the fluent of $\frac{\dot{x}\times\sqrt{na^{n-1}}}{\sqrt{4sF\times p^n-x^n}}$. Or, if $n-1$ be negative and $=-m$, then $\dot{T}=-\dot{x}\times\sqrt{\frac{1-m\times p^{m-1}x^{m-1}}{4sFa^m\times p^{m-1}-x^{m-1}}}$, and $T=$ its fluent. Q. E. I.

574. Cor. 1. Let the force vary as the distance from the center of force S, or $n=2$, and $T=\sqrt{\frac{2a}{4sF}}\times$ into the fluent of $\frac{-\dot{x}}{\sqrt{p^2-x^2}}=$ cor. — length of a circular arc, whose radius is unity and right sine $\frac{x}{p}$. If the time vanish, or $x=p$, the cor. = length of a circular arc whose radius is 1 and right sine is $\frac{p}{p}$, or 1, and therefore = a quadrantal arc; consequently the time of describing any space PO is equal to $\sqrt{\frac{2a}{4sF}}\times$ into a quadrant $-\sqrt{\frac{2a}{4sF}}\times$ into the arc whose right sine is $\frac{x}{p}$, the radius being unity. When the body descends to the center, or $\frac{x}{p}=0$, the time is equal to $\sqrt{\frac{2a}{4sF}}\times$ into a quadrant. If the radius of the circle $=r$, and a quadrantal arc of that circle $=\mathcal{Q}$, the quadrant of a circle whose radius is unity $=\frac{\mathcal{Q}}{r}$, and the time of descent to $S=\sqrt{\frac{2a}{4sF}}\times\frac{\mathcal{Q}}{r}=$ a given quantity.

575. Cor. 2. If the force vary inversely as the square of the distance from S, then $V=\sqrt{\frac{4sFa^2\times p-x}{px}}$ and $\dot{T}=\dot{x}\times$

FIG. CLXVI. $\sqrt{\frac{p}{4sFa^2}} \times \sqrt{\frac{x}{p-x}}$, and $T = \sqrt{\frac{p}{4sFa^2}} \times$ into the fluent of $\dot{x} \times \sqrt{\frac{x}{p-x}}$. With SP or p as a diameter, and from the center C describe a circle, and the fluent of $\dot{x} \times \sqrt{\frac{x}{p-x}}$ is as the area PRS; for this area = sector PCR + the triangle SRC = arc PR + sine $OR \times \frac{SP}{4}$, and the fluxion of the area = $\overline{Rr+mr}\,(P\dot{R}+O\dot{R}) \times \frac{SP}{4} = \frac{\dot{x} \times CR}{OR} + \dot{x} \times \frac{CO}{OR}$ (sim. triang.), (or $\frac{\dot{x} \times x}{OR}$, or $\frac{x \times \dot{x}}{\sqrt{x \times p-x}}$, or $\dot{x} \times \sqrt{\frac{x}{p-x}}$) $\times \frac{SP}{4}$; and consequently the fluent of $\sqrt{\frac{p}{4sFa^2}} \times \dot{x} \sqrt{\frac{x}{p-x}} = \sqrt{\frac{p}{4sFa^2}} \times$ arc $\overline{PR}$ + sine OR. And because $\frac{p}{4sFa^2}$ is given, the time of describing PO varies as $PR+OR$, or as $\overline{PR+OR} \times \frac{SP}{4}$, or as the area PRS.

FIG. CLXIII. 576. PROP. *If a body begin to fall from* A, *it is required to determine the space* AO *through which it must descend, when impelled by a force varying as that power of the distance whose exponent is* n — 1, *to acquire a velocity,* V, *equal to that communicated, whilst the body describes a space equal to* $\frac{AS}{2}$, *by the constant action of a force equal to that at* A.

Let the force at $A = 1$
$SA = r$
$SO = x$. And the force at A, or 1 : the force at $O :: r^{n-1} : x^{n-1}$, and the force at $Q = \frac{x^{n-1}}{n-1}$, and $\frac{r}{2} \times$ the force at A =

$= \frac{r}{2} =$ the fluent of $- \frac{x^{n-1}\dot{x}}{r^{n-1}}$ (567) $= - \frac{x^n}{n \times r^{n-1}}$ + cor. $= \frac{r^n - x^n}{n \times r^{n-1}}$, and $r^n - x^n = \frac{nr^n}{2}$, and $x = \frac{\overline{2 - n}^{\frac{1}{n}} \times r}{2^{\frac{1}{n}}}$. Q. E. I.

577. Cor. 1. If the force vary directly as the distance from the center S, or $n = 2$, $x = \frac{\overline{2 - 2}^{\frac{1}{2}} \times r}{2^{\frac{1}{2}}} = 0$, and the body must fall to the center to acquire a velocity equal to that acquired in descending through $\frac{SA}{2}$, when acted upon by a constant force equal to that at A.

578. Cor. 2. If the force vary inversely as the distance, or $n = 0$, $x = \frac{\overline{2 - 0}^{\frac{1}{0}} \times r}{2^{\frac{1}{0}}}$, which expression does not shew the value of x: but it may be found by the following process. The force at A or 1 : force at $O :: x : r$, and the force at $O = \frac{r}{x}$, and $\frac{r}{2} = r \times$ flu. $- \frac{\dot{x}}{x} = r \times -$ hyp. log. of x + cor. $= r \times -$ log. of x $+ r \times$ log. of $r = r \times$ log. of $\frac{r}{x}$.

579. Cor. 3. If the force vary inversely as the square of the distance from S, or $n = -1$; $x = \frac{\overline{2 + 1}^{-1} \times r}{2^{-1}} = \frac{1}{3} \times r$, and the body must fall through a third of the distance SA, or to m. If the force vary according to any law therefore between the direct simple and inverse duplicate ratio of the distance from S, the body must fall to some intermediate space between S and m. If the force vary as that power of the distance inversely, whose exponent

ponent is 3, 4, &c. or $n = -2, -3$, &c.; then $x = \frac{\overline{2+2}^{-\frac{1}{2}} \times r}{2^{-\frac{1}{2}}}$, $\frac{\overline{2+3}^{-\frac{1}{3}} \times r}{2^{-\frac{1}{3}}}$, &c. $= \frac{2^{\frac{1}{2}} \times r}{\overline{2}^{\frac{1}{2}}}$, $\frac{2^{\frac{1}{3}} \times r}{5^{\frac{1}{3}}}$, &c.; and the space fallen through $= r - \frac{2^{\frac{1}{2}} \times r}{4^{\frac{1}{2}}}$, $r - \frac{2^{\frac{1}{3}} \times r}{5^{\frac{1}{3}}}$, &c.

FIG. CLXIV.

580. PROP. *Suppose the force at* A *to be* 1, *and in other places to vary as that power of the distance whose exponent is* n — 1, *it is required to determine the height to which a body will ascend when acted upon by this variable force, and projected from* A *in the direction* SA *with a velocity equal to that acquired in falling through* $\frac{SA}{2}$, *as in the last proposition.*

Let *B* be the place to which the body ascends, and the force at *A* or 1 : force at any other point $O :: r^{n-1} : x^{n-1}$, and the force at $O = \frac{x^{n-1}}{r^{n-1}}$. But if the body were impelled by a constant force equal to that at *A*, it would ascend through a space equal to $\frac{r}{2}$, and (567) $\frac{r \times 1}{2}$ = the fluent of $\frac{x^{n-1}\dot{x}}{r^{n-1}} = \frac{x^n}{a \times r^{n-1}}$ + cor. $= \frac{x^n - r^n}{n \times r^{n-1}}$, and $\frac{n \times r^n}{2} = x^n - r^n$, and $x = \overline{\frac{n+2}{2}}^{\frac{1}{n}} \times r$. Q. E. I.

581. Cor. 1. Suppose the force to vary directly as the distance from *S*, or $n = 2$; then $\frac{\overline{n+2}^{\frac{1}{n}} \times r}{\overline{2}^{\frac{1}{n}}} = \frac{4^{\frac{1}{2}} \times r}{2^{\frac{1}{2}}}$, and $x\ (= SB) : r :: 4^{\frac{1}{2}} : 2^{\frac{1}{2}} :: 2^{\frac{1}{2}} : 1$.

582. Cor. 2. If the force vary inversely as the distance from *S*, or $n = 0$, $x = \frac{\overline{0+2}^{\frac{1}{0}} \times r}{2^{\frac{1}{0}}}$, which expression does not shew the mag-

magnitude of x; but it may be determined by the proceſs in (578).

583. Cor. 3. If the force vary inverſely as the ſquare of the diſtance from S, or $n = -1$, then $x = \frac{\overline{2-1}^{\frac{1}{-1}} \times r}{\overline{2}^{\frac{1}{-1}}} = 2r = SD$.

If the force therefore vary according to any law between the direct ſimple and inverſe duplicate ratio of the diſtance from S, the body will riſe to ſome intermediate altitude between B and D. If the force vary inverſely as the cube of the diſtance, or $n = -2$, then $x = \frac{\overline{2-2}^{\frac{1}{-2}} \times r}{2^{-\frac{1}{2}}} = \frac{2^{\frac{1}{2}} \times r}{0}$, and is infinitely great. If therefore the force vary according to a law between the inverſe duplicate and triplicate, the body will aſcend to ſome intermediate ſpace between D and infinity.

CHAP.

CHAP. XIII.

PENDULOUS MOTION*.

FIG. CLXV. 584. DEF. *A BODY, or number of bodies, connected to a right line* SP, *and moving about a point* S *to which it is ſuſpended, by the force of gravity, is called a pendulum.*

585. DEF. *The motion of a pendulum in the ſame direction, from a ſtate of reſt, till it begins to return in an oppoſite direction, is one vibration or oſcillation.*

586. Cor. The velocity acquired in deſcending from P to V, ſuppoſing SV to be perpendicular to the horizon, will make the body P deſcribe an arc Vp, whoſe perpendicular altitude is equal to that of VP (542), and the motion through VP is equal to one half of a vibration. And becauſe a pendulum, compoſed of any number of bodies, will perform a given vibration, or equal parts of a vibration, in the ſame time as if they were collected in their center of oſcillation (512); a pendulum compoſed of any number of bodies may be reduced to one, where one body is connected to a right line SP, and the length of a pendulum is always underſtood to mean the diſtance between the centers of oſcillation and ſuſpenſion.

587. PROP. *If a body vibrate in a circular arc* PV p, *that part of the force of gravity, which accelerates and retards it, varies as the right ſine of the arc intercepted between the body and the loweſt point.*

DEM.

* Keil's Phyſ. Lect. XV. Helſham, Lect. X. Muſchenb. Ch. XII. CCXCVII. Maclaurin's Newt. Book II. Ch. V. Emerſon's Mechan. Prop. XI. Hugen. Horol. Oſcil. Part II. Prop. XVI. Rohault's Notes, Part II. Ch. XXVIII.

DEM. Let the force of gravity be repreſented, in quantity and direction, by a given line LP perpendicular to the horizon, and be reſolved into two forces, LM parallel to the ſtring SP, and PM touching the circle in P, of which LM is the tenſion of the ſtring and has no effect upon the body's motion, and it is accelerated and retarded by PM only; therefore the accelerating force (A) : force of gravity (G) :: PM : PL :: PN : SP (ſim. triang.), and $A = \frac{G \times PN}{SP}$, and conſequently A varies PN. Q. E. D.

588. Cor. 1. The velocity at any point Q is equal to that acquired in falling through the perpendicular altitude ND (541), and varies as $\sqrt{ND}$, or $\sqrt{NV - VD}$, or $\sqrt{NV \times 2VS - VD \times 2VS}$, or $\sqrt{PV^2 - QV^2}$ (PV and QV being chords of the arcs PV and QV), or as the right ſine of a circular arc, whoſe radius is the chord PV, and verſed ſine the difference between the chords PV and QV.

589. Cor. 2. If the arc mV be ſuppoſed equal to two inclined planes mn and nV, touching it at m and V, theſe planes are equal, and the velocity in the horizontal plane nV is uniform, and the time of deſcribing it is equal to half the time of falling down mn (527): but the time of falling down mn (t) : time (T) of falling down mV ($= 2\,mn$), or down the diameter $2SV$ :: mn : mV :: 1 : 2, and $t = \frac{T}{2}$; and conſequently the times of deſcribing mn and nV, or the time of half a vibration $= \frac{3T}{4}$, and the time of a whole vibration : T :: 3 : 2.

SCHOLIUM.

590. If the body P be acted upon by a force F, perpendicular to the horizon, which is to the force of gravity as the arc PV to its ſine PN, all vibrations are iſochronal; for, let SP repreſent the conſtant force of gravity, and $PL = F$; and by a reſolution of

PL

PL into two, LM parallel to SP, and PM coincident with the tangent, this laſt is the only part of F that accelerates P. But $PL : PS :: PM : PN$ (ſim. triang.), and

$PL : PS :: PV : PN$ (hypoth.); and conſequently $PM = PV$, or the accelerating force is as the diſtance from the loweſt point V, and P will always arrive at V in the ſame time (563). The iſochronal force in a circle, F, is therefore equal to $G \times \frac{PV}{PN}$; and, when a pendulum is urged by the force of gravity, the time of a vibration will be encreaſed with the arc of vibration, becauſe the exceſs of F above G is encreaſed with that arc.

FIG. CLXVII.

591. DEF. *If a circle* FPE *revolve upon the right line* BA, *the curve line* BVA *deſcribed by any point* P *of the periphery in one revolution, is called a cycloid:* BA *is the baſe,* VD *biſecting* BA *at right angles is the axis,* V *the vertex,* PM *parallel to the baſe is an ordinate, and* FPE *is the generating circle, of the cycloid.*

592. Cor. Becauſe every point of the periphery of the generating circle has been applied to the baſe BA, it is evident that BA is equal to the periphery, and BD to the ſemiperiphery of the generating circle.

593. LEMMA. *If a circle be deſcribed upon the axis as a diameter, and an ordinate* PM *be drawn from any point* P, *the part of this ordinate* PL, *contained between the point from whence it is drawn and the periphery of the circle, is equal to the circular arc* VL, *contained between the vertex and the interſection of the ordinate and periphery.*

DEM. Let FPE be any poſition of the generating circle, and, becauſe every point of the arc PE has been applied to BE, the right line $BE =$ the arc $PE =$ the arc LD, and the arc $LV = ED = MN = PL$. Q. E. D.

594. Cor.

594. Cor. Becauſe PL is always equal to the arc LV or PF, their cotemporary increments or decrements are equal, that is, the initial motions of the point P, which traces out the cycloidal arc, one parallel to the baſe BA, and the other in the direction of the tangent to the circle at P, are equal to each other.

595. LEMMA. *An ordinate being drawn from any point* P *of the cycloidal arc, cutting the periphery of the generating circle, whoſe diameter is the axis* VD, *in* L, *the chord* VL *of the circular arc is parallel to the line touching the cycloid at* P.

DEM. Let GP be a tangent to the circle at P, and producing EP to C, the $\angle CPq = \angle GPE = \angle GEP$ (the tangents DP and DE being equal, EUC. B. III. p. 36.) $= \angle EPN$: but the initial motions of P, in the directions GP and PL, being equal (594), the path of P or Pb will biſect the angle qPp (compoſition of motion), and conſequently the $\angle CPb = \angle EPb$ or Pb is at right angles to EP, and FP, which is parallel to LV, is alſo perpendicular to EP, and therefore is a tangent to the curve at P. Q. E. D.

596. Cor. A tangent to the cycloid at the vertex V is therefore perpendicular to the axis VD and parallel to the baſe BA.

597. LEMMA. *The generating circle being deſcribed upon the axis as a diameter, and an ordinate being drawn from any point* O, *cutting the periphery of the circle in* R, *the cycloidal arc* VO *is equal to twice the correſponding chord of the circular arc* VR.

DEM. Draw an ordinate ro infinitely near to RO, and rs perpendicular to VR produced, and $Vr = Vs$, the angle at V being evaneſcent; and Oo $(= Rv)$ and Rr are cotemporary increments of the arc VO and chord VR: drawing tangents to the circle at V and R, the triangles VRT and Rrv, having the angles at R ver-

 tical,

tical, and the angles TVR and Rvr alternate, are ſimilar, and conſequently $Rr = vr$ and Rv or $Oo = 2Rs$. The increment of the arc VO is therefore equal to twice the correſponding increment of the chord VR, and, becauſe they are naſcent and evaneſcent at the ſame time, $VO = 2VR$. Q. E. D.

FIG. CLXVIII.

598. PROP. *To make a body oſcillate in a given cycloid* AVB.

Produce the axis VD making $SD = DV$, and through S draw a line KSH parallel to BA, and let two circles, each equal to DLV, generate two ſemicycloids SA, SB, each equal to BV or AV; and if one extremity of a ſtring SCX, whoſe length is equal to SV or SCA, be fixed at S, a body collected in the other extremity X, will always be found in the cycloidal arc XVP; for, becauſe SCX is a tangent to the cycloid at C, CX is parallel to EA (595); and $CG = EA$; but $CX = 2AE$ (597) $= 2CG$ and $CG = GX$, and the ordinates XL and CE are equidiſtant from AD, or $AF = DN$, and the arc $AE =$ the arc ED, and the chord LD is parallel to AE or XG: but $AG = CE =$ the arc AE, and conſequently the remaining arc $HE (= LV) = GD = XL$, and X is in the cycloidal arc AXV (593).

599. Cor. Becauſe SCX touches the curve at C, it is evident that whilſt X deſcribes a very ſmall arc, C may be deemed quieſcent, or CX is always perpendicular to the cycloidal arc at X; and very near the vertex V, where SV is perpendicular to the curve, a circular arc, whoſe radius is SV, will coincide for a ſmall diſtance with the cycloidal arc.

600. PROP. *If a pendulum begin to vibrate from any point* P, *and a circle be deſcribed, whoſe radius is equal to the cycloidal arc* VP, *the velocity in different points will vary as the right ſine of an arc of this circle, whoſe verſed ſine is the ſpace deſcribed by* P.

DEM.

DEM. The velocity at any point Q is equal to that acquired in falling through the same perpendicular altitude NR (545), which is as $\sqrt{NR}$, or $\sqrt{NV - RV}$, or $\sqrt{NV \times VD - VR \times VD}$, or $\sqrt{VF^2 - VE^2}$, or $\sqrt{VP^2 - VQ^2}$, or $\sqrt{Vn^2 - Vq^2}$, or qn. Q.E.D.

601. Cor. 1. The velocity therefore encreases from P to V, where it is the greatest, being as Vv, and then it decreases in such a manner that the velocities at all equal distances from V are equal, and consequently the cotemporary changes, or increments as the body accedes to V, and decrements as it recedes from it, are equal at equal distances from V.

602. Cor. 2. If s be the number of inches through which a body descends from rest by the force of gravity in $1''$, and V be the velocity acquired in vibrating through PQ, or in falling down the perpendicular NR, then $NR : s$ as the squares of the velocities acquired in falling through NR and s, or as $V^2 : 4s^2$, and $V = \sqrt{NR \times 4s}$

$$= \sqrt{4s \times NV - VR} = \sqrt{\frac{4s}{DV}} \times \sqrt{FV^2 - EV^2} = \sqrt{\frac{4s}{DV}} \times$$

into the right sine $\frac{qn}{2}$; and at the vertex the velocity is equal to $\sqrt{\frac{4s}{DV}} \times \frac{Vv}{2}$ inches in $1''$.

603. Cor. 3. If a pendulum begin to oscillate at different distances PV and QV from the vertex V, the velocities acquired at V in these different vibrations, are as $\sqrt{NV} : \sqrt{RV}$, or as $VF : VE$, or as $VP : VQ$; or the velocity acquired in vibrating from rest is as the distance from the vertex at which it begins to vibrate.

604. Cor. 4. If s be the space described from rest, as in cor. 2. the velocities acquired at V, in vibrating from different points P

and $\mathcal{Q}$ are equal to $\sqrt{\frac{4s}{DV}} \times \frac{VP}{2}$, and $\sqrt{\frac{4s}{VD}} \times \frac{qn}{2}$, or such as would carry the body over these numbers of inches in 1″.

605. PROP. *The time in which the pendulum vibrates through any arc* QH, *is equal to the time in which a body would describe the corresponding circular arc* no *with the greatest velocity* Vv, *continued uniformly.*

Let M be infinitely near to $\mathcal{Q}$, so that $\mathcal{Q}M$ may be described uniformly, and taking Vp, Vq, Vm, Vh, respectively equal to VP, $V\mathcal{Q}$, VM, VH, and drawing the right line nk parallel to Vp; nk (qm or $\mathcal{Q}M$) : nl :: qn : Vv, or $\mathcal{Q}M$ and nl are to each other as the velocities with which they are described, and are therefore described in the same time. The same may be proved of the other corresponding parts of $\mathcal{Q}H$ and no, which are, consequently, described in the same time. Q. E. D.

606. Cor. 1. The times of describing any arcs $P\mathcal{Q}$, $\mathcal{Q}V$ are to each other as the corresponding circular arcs pn, nv, because they are described with the same uniform velocity; and the time of a whole vibration is as the semiperiphery pvx.

607. Cor. 2. The velocity, with which $\mathcal{Q}M$ is described, is actually equal to $\sqrt{\frac{4s}{DV}} \times \frac{qn}{2}$, and the velocity with which nl is described, is equal to $\sqrt{\frac{4s}{DV}} \times \frac{Vp}{2}$. The time of a vibration is equal to the time in which a body would describe the semiperiphery pvx with an uniform velocity equal to $\sqrt{\frac{4s}{DV}} \times \frac{Vv}{2}$ inches in a second.

608. Cor.

608. Cor. 3. If a body begin to oſcillate from different points P and Q, the times of their vibrations are equal to the times of deſcribing the ſemiperipheries of circles, whoſe radii are VP and VQ, with uniform velocities, which are reſpectively equal to $\sqrt{\frac{4s}{DV}} \times \frac{VP}{2}$, and $\sqrt{\frac{4s}{DV}} \times \frac{VQ}{2}$; but theſe times are as the ſpaces or ſemiperipheries divided by the velocities, or divided by their radii VP and VQ, and are conſequently equal: the times therefore of all vibrations, however different, are equal.

609. PROP. *Suppoſing a pendulum to begin to oſcillate from any point* P, *the time in which it performs one vibration, is to the time of deſcent down the axis as the periphery of a circle to its diameter.*

DEM. The times of deſcent down DV and FV are equal (538); and in this time a body would deſcribe a ſpace equal to $2FV$ or PV or pV, with the velocity acquired in falling down FV or PV continued uniform; but this is to the time of deſcribing pvx with the ſame velocity, or to the time of one vibration (607) as the ſpaces deſcribed, or as $pV : pvx$, or as the diameter of a circle to its periphery. Q. E. D.

610. Cor. 1. The periphery of a circle, being to its diameter in a given ratio, it appears again that the times of all vibrations are as the times of deſcent down the axis, and conſequently are given.

611. Cor. 2. The velocity with which the ſemiperiphery pvx is deſcribed, when the time of deſcribing it is equal to a vibration, is equal to $\sqrt{\frac{4s}{DV}} \times \frac{PV}{2}$ (607), and the time is equal to $\frac{pvx}{\text{vel.}} = \frac{2pvx}{PV} \times \sqrt{\frac{DV}{4s}}$; and the time of falling down $DV = \sqrt{\frac{DV}{s}}$;

con-

conſequently the time of a vibration is to the time of falling down $DV :: \frac{2 \times pvx}{PV} \times \sqrt{\frac{DV}{4s}} : \sqrt{\frac{DV}{s}} :: \frac{2 \times pvx}{PV} \times \frac{1}{\sqrt{4}} : 1 :: 2pvx : 2PV$, the ſame analogy as that in this propoſition.

612. Cor. 3. The times of vibrations in different cycloids, being equal to the times of deſcent down their axes multiplied into the ſame given quantity, will vary as the times of deſcent down the axes, or lengths of the pendulums; or ſuppoſing L to be the length of a pendulum, and T the time of a vibration, and to be variable; T will be as $\sqrt{L}$, and L as T^2.

613. Cor. 4. If the vibrations be very ſmall, the ſtring is not ſenſibly affected by the cycloidal arcs SA, SB, and the pendulum SP will deſcribe a circular arc, and hence it appears that very ſmall vibrations in circular arcs are performed in equal times. The times of deſcent down a circular arc and its chord are therefore unequal, the former being to the time of deſcent down the axis as half the periphery of a circle to its diameter; and the latter is equal to the time of deſcent down the diameter, or four times the axis, and is conſequently to the time of deſcent down the axis as 2 : 1.

614. Cor. 5. If the pendulum begin to oſcillate from B, half the time of a vibration is to the time of deſcent down the inclined plane BV as $BD : BV$; for half the time of a vibration is to the time of deſcent down the axis, as the ſemiperiphery of a circle to its diameter, or as $DFEV (BD)$ is to DV, and the time down DV : time down the inclined plane $BV :: DV : BV$, and, ex æquo, half the time of a vibration : time of deſcent down the inclined plane $BV :: BD : BV$.

615. Cor. 6. The ſpace deſcribed by a falling body in 1″, may be diſcovered, if the length of a pendulum performing one vibration

tion in a ſecond be known: let the length of this pendulum be 39.2 inches, and (609) 1″: time of deſcent through $\frac{39.2}{2}$ (t) :: periphery of a circle to its diameter :: 3.14159, &c. (p) : 1 and $t = \frac{1}{p}$. But $\frac{39.2}{2}$: ſpace fallen through in 1″ (x) :: $\frac{1}{p^2}$: 1^2 and $x = \frac{39.2}{2} \times p^2 = 193.1$ inches nearly $= 16.1$ feet nearly.

616. Cor. 7. The length of the pendulum performing one vibration in 1″, may be diſcovered, if the ſpace through which a body deſcends in 1″ be known; for let this ſpace $= 193.1$ inches, and (laſt cor.) the length of the pendulum $= \frac{2 \times 193.1}{3.14159}$ inches $= 39.2$ inches.

617. Cor. 8. The time of one vibration may be diſcovered, if the length of the pendulum be known; for let L be the given length of the pendulum in inches, and T be the time of one vibration, and T : time of deſcent through $\frac{L}{2}$ :: 3.14159, &c. : 1 :: p : 1; but the time of deſcent through $\frac{L}{2} = \sqrt{\frac{L}{2s}}$, and conſequently $T = p \times \sqrt{\frac{L}{2s}}$ ſeconds, or parts of ſeconds. If $L = 39.2$ inches, and $s = 16.1 \times 12$ inches, $T = 3.14159 \times \sqrt{\frac{39.2}{2 \times 16.1 \times 12}}$ $= 1''$ nearly.

618. Cor. 9. The number of vibrations in a given time, being inverſely as the time of one vibration, will be as $\frac{1}{\sqrt{L}}$. If N be the

the number of vibrations in a given time T, and t be the time of one vibration, then $N = \frac{T}{t}$; but, the length of the pendulum being L, $t = p \times \sqrt{\frac{L}{2s}}$, and consequently $N = \frac{T}{p} \times \sqrt{\frac{2s}{L}}$. If $T = 60 \times 60''$, then $N = \frac{60 \times 60}{3{\cdot}14159} \times \sqrt{\frac{193 \times 2}{39{\cdot}2}} = 60 \times 60$ very nearly.

619. PROP. *Let a pendulum, whose length is* L *inches, lose or gain any number of vibrations expressed by* n, *in any number of hours,* h, *to find the length of a pendulum that shall vibrate once in* 1″.

Let Y = the length of the pendulum required, and let the number of vibrations performed by Y in h hours, or $h \times 60 \times 60'' = m$, and L performs $m \pm n$, vibrations in h hours; but $L : Y :: m^2 \overline{m \pm n}^2$ (618) and $Y = L \times \frac{\overline{m \pm n}^2}{m^2}$.

620. PROP. *If the length of a pendulum and the force of gravity be variable, the time of an oscillation varies as the square root of the length of the pendulum* L, *divided by the square root of the force of gravity,* G.

DEM. The time of one oscillation varies as the time of descent down the axis, and that is as the square root of the axis directly and force inversely, or as $\sqrt{\frac{DV}{G}}$ (532), or as $\sqrt{\frac{L}{G}}$. Q.E.D.

621. Cor. 1. Therefore L is as $T^2 \times G$, and T being given, L is as G: if therefore the lengths of two pendulums, performing a vibration, or the same number of vibrations, in the same time be known,

known, the ratio of the forces of gravity, being the ſame with the ratio of theſe lengths, will conſequently be known.

622. Cor. 2. Or, if the number of vibrations performed in the ſame time, in different places, by two pendulums whoſe lengths are L and l, be equal to m and $m+n$ reſpectively, the forces of gravity G and g, in the places of obſervation, may be found: for, the times of one vibration of the pendulums L and l are as $\sqrt{\frac{L}{G}}$: $\sqrt{\frac{l}{g}}$, and $m+n : m$ inverſely as the times of one vibration, or as $\sqrt{\frac{L}{G}} : \sqrt{\frac{l}{g}}$ (620), and conſequently $G : g :: m^2 \times L : \overline{m+n}^2 \times l$.

623. PROP. *That part of the force of gravity, which accelerates or retards a pendulum vibrating in a cycloid, varies directly as its diſtance from the vertex.*

DEM. Let the force of gravity be repreſented, in quantity and direction, by the line DV, and be reſolved into two forces, DF parallel to the ſtring, and VF parallel to the tangent at P, or direction in which P moves; and $VF (= 2VP)$, which only accelerates P, varies as VP. Q. E. D.

Otherwiſe: The velocities at Q and M are as qn, ml; the time of deſcribing QM is as the included circular arc nl; and, ſuppoſing the time nl to be very ſmall and given, the accelerating force at Q is as the change of velocity kl (453), or as $\frac{Vq \times nl}{Vn}$ (ſim. triangles), or, becauſe nl and Vn are given, as Vq or VQ. Q. E. D.

624. Cor. 1. The force accelerating or retarding the pendulum (A): whole force of gravity (G) :: $VF : VD :: VP : VB$ and $A = \frac{G \times VP}{VB}$.

$\frac{G \times VP}{VB}$; and the tenſion of the ſtring : $G :: DF : DV$, and this tenſion $= \frac{G \times DF}{DV}$. At the vertex where $DF = DV$ the tenſion of the ſtring $= G$, and $A = o$; and, at B, $VP = VB$ and $A = G$, and the tenſion $= o$.

625. Cor. 2. The tenſion of the ſtring : force accelerating or retarding $P :: DF : VF :: \sqrt{DN} : \sqrt{VN}$ (ſim. triangles).

CHAP.

Plate XXI.

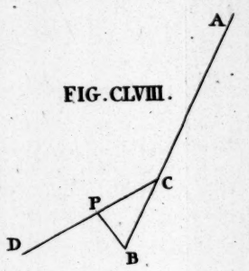


FIG. CLVIII.

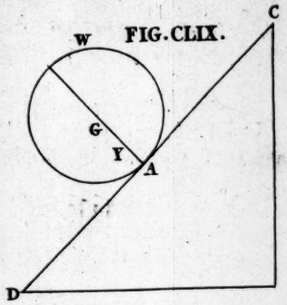


FIG. CLIX.

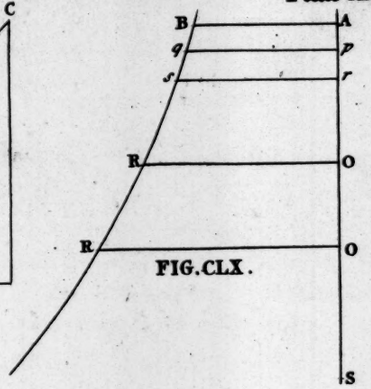


FIG. CLX.

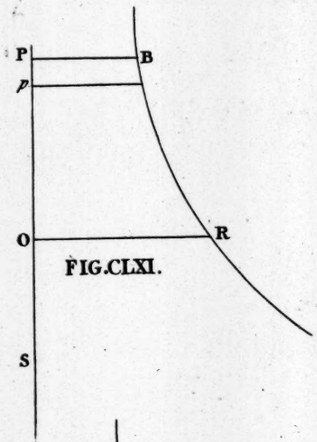


FIG. CLXI.

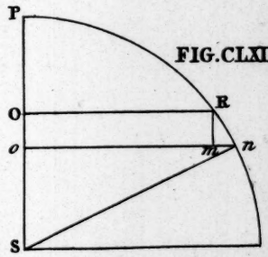


FIG. CLXII.

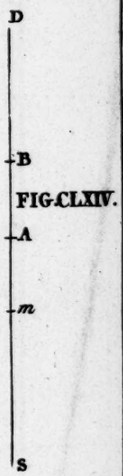


FIG. CLXIV.

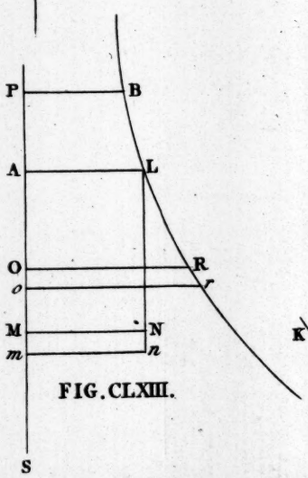


FIG. CLXIII.

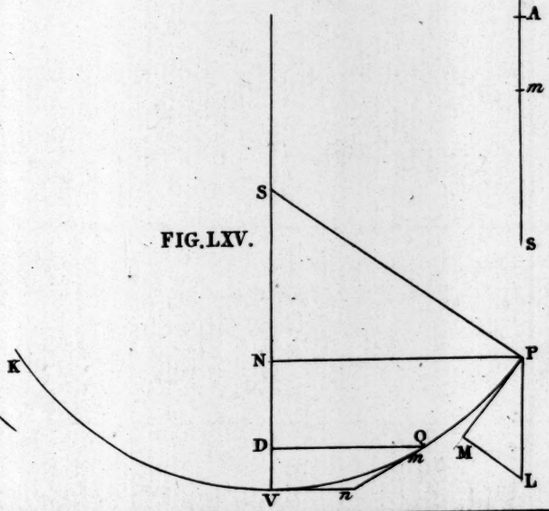


FIG. LXV.

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CHAP. XIV.

PROJECTILE MOTION.

626. DEF. *THE range or random of a projectile, is the rectilineal distance between the point of projection and impulse upon any obstacle; the horizontal range is called the amplitude; and the angle of elevation is the angle contained between the horizon and the direction of projection.*

627. PROP. *A body projected in any direction* DN, *and acted upon by a constant force,* G, *in a direction parallel to a right line* DO *inclined in any angle to* DN, *will describe a parabola.* FIG. CLXIX.

DEM. Let DO be described from rest, by the action of G, in the same time, T, in which DE is described by the velocity of projection, and, completing the parallelogram, the body will, at the end of the time T, evidently be at R; but DO varies as T^2 (526), or (because the velocity in DN is uniform from the first law of motion, and T varies as DE) as DE^2 or OR^2, which is a property peculiar to the common parabola. Q. E. D.

628. Cor. 1. The parameter belonging to any diameter DO is equal to $\frac{OR^2}{DO}$ or $\frac{DE^2}{ER}$ (conic sect.); and if the parameter belonging to any point D be equal to $\frac{DE^2}{ER}$, the parabola will pass through the point R.

629. Cor. 2. If the velocity of projection be the same, the parameter belonging to DO will be the same, whatever be the angle of projection; for the projectile velocity and G being given, DE, ER, and consequently $\frac{DE^2}{ER}$, or the parameter, are given.

630. Cor. 3. Let Υ be the space through which a body falls from rest, when acted upon by the constant force G, to acquire a velocity, V, equal to that in any point D of the curve, and let v be the velocity acquired in falling from E to R, by the action of same force G, and $V : v :: DE : 2ER$ (106): but $\Upsilon : ER :: V^2 : v^2$ (526) $:: DE^2 : 4ER^2$, and $\Upsilon = \frac{DE^2}{4ER} = \frac{1}{4}^{th}$ of the parameter belonging to DO.

631. Cor. 4. If G be the force of gravity, the axis and all diameters of the parabola, are perpendicular to the horizon; the velocities are equal at equal distances from the principal vertex; the time of arriving at the vertex of the axis, or at the greatest altitude, is equal to half the time of the flight; the parabola cuts the horizon, and all lines parallel to the horizon, in equal angles; and DE, which is parallel to the ordinate OR, is a tangent to the parabola at D.

FIG. CLXX.

632. Cor. 5. The velocity in different points varies in a subduplicate ratio of the parameter belonging to those points, or, if IX be the directrix and DI perpendicular to it, in a subduplicate ratio of DI or DS, S being the focus, or, in the same parabola, as the perpendicular $S\Upsilon$ upon the tangent DT. The horizontal velocity DH, determined by drawing perpendiculars from the body upon the horizon, varies as DE, and is consequently uniform; and the velocity, therefore, in any point D, being as DE, is as the secant of the angle of elevation; and at the principal vertex, the velocity in the curve and the horizontal velocity, are equal.

633. PROP.

FIG. CLXVI.

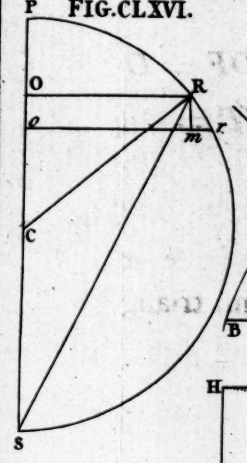


FIG. CLXVII.

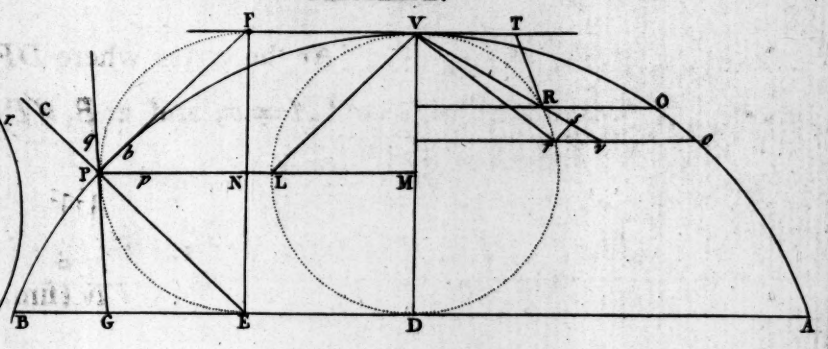
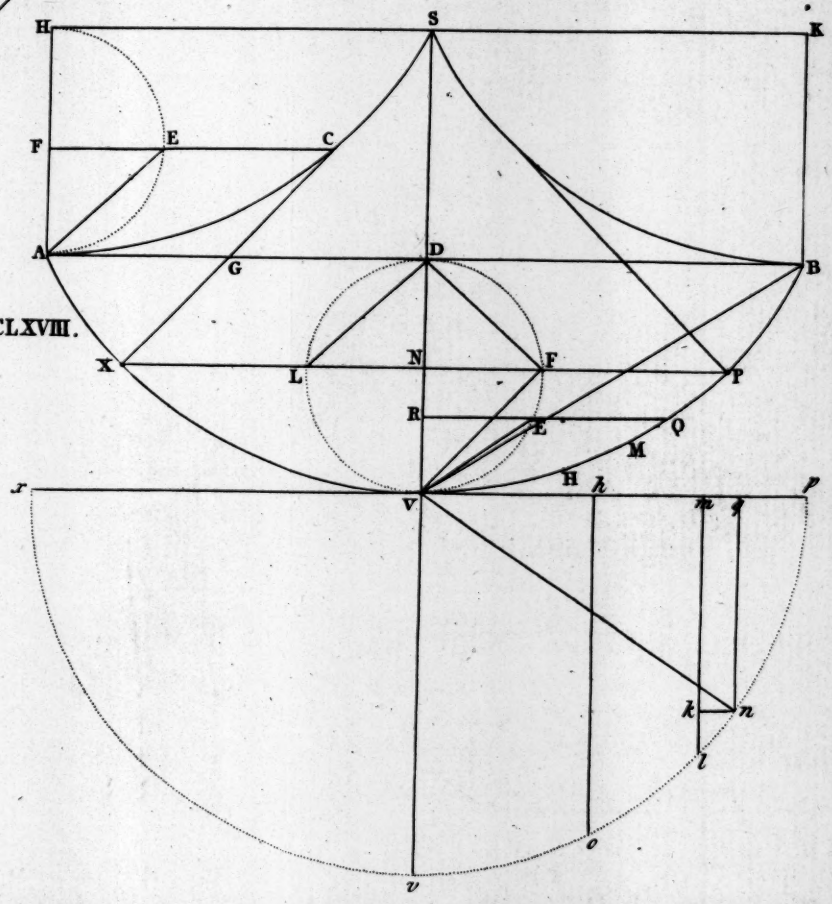


FIG. CLXVIII.




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633. PROP. *To find the directions in which a body, being projected with any velocity,* V, *from a given point* D, *will paſs through any point* P. FIG. CLXXI.

Becauſe V, or the number of feet uniformly deſcribed in 1″, with the velocity of projection, is given, the ſpace DE, deſcribed from reſt to acquire this velocity, is known, being equal to $\frac{V^2}{64}$ (art. 529). Draw DA, equal to $4DE$, perpendicular to the horizon, BC perpendicular to DA through its biſection G, and DC perpendicular to DP; and from C as a center, at the diſtance CD, deſcribe a circle, and, if R, r, be the interſections of its periphery with a right line, paſſing through P perpendicular to the horizon, DR, and Dr will be the directions required; for (ſim. triang.) $DA : DR :: DR : RP$, and $DA : Dr :: Dr : rP$, and DA (= the parameter) $= \frac{DR^2}{RP}$ or $\frac{Dr^2}{rP}$; and the parabola deſcribed by a body projected in either of the directions DR or Dr with the velocity V, will paſs through P (628). Q. E. I.

634. Cor. 1. Becauſe the arcs BR and Br are equal, the line DB makes equal angles with the correſponding directions DR and Dr. Drawing $B\mathcal{Q}$ a tangent to the circle at B, a body projected in the direction DB, will paſs through $\mathcal{Q}$, and $D\mathcal{Q}$ is the greateſt diſtance upon that line to which the body can be projected with the velocity V.

635. Cor. 2. If DP be horizontal, the range upon the horizon $= DF = RN$, the ſine of twice the angle of elevation FDR; for, by the property of the circle $\angle FDR = \angle RAD = 2\angle RCD$. The horizontal range is therefore the greateſt, when the angle of elevation is 45°, the ſine of twice that angle, or of 90°, being equal to $\frac{1}{4}DA$, or $\frac{1}{4}$ of the parameter at D; or it is equal to the latus rectum, for that is always equal to $\frac{DM^2}{MV}$, which, in this caſe, is equal to $\frac{DC^2}{4\times\frac{1}{4}DC} = DC$. FIG. CLXXII.

Becauſe

any angle varies as the radius or diameter, if the upon any other diameter DE or $\frac{1}{4}^{th}$ of DA, the will be equal to the ſine of twice the angle of rcle multiplied into $\frac{DA}{DE}$, or into 4.

the axis MV, produced, be interſected by the greateſt altitude $MV = \frac{1}{4} MT$ (conic ſect.) = erſed ſine of twice the angle of elevation; and right, $\frac{1}{4}^{th}$ of the verſed ſine $= \frac{1}{4} DA$. Becauſe any angles are as the diameters of the circles rcle be deſcribed with a diameter equal to DE, l to the verſed ſine of twice the angle of ele-

e time of the flight is equal to the time of de- he projectile velocity, and varies as DR, or as ſine of the angle of elevation; and conſequent- greateſt when the angle $FDR = 90°$, and it is ime of deſcent from reſt through the parame- e is alſo equal to the time of falling from reſt herefore varies as $\sqrt{RF}$, or as the ſquare root f twice the angle of elevation, or as the ſquare e greateſt altitude.

velocity of projection, V, *and the angle of eleva- deſcribe the path of the projectile.*

umber of feet uniformly deſcribed in 1″, and

f the parameter, is the ſame whatever be the

and the periphery of a circle, whoſe radius is D, will paſs through the foci of all parabolas y projected from D with the velocity V. Let ular to the horizon, and DR the direction of pro-

Becauſe the ſine of any angle
circle be deſcribed upon any
horizontal range will be equ

elevation of this circle multip

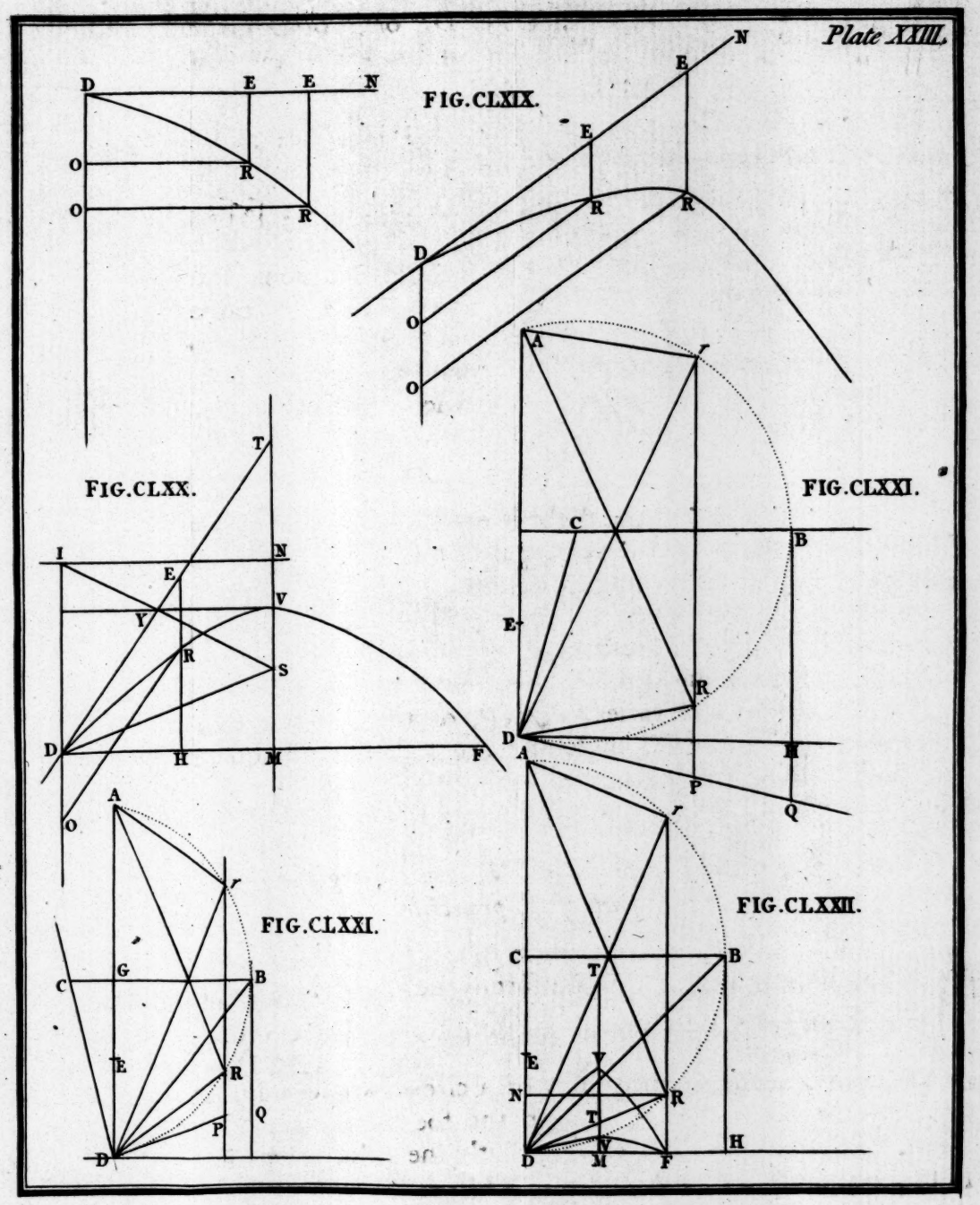
636. Cor. 3. If the axis *M*
tangent in *T*, the greateſt a
$\frac{1}{4}FR = \frac{1}{4}^{\text{th}}$ of the verſed ſine
when this angle is right, $\frac{1}{4}^{\text{th}}$ o
the verſed ſines of any angle
deſcribed, if the circle be deſc
the altitude is equal to the v
vation of this circle.

637. Cor. 4. The time of
ſcribing *DR* with the projecti
$\frac{1}{2}DR$, which is the ſine of the
ly the time is the greateſt whe
then equal to the time of deſ
ter *AD*. The time is alſo eq
through *RF*, and therefore va
of the verſed ſine of twice the
root of $\frac{1}{4}RF$, or the greateſt al

638. PROP. *The velocity of*
tion being given, to deſcribe the

FIG. CLXXIII. Let *V* be the number of
$\frac{V^2}{64} = DE = \frac{1}{4}^{\text{th}}$ of the paran
angle of elevation, and the p
DE, and center *D*, will paſ
deſcribed by a body projected
DE be perpendicular to th

Plate XXIII.


D
E
E
N
O
R
O
R
FIG.CLXIX.
N
E
E
R
R
D
O
O
T
FIG.CLXX.
I
N
E
V
Y
R
S
D
H
M
F
O
A
r
C
G
B
E
R
P
Q
D
FIG.CLXXI.
A
r
C
B
E
D
P
Q
FIG.CLXXI.
C
B
E
D
M
F
A
r
C
T
B
E
N
R
T
V
D
M
F
H
FIG.CLXXII.


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APR

projection, and making the angle $SDR = \angle RDE$; S will be the focus: EX drawn perpendicular to DE is the directrix; V the biſection of SX drawn parallel to DE, is the principal vertex, and SV the axis; DM perpendicular to VM is an ordinate; SE biſects DT in R and is perpendicular to it, and RV is a tangent to the parabola at V (conic ſect.).

639. Cor. Again, it appears that the amplitude DF is (as in art. 635) equal to four times the ſine of twice the angle of elevation, for $\angle MDR = \angle RDC = 2\angle RCN$, and $DF = 2DM = 2NV = 4RN$.

640. PROP. *To find the leaſt velocity with which a body projected, from a given point* D, *will hit a given point* P. FIG. CLXXIV.

Draw BPH perpendicular to the horizon, and, making PB equal to PD joined, biſect the angle DPB by the right line PC, meeting DC perpendicular to DP, in C; and, becauſe the triangles CBP and CDP are equal and ſimilar, $CB = CD$, the $\angle$s CBP and CDP are right angles, and BP is a tangent to the circle deſcribed from C, as a center, at the diſtance CB. Biſect DG, drawn perpendicular to the horizon, in E, and a body projected in the direction DB, with a velocity equal to that acquired in falling through ED, will paſs through P (634), and, becauſe DP is the greateſt range (634), the velocity muſt be the leaſt poſſible. Q. E. I.

641. Cor. If P be in the horizon, CD and GD coincide, and the angle of elevation BDH is equal to 45°.

642. PROP. *The range upon any plane* DP, *is equal to the product of the parameter and ſines of the angles formed by the plane and direction of projection, and the plane and a perpendicular to the horizon, divided by the ſquare of the coſine of the plane's elevation.* FIG. CLXXI.

DEM.

DEM. From trigon. $DP : DR :: \text{sin.} \angle DRP \text{ or } \angle ADR : \text{sin.} \angle DPF$,
$DR : DA :: \text{sin.} \angle DAR \text{ or } \angle PDR : \text{sin.} ARD \text{ or } \angle RPD \text{ or } DPF$;
and consequently $DP : DA :: \text{sin.} \angle ADR \times \text{sin.} \angle PDR : \text{sin.}^2 \angle DPF$ or $\text{cos.}^2 \angle PDF$, and $DP = \frac{DA \times \text{sin.} \angle ADR \times \text{sin.} \angle PDR}{\text{cos.}^2 \angle PDF}$.
Q. E. D.

FIG. CLXXV.

643. Cor. 1. Let $S =$ the sine of the angle contained between the plane and the direction; $\mathcal{Q} =$ the sine of the angle contained between the direction and the parameter DA perpendicular to the horizon; and $C =$ the cosine of the angle contained between the plane and the horizon; and the range $DP = \frac{DA \times S \times \mathcal{Q}}{C^2}$, and, because DA varies as the square of the velocity, or V^2, DP is as $\frac{V^2 \times S \times \mathcal{Q}}{C^2}$. This corollary may be demonstrated differently by the following process: let V be the velocity of projection, or number of feet described uniformly in 1″, and $V : DR :: 1'' : \frac{DR''}{V} =$ the time of describing DR or RP, and $RP = 16 \times \frac{DR^2}{V^2}$ (529) $= DP \times \frac{S}{\mathcal{Q}}$; but $DR = DP \times \frac{C}{\mathcal{Q}}$, therefore $DP \times \frac{S}{\mathcal{Q}} = 16 \times \frac{DP^2 \times C^2}{V^2 \times \mathcal{Q}^2}$, and consequently $DP = \frac{S \times \mathcal{Q} \times V^2}{16 \times C^2}$.

644. Cor. 2. If DP be bisected in M and MV be drawn perpendicular to the horizon, the greatest altitude $MV = \frac{1}{2} MT = \frac{1}{4} PR$; but $PR : DP :: S : \mathcal{Q}$ and $PR = \frac{DP \times S}{\mathcal{Q}} =$ (substituting the value of DP) $\frac{V^2 \times S^2}{C^2}$; and consequently MV, or the greatest altitude varies as $\frac{V^2 \times S^2}{C^2}$.

645. Cor.

645. Cor. 3. The time of the flight is equal to the time of describing DR, and is as $\frac{DR}{\text{vel.}}$; or, if V = the number of feet described uniformly in 1″ with the velocity of projection, the time is actually equal to $\frac{DR''}{V}$ = (fig. 171.) $\frac{DA \times S}{C \times V}$; and because DA is as V^2, the time varies as $\frac{V \times S}{C}$. The time is also equal to the time of descent from rest through RP, and consequently varies as $\sqrt{PR}$, or as $\sqrt{MV}$. FIG. CLXXV.

646. Cor. 4. Supposing V to be given, the range DP is the greatest when $S = Q$, or when the direction bisects the angle contained between the plane and the vertical DA; and if the plane DP coincide with the horizon, the amplitude $DP = DF$ $\left(= \frac{DA \times S \times Q}{C^2}\right) = \frac{DA \times FR \times DF}{DR^2}$ (making DR the radius) = DF or RN the sine of twice the angle of elevation, DA being equal to $\frac{DR^2}{FR}$, and $\frac{DA \times FR}{DR^2}$ being given. The greatest altitude $= \frac{DA \times S^2}{4C^2} = \frac{DA \times FR^2}{4DR^2} = \frac{1}{4}FR$.

647. Prop. *If both the velocity of projection,* V, *and the angle of elevation, vary, the horizontal range or amplitude will vary as the square of the velocity and sine of twice the angle of elevation.*

Dem. The range, being generally as $\frac{V^2 \times S \times Q}{C^2}$ (643), will, when DP coincides with the horizon, be as $\frac{V^2 \times S \times Q}{\text{rad.}}$; and, if the radius be unity, $S \times Q = \frac{1}{2}$ of the sine of twice the angle of elevation (trigonom.), and the amplitude consequently is as $V^2 \times$ sine of twice that angle. Q. E. D. FIG. CLXXI.

Another demonſtration:

FIG. CLXXVII. Let V = the number of feet deſcribed uniformly in 1", by the velocity of projection, and $V : DL :: 1'' : \frac{DL''}{V}$, or the time of deſcribing DL with the velocity of projection, which is equal to the time of deſcribing LF from reſt, and $LF = \frac{s \times DL^{2''}}{V^2}$, ſuppoſing s to be the deſcent from reſt in the firſt ſecond (529); but, deſcribing a circle with any diameter DA, $LF = \frac{DL \times DR}{DA}$ (ſim. triang.), and conſequently $\frac{s \times DL^2}{V^2} = \frac{DL \times DR}{DA}$, and $DL = \frac{DR \times V^2}{s \times DA}$; and $DF \left(= \frac{DL \times RN}{DR} \text{ from ſim. triang.}\right) = \frac{RN \times V^2}{s \times DA}$, and varies as $V^2 \times RN$, becauſe s and DA are given. Q. E. D.

648. Cor. The greateſt altitude is generally as $\frac{V^2 \times S^2}{G^2}$ (644), or when DP is horizontal, as $\frac{V^2 \times S^2}{\text{rad.}^2}$, or, the radius being unity, as $V^2 \times$ verſed ſine of twice the angle of elevation; for $S^2 = \frac{1}{4}$ of this verſed ſine.* This corollary is alſo deducible from the ſecond demonſtration; for the greateſt altitude $= \frac{LF}{4} = \frac{DL \times DR}{4DA} =$ (ſubſtituting the value of DL) $\frac{DR^2 \times V^2}{4s \times DA^2} = \frac{V^2 \times DN}{4s \times DA}$, and conſequently varies as $V^2 \times DN$, s and DA being given.

649. PROP.

FIG. CLXXVI. * Let the arcs DF, EF, and the angles DCF and ECF, be equal, and, FCD being the angle of elevation, EL drawn perpendicular to the radius CD, is the ſine, and DL the verſed ſine, of twice that angle; and, joining ED and drawing GH from G, the interſection of CF and ED, parallel to DL, $EH = \frac{1}{2}$ the ſine, and $GH = \frac{1}{2}$ the verſed ſine. But the triangles EGH and ECG are ſimilar, the angles at H and G being right, and the $\angle EGH = \angle EDC = \angle CED$; therefore $EG \ (= FN = S) : EH :: CE \ (= 1) : CG \ (= CN = 2)$, and $EH = S \times 2$. And EG or $S : GH \ (= \frac{1}{2} LD) :: GE : EG$, and conſequently $GH = S^2$.

649. PROP. *The velocity of projection*, V, *or number of feet described uniformly in* 1″ *with that velocity, and the angle of elevation*, E, *being given, to find the amplitude, altitude, and time of flight.*

1. The space described from rest to acquire the velocity of projection is equal to $\frac{V^2}{4s}$, and the amplitude, when the angle of elevation $= 45°$, is equal to $\frac{V^2}{2s}$ (635); but the amplitudes are as the times of twice the angles of elevation, and consequently sin. of 90° : sin. of $2E :: \frac{V^2}{2s}$, or $\frac{1}{4}$ parameter : amplitude, which is therefore known.

2. When the angle of elevation $= 45°$, the altitude $= \frac{1}{4}$ of the parameter $= \frac{V^2}{8s}$, and, the altitude being always as the versed sine of twice the angle of elevation, the versed sine of 90° : versed sine of $2E :: \frac{V^2}{8s}$: altitude, which is therefore known.

3. When the angle of elevation is 90°, the time of the flight is equal to the time of falling, from rest, down the parameter, $\frac{V^2}{s}$, and $= \sqrt{\frac{V^{2\prime\prime}}{s^2}} = \frac{V''}{s}$, and the times of flight being as the sines of angles of elevation (637), the sine of 90° : sine of $E :: \frac{V''}{s}$: time of the flight, which is therefore known. Q. E. I.

650. PROP. *The amplitude and the angle of elevation*, E, *being given, to find the velocity of projection, and the altitude.*

1. Let the parameter $= P$, and, when the angle of elevation is 45°, the amplitude $= \frac{P}{2}$; therefore, the sin. of $2E$: sin. of 90° :: given amplitude : $\frac{P}{2}$, which is therefore known; and the velocity of projection, being equal to that acquired in falling from rest through $\frac{P}{4}$, is also known, being equal to $\sqrt{4s \times \frac{P}{4}}$ feet in 1" (529).

2. The altitude is found when the parameter is known, by the last proposition.

Or, The altitude is easily found by the following process: Bisect the amplitude DF in M, and, drawing MT perpendicular to DF, the angles in the triangle DMT, and the side DM, are known, and MT may be found from trigonometry; and consequently $MK = \frac{1}{4}MT$ is known. Q. E. I.

FIG. CLXXVII.

SCHOLIUM.

651. By a similar process the converse of these propositions are easily solved, that is, 1. the amplitude and altitude being given, to find the velocity of projection and angle of elevation; 2. the velocity of projection and altitude being given, to find the amplitude and angle of elevation; and, 3. and the angle of elevation and altitude being given, to find the amplitude and velocity.

SCHOLIUM II.

652. In the theory of projectiles, the medium is supposed to be void of resistance, which supposition does not obtain in practice, the resistance of the air being variable according to the different velocities and magnitudes of the projectiles, and always considerable.

able. 1. If a musket ball, of $\frac{1}{40}$th of an inch diameter, be fired, from a piece forty-five inches long, with half its weight of powder, its velocity is nearly 1700 feet in 1″, and its horizontal range, when the angle of elevation is 45°, ought to be about seventeen miles; but from practical writers it appears, that the range is less than half a mile. An iron ball of 24 lb. weight, discharged with a full charge of powder, has a velocity of 1650 feet in 1″, and its horizontal range at 45° would be about sixteen miles, were the path described a parabola; but, from experiments, it appears not to be three miles, and not $\frac{1}{5}$th part of the space investigated by the theory. When the velocity of the shot is about 400 feet in 1″, the resistance of the air is still considerable, and neither the amplitude, height, or time of flight, correspond with the theory. This resistance is confirmed by the constant observation of all conversant in the projection of bombs; for the ranges at elevations equally distant from 45°, ought, according to the theory, to be equal; but the ranges of a shell projected at an elevation of 15° or 20°, are always found to be greater than those projected at elevations equal to 60° or 65°, though equally distant from 45°.

Projectiles, whose motion is sensible to the eye, are seen to descend in a curve obviously shorter, and inclined in a greater angle to the horizon, than that in which they ascended; and whoever views, in a proper situation, the flight of stones, arrows, shells, &c. projected to any considerable distance, may see evidently that the vertex of the curve or greatest altitude, divides the path described into two unequal parts, and is more remote from the point of projection, than where the projectile falls to the ground.

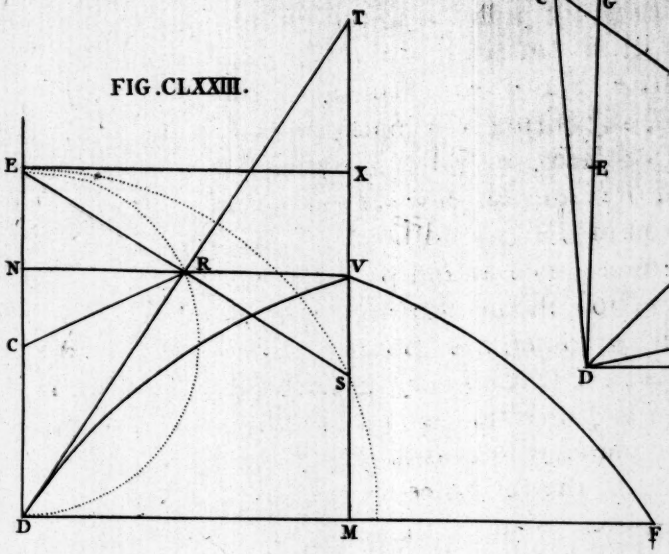
SCHOLIUM III.

653. A body, projected with a considerable velocity, is often not only affected by the resistance of the air, and deflected from a parabolic path in a direction perpendicular to the horizon, but is made to deviate laterally and change the plane of motion. The path of a tennis-ball, struck with great force, is plainly observed to be incurvated sideways as well as downwards, and to move in a different plane from that arising from the combined

bined action of gravity and force of projection. Bullets are not only depreſſed beneath the line of projection, but deflected to the right or left of that direction by the reſiſtance of the air, or action of ſome other force. Mr. Robins fixed a barrel, carrying a ball of $\frac{1}{4}$th of an inch diameter, and, firing at a mark, one foot and $\frac{1}{2}$ſquare, at the diſtance of 180 feet, miſſed it only once in ſixteen ſucceſſive trials; but when the ſame barrel was fired with a ſmaller quantity of powder, he found the ball to be deflected 100 yards to the right or left of the line aimed at and placed at the diſtance of 760 yards; and its direction in the perpendicular was equally uncertain, its range differing ſometimes 200 yards. Becauſe the force of gravity acts always in a direction perpendicular to the horizon, a body projected in any direction would, if unreſiſted, be always in the ſame vertical plane, which, as is obſerved, is not true in fact; and, therefore, beſides the reſiſtance of the air to the progreſſive motion of the body, there is another lateral force producing a deviation from the plane of projection, which is probably the inequality of reſiſtance upon its ſurface: for, if the ſurface of a projectile, protruding the particles of a fluid, be equally reſiſted in every point, it is obvious that the plane in which it firſt moves is not altered. But ſuppoſing the reſiſtances upon different ſides of a body to be unequal, it will be impelled, by the greater reſiſtance, towards thoſe parts where the reſiſtance is leaſt; and thus either diverge laterally from the firſt plane of motion, or aſcend, or deſcend in that plane, beyond the altitude inveſtigated by the theory. A motion of rotation round an axis will produce this inequality of reſiſtance upon the different ſides of a projectile, and conſequently its irregular motions; for the parts of a revolving body are expoſed to the air, which is protruded, in different angles of obliquity and with different forces, and muſt therefore be differently reſiſted. If the axis of rotation be perpendicular to the direction, the body will be deflected naturally from the vertical plane, and becauſe the velocity of the body, and conſequently reſiſtance of the air, are variable, this deflection will be different in different points, or it will be a curve line. That this lateral deviation is produced by a motion of rotation ſeems to be confirmed by experiments. Let a ſphere of wood, ſuſpended freely by a cord in a

current

FIG. CLXXIII.



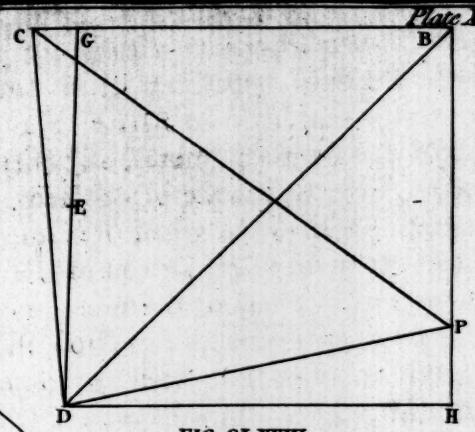
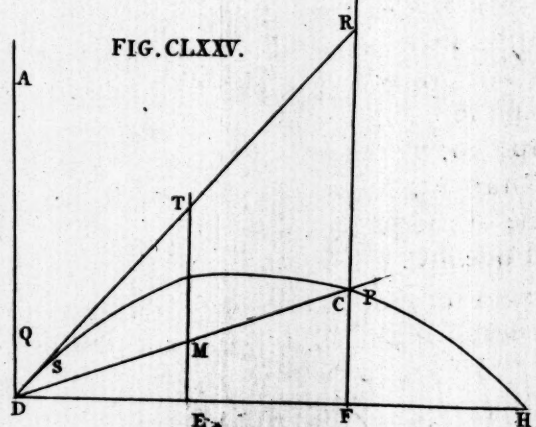


FIG. CLXXIV.

FIG. CLXXV.



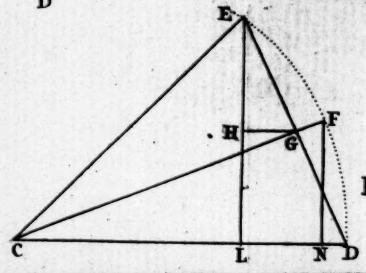
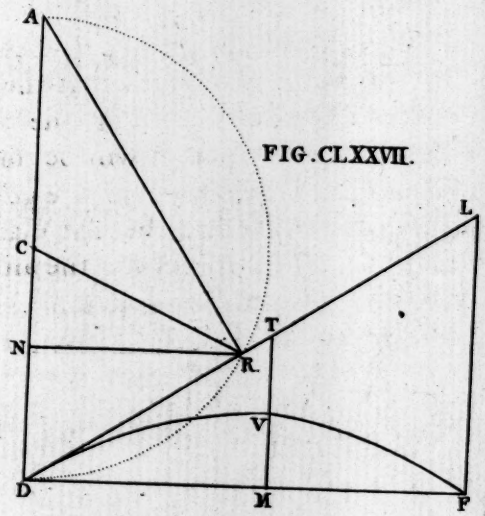


FIG. CLXXVI.

FIG. CLXXVII.



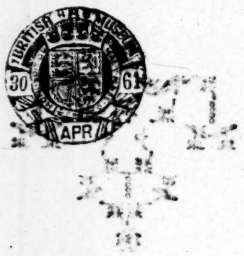
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current of air, be made to revolve round a ſtring as an axis, and the parts of its ſurface, on oppoſite ſides, oppoſing or conſpiring with the motion of the particles of the fluid, are unequally impelled by it, and the ſphere is always deflected towards that ſide whoſe reſiſtance is leaſt. Or, let a wooden ball, loaded with lead, be ſuſpended freely in a ſtream of water by a twiſted cord; and, as the cord returns to its natural ſtate, the ball revolves round it, and moves gradually towards the ſide where the reſiſtance is leaſt, or where the parts conſpire with the motion of the water. When the ball arrives at its utmoſt extent, it is quieſcent for a moment, and returns gradually to its firſt ſituation, and is again quieſcent till the motion of the ball twiſt the cord the contrary way, and it then moves alſo towards the other ſide; and in this manner the ball continues to vibrate till its motion be deſtroyed by friction, and it remain quieſcent in its firſt ſituation. The magnitude and direction of reſiſtance of the air, and conſequent deviations from theory, can only be aſcertained by a ſeries of experiments, which, reſulting from the operation of cauſes apparently fluctuating and uncertain, produce an inconſiſtency in the ſame experiments, and render the ſubject at leaſt very complicated and difficult *.

* See Robins's Tracts of Gunnery, p. 183, &c. and Euler's True Principles of Gunnery, by Mr. Brown.

ERRATA et ADDENDA.

Pag. 2. lin. 7. *for* of that, *read* that.
10. art. 15. *for* the exiſtence of ratios, *read* the exiſtence of the meaſures of ratios.
24. art. 48. ex. 1. for *La*, read *Lm*, and for *La*, read *LA*.
29. ex. 2. *dele* or cloſeneſs.
37. art. 78. *for* all infinitely great, &c. *read* all infinitely great or ſmall magnitudes compared with each other, admit of the ſame inequality of ratios with finite magnitudes.
42. art. 85. *dele* the remoteneſs of the elementary particles.
67. lin. 8. *after* velocity, *read* when in motion.
fig. 26. the letters *L*, *M*, wanting.
81. art. 194. *for MBN*, *read PBN*.
82. art. 200. *after BD*, *put BE*.
83. art. 204. *dele* in the ſame direction.
fig. 48. the letter *D* wanting.
111. ex. 3. *for* fire, *read* fibre.
115. line the laſt. *after* velocities, *put* in oppoſite directions.
148. . *for* fig. 16. *read* fig. 106.
152. l. . by the direction of the reſiſtances is underſtood the right line in which the reſiſting forces act, and by *the quantities of the reſiſtances* are underſtood, not the quantities really exerted upon the wedge, but which would be exerted if no part were loſt.
186. the following corollary wanting to art. 470. If the circle does not cut or touch the line *DH*, the bodies will never meet.
197. art. 500. *for* $\pm$ velocity of $V \times SA$, *read* $= \frac{\text{velocity of } V \times SA}{SV}$.
198. art. 500. l. 2. for $m \times SV$, read $m \times SV \times SC$.
231. art. 589. *for* arc, *read* chord, in which corollary, the body is ſuppoſed to vibrate in the chords.



A SYSTEM OF NATURAL PHILOSOPHY.

HYDROSTATICS.

1. THE ſcience deſcribing the mechanical affections of fluids, and their efficacy in the production of thoſe natural phenomena, of which they are the immediate cauſes, is very properly entitled Hydrodynamics; which, according to the nature of the ſubjects conſidered, is divided into ſeveral branches; as, Hydraulics, which treats of the motion of water through pipes or conduits; Hydroſtatics, whoſe object is the relation between the weight of water and other bodies; Pneumatics, which is employed in inveſtigating the properties of air, &c. but the term, Hydroſtatics, is now uſed, with greater latitude, to denote the ſcience deſcribing the properties of all fluids, excepting light and electric effluvia, and the effects reſulting from them. The primary diviſion of hydroſtatics, like that of other branches of natural knowledge, is analytic and ſynthetic: the firſt inveſtigates, from a ſeries

of concurring experiments, thoſe affections of fluids which ſeem to be general natural cauſes, as gravity, elaſticity, attraction of coheſion, &c.; and the ſynthetic, aſſuming theſe as eſtabliſhed principles that operate generally in the part of the creation contemplated, demonſtrates them ſcientifically to be adequate to the production, not only of the particular effects adduced to prove their exiſtence and power, but of all ſimilar phenomena. An obvious diviſion of the ſynthetic part is derived from thoſe different principles, which are ſufficiently diſcovered to ſupply data for ſynthetic reaſoning, and the following arrangement is therefore adopted.

Firſt, The preſſure, ſpecific gravity, and motion, of fluids are placed in ſucceſſive order, as being dependent upon the ſame principle, their gravity.

Secondly, The properties of the air-pump, different denſities of the air, winds, and other phenomena ariſing from elaſticity.

Thirdly, The reſiſtance of fluids to bodies moving in them.

Fourthly, The aſcent of vapours, and other phenomena ariſing from the attraction of coheſion.

CHAP. I.

OF FLUIDITY.

2. DEF. *A FLUID is a body whoſe parts eaſily yield to any force impreſſed upon them, and move freely amongſt each other; as fire, air, mercury, water, vapours, &c.*

SCHOLIUM I.

3. Fluidity is a property of matter obtaining in different bodies, and often in the ſame body at different times, in different degrees. Light, fire, electric effluvia, air, water, &c. are the moſt fluid bodies with which we are acquainted; and of theſe ſome are more fluid than others. Water is more fluid than wine, this laſt than ſyrup, and ſyrup than oils, and at length we may arrive at bodies whoſe deſcription it will be difficult to define, whether they be fluid or ſoft. The ſubject does not admit of more preciſion; but a body may however be defined to be fluid, when its component parts are ſo diſunited and incoherent, that no ſenſible reſiſtance is oppoſed to their ſeparation and motion in any direction, but which are ſo compreſſed, or whoſe mutual attraction is ſo ſtrong, as to preſerve their contiguity and make them form one aggregate maſs. Fluidity is therefore an intermediate ſtate between hardneſs or ſoftneſs, in which the parts adhere together firmly or with very ſenſible tenacity, and an entire want of aggregation, in which the component parts are ſo ſeparated and removed from each other, that they do not form one maſs. The difference between ſoft and fluid bodies ſeems to conſiſt in this: the parts of the firſt, though eaſily diſplaced, do not propagate motion to the

diſtant parts, but the whole remains nearly quieſcent when the force is removed: the parts of the ſecond, when moved by any force, communicate motion to the particles immediately contiguous, and theſe to the next ſucceſſively, till the fluid become agitated to a conſiderable diſtance, and, after the force is removed, it returns to its former figure. Clay, ſnow, butter, are ſoft bodies, becauſe their parts are eaſily diſplaced, and then remain nearly quieſcent. Air, water, vapours, oil, &c. are fluids, becauſe motion is eaſily communicated and diffuſed in every direction, and after many tremulous agitations deſtroyed, when the particles are again quieſcent. That air is a fluid appears from the eaſy paſſage it affords to light, and the motion communicated to it, in every direction, by the vibrations of ſounding bodies. It is certain that the particles of ſome fluids are not in immediate contact, becauſe they are compreſſible, and their dimenſions may be much diminiſhed; and whether thoſe of any fluid touch each other is not known, and cannot be obſerved by the ſenſes, however aſſiſted with glaſſes. It is therefore unknown in what manner the particles of fluids act upon each other, whether by actual impulſe, or a repulſive force, and occaſion the inteſtine motion which diſcriminates them from ſoft bodies. Fluidity may be conceived to ariſe from the following cauſes:

1. Let the particles, conſtituting any maſs of matter, be very ſmall, equal, lubricous, ſpherical globules, and if they be in actual contact, the ſurfaces of contact will be ſmall, and conſequently their attraction alſo ſmall (Mechan. art. 248). Motion would be eaſily communicated to a maſs of matter compoſed of ſuch parts, and diffuſed to ſome ſmall diſtance. A fluid muſt be tenacious when its component parts are large, heavy, and ſtrongly attract each other or have plane ſurfaces.

2. If the component parts of a body be very ſmall globules, which are not in contact, but impreſſed at different diſtances with principles of alternate attraction and repulſion (which are ſeen to obtain in ſome natural ſubſtances), they will be fluid, becauſe a

ſmall

ſmall force would communicate motion to the parts of ſuch a body, which would be propagated neceſſarily in all directions.

3. A body may be rendered fluid by diminiſhing the ſurfaces of contact, or diſtances of its conſtituent parts, by the interpoſition of a fluid; thus, ſalts are rendered fluid by the interpoſition of water between its parts; gold by aqua regia, &c. and theſe bodies are ſaid to be in a ſtate of ſolution. Fluidity in every body is affected by heat: oils, balſam, honey, &c. are moſt fluid when hot; firm bodies, as ſalts, metals, &c. are rendered fluid by the application of fire, when they are ſaid to be fuſed; and their component parts ſeem to be ſeparated, as the parts of ſalts are by water, and to be ſuſpended in a very ſubtle elaſtic inviſible fluid, which is perhaps the only ſubſtance in nature eſſentially fluid.

SCHOLIUM II.

4. If fine ſand, or plaſter of Paris finely powdered, be put into a veſſel over a fire, it will ſoon begin to boil like water, and exhibit all the motions and appearances of a fluid; and hence ſome people have ſuppoſed that the parts of a fluid are in a conſtant inteſtine motion. That this opinion is not true in all fluids appears from many facts. 1. The parts of ſome fluids attract each other, and, in an equilibrium of the parts, they muſt be quieſcent. 2. In pure fluids no inteſtine motion is diſcernible. 3. Particles of duſt, ſand, &c. mixed with water, ſwim when the fluid is agitated, and are ſeen to ſubſide when the agitation ceaſes. And if a ſubſtance be diſſolved in water, ſmall particles of it near the bottom are diſtinctly ſeen to be quieſcent, and the ſurrounding fluid cannot therefore have any inteſtine motion, &c. Moſt fluids are an aggregate of the ſimple elementary fluid and extraneous matter. Rain-water, ſpring-water, river-water differ in degrees of purity, but the firſt or moſt pure, is impregnated with ſeeds of plants, animalcules, earth, and other ſubſtances. Air, or the atmoſphere, is a heterogeneous aſſemblage of air, vapours of various

various kinds, exhalations, ſalts, ſulphur, metals, &c. When theſe adventitious mixtures are conſiderable, the concluſions derived, from ſuppoſing the element nearly homogeneous, muſt be affected, and proper allowances muſt be made, as the fluid nearly pure, and thus adulterated, may have very different properties.

CHAP.

CHAP. II.

OF THE WEIGHT OF FLUIDS.

5. PROP. *FLUIDS gravitate in proprio loco, or when immerſed in the ſame, or a different, fluid.*

EXP. 1. A quantity of water, or any of the groſs fluids, whoſe weight is *C*, being put into a veſſel of water, mercury, or other fluid, whoſe weight is *A*, the weight of the whole is $A+C$.

2. Suſpend a hollow ſphere or bottle, whoſe weight, when empty and immerſed in water, is *A*, from one arm of a balance, and water being admitted into it, its weight is found to be encreaſed by the preciſe weight of the included fluid.

6. COR. Any portion of a fluid, immerſed in its own or a different element, preſſes upon the contiguous parts, in a direction perpendicular to the horizon, with its whole weight.

7. DEF. *Air is a fluid that is elaſtic, inviſible, inodorous, &c. and is frequently uſed ſynonimouſly with the atmoſphere, or that tranſparent fluid which encompaſſes the earth* *.

8. PROP. *Air is influenced by gravity.*

EXP. 1. An exhauſted receiver is fixed to the ſtand by the preſſure of the external air.

2. If

* Prieſtley, or Cavallo, on Air.

2. If a ſmall receiver be placed under a larger, and both be exhauſted, the ſmaller is eaſily moved, and the larger one remains fixed.

3. If the ſmaller be of thin glaſs and have a valve on the top opening outwardly, they will both be exhauſted, and the ſmaller will be broken by the admiſſion of air into the larger.

4. If the upper orifice of a receiver be covered by the hand, the preſſure of the external air will be ſenſibly felt, and if covered with thin glaſs, the glaſs will be broken upon exhauſting the receiver.

5. If mercury be poured into a wooden cup, fixed cloſely to the upper orifice of an exhauſted receiver, it will be forced through the pores of the wood by the preſſure of the external air, and deſcend like a ſhower.

6. The mercury riſes in the gage as the receiver is exhauſted, and the mercury in a barometer placed in an exhauſted receiver, deſcends to the level of that in the baſon, and the air being readmitted, the mercury deſcends in the gage and riſes in the barometer to its former altitude.

7. Let the open orifice of a barometer tube, filled with mercury, be inſerted into a baſon of mercury, and that in the tube will deſcend till it be about 30 inches, and there remain quieſcent. Suſpend this tube from the beam of a balance, and the weight neceſſary to counterpoiſe the air incumbent upon the top of the tube, is exactly equal to the weight of the tube and mercury included. If the upper orifice be opened, the mercury will deſcend to the level of that in the veſſel, and the counterpoiſe in the balance be equal to the weight of the tube.

8. If a piſton exactly filling a hollow cylindrical tube, inſerted in a veſſel of water, be drawn upwards, the water will aſcend to an altitude of about 34 feet and there remain ſuſpended; and, a ſmall orifice being made between the altitude and the piſton to admit the air, the water will deſcend to the level of that in the veſſel.

9. A tube *AB*, whoſe orifice *A* is covered with a wet bladder

and put into a baſon of mercury, is inſerted into a ciſtern *VGHI*, whoſe bottom is covered by a hard cement *GH*, into which alſo another tube *TC* is inſerted, and the inſertions at *I* and *K* are ſtopped to prevent the admiſſion of air. Let the ciſtern *VGHI*, and tubes *BA*, and *CT*, be filled with mercury, and the orifice *T* be covered with bladder; and, the orifice at *A* being opened, the mercury will deſcend through *TC*, and then through *BL*, till *AL* be about 30 inches, when it will remain ſuſpended. If air be admitted into the upper part of the ciſtern *VGHI*, the mercury in *BA* will deſcend to the level of that in the ciſtern, and that in *TC* will aſcend to an altitude, *CD*, preciſely equal to *AL* *. Q.E.D. PLATE I. FIG. I.

9. PROP. *The weights of equal magnitudes of rain-water and air are to each other as* 850 : 1 *nearly.*

EXP. 1. Let a cork, with a hole perforated in it lengthways, be ſqueezed into a glaſs bottle, whoſe bottom is covered with water; and let a tube, open at both ends, paſs through the hole in the cork into the water, and air be forced through the tube, till the air in the bottle be denſer than in its natural ſtate, and, by its increaſed elaſticity, make the water aſcend in the tube: then placing the bottle in a veſſel, and covering it with ſand or ſalt, that the included air may remain of the ſame temperature, and obſerving the altitude of the water in the tube at the ſurface of the earth, and at an altitude above it of 54 feet, the water is found to riſe about $\frac{3}{4}$ of an inch, and conſequently, the weight of a column of air whoſe length is 54 feet, is equal to that of water whoſe length is $\frac{3}{4}$ of an inch, or $\frac{1}{16}$ of a foot; and the weights of equal columns of water and air, are to each other as 864 : 1.

2. Let a large glaſs bottle be exhauſted by an air-pump; or, a little water being put into it, let a lamp be holden under it till it boil,

* More experiments are frequently deſcribed than are neceſſary to eſtabliſh the truth of a propoſition, merely for the ſelection of the experimental philoſopher.

boil, and the air be expelled by the vapour; and the mouth being ſtopped by a cork, weigh the bottle thus exhauſted, and when the air is readmitted, and the weight of a given portion of air may, by this proceſs, be actually weighed. If the bottle be capable of containing two pounds of rain-water, the weight of the air, contained in it, will be little more than 142 grains, which is to two pounds as 1 to a number a little above 800. Q. E. D.

SCHOLIUM.

10. Not only air, water, mercury, but all ſubtle effluvia acceſſible to experiments, are found to be heavy; but whether fire, light, electric and magnetic effluvia are ſubject to the ſame influence, can only be inferred from analogy. Any aſſignable portions of theſe fluids, could they be included and confined in any veſſel, and ſubmitted to the niceſt and moſt ſenſible modes of eſtimating the weights of bodies, would, from the extreme minuteneſs of their parts, bear no aſſignable ratio to a finite magnitude.

11. PROP. *Fluids exert a preſſure upwards, and laterally in every direction, which, at the ſame depth, is equal to the preſſure perpendicular to the horizon.*

EXP. 1. If a glaſs veſſel be filled with water, and its orifice be cloſely covered with a braſs plate, the upward preſſure of the air will keep the water from running out, when the glaſs is inverted.

2. If ſmall holes be perforated in the bottom, or ſide, of a veſſel, whoſe top is cloſely covered, the upward, or lateral preſſure of the air will prevent the deſcent of the water: but if a ſmall hole be made in the top, the water will then run out.

3. Two hollow braſs hemiſpheres kept contiguous will adhere together, when the air is exhauſted from the cavity, with a force, which requires a weight of about 15 pound, for every ſquare inch of

of a great circle of the biſected ſphere, to ſeparate them. If the exhauſted hemiſpheres be ſuſpended under the receiver of an air-pump, upon exhauſting the air the lower hemiſphere will fall to the bottom of the receiver.

4. A ſyringe being faſtened to a plate of lead, and the piſton being drawn upwards from the lead, the air will make the lead aſcend and drive it againſt the piſton. The piſton being hung in an exhauſted receiver, the lead and ſyringe will deſcend, and upon the readmiſſion of air, be impelled upwards againſt the piſton.

5. Soft bodies ſuſtain the preſſure of the air without any change of figure, brittle bodies without being broken, and a man does not feel the preſſure upon any part of his body, though the preſſure, which they ſupport, be equal to a column of mercury 30 inches high, or a column of water 34 feet. This preſervation of figure in the ſoft and brittle ſubſtances, and preſſure not perceived, are owing to the equality of preſſure on every ſide; for, if the preſſure on one ſide be removed or diminiſhed, the effect of preſſure upon the other is immediately perceived by a change of figure.

6. A ſmall column of mercury included in a tube, open at both ends, which is immerſed perpendicularly in water to a depth of a little more than 14 times the length of the column, will be ſupported by the preſſure of the water upwards: or, if a circular braſs plate be cloſely fitted to the orifice of an open cylindrical tube, by a cord or wire fixed to the middle of it, and the tube and plate be immerſed in water to a depth equal to 8 times the thickneſs of the plate, it will be ſupported by the preſſure upwards.

7. Let the lower extremities of a number of bent tubes *A*,*B*,*C*,*D*,*E* be inclined, in any different angles, to their longer legs, and ſtopping the higher orifices immerſe them in water to the ſame perpendicular depths; upon opening the upper orifices, the water will riſe in all to the level of the external water. A globule of mercury is equally preſſed upwards, at equal perpendicular depths, in all theſe tubes. FIG. II.

8. The ſurface of paper or thin glaſs, or any thin ſoft ſubſtance immerſed to any depth in water or mercury, being placed parallel,

or inclined in any angle, to the ſurface of the fluid, is not broken or bent by the incumbent preſſure, though equal to many pounds, which muſt therefore be ſupported by an oppoſite and equal preſſure. Q. E. D.

SCHOLIUM.

12. It abundantly appears, by theſe experiments, that the perpendicular preſſure of any column of air or water is, from ſome unknown connexion of the parts, diffuſed laterally in every direction, and at the ſame depth, the preſſures eſtimated in any direction, whether perpendicular or inclined in any different angles, to the horizon, are equal to each other. What has been proved of air, water, mercury, muſt obtain alſo in all other ſubſtances, that are fluid, and under the influence of gravity.

CHAP. III.

OF THE ELASTICITY OF FLUIDS.

13. DEF. *AN elaſtic fluid is that, whoſe dimenſions are encreaſed by diminiſhing, and diminiſhed by encreaſing, the preſſure*.*

14. PROP. *Air is elaſtic.*

EXP. 1. A full blown bladder will yield to a preſſure, and recover its figure when that preſſure is removed; and a bladder not full blown, when put into an exhauſted receiver, will be diſtended and appear to be full.

2. A bladder not quite full blown, being put into a receiver with a ſmall weight hanging from it which juſt touches the bottom, will elevate the weight by the expanſion of the air, when the receiver is exhauſted.

3. If the open end of a tube, whoſe other orifice is cloſed, be immerſed perpendicularly in water, the ſpace occupied by the air will be diminiſhed as the depth of the tube, or preſſure upwards, is encreaſed; or, if the ſhorter leg of a bent tube be cloſed, and mercury be poured into the longer, the air will be compreſſed in the ſhorter into a ſpace continually decreaſing, as the quantity of mercury or preſſure is encreaſed.

4. Warm water, or newly fermented liquors, placed under a receiver, will appear to boil, when the air is exhauſted.

5. If one of two communicating receivers be exhauſted whilſt the communication is ſtopped by a cock, the air will ruſh violently out

* Newt. Princip. Lib. II. Sect. 5. Muſſ. Vol. II. Ch. 39. Graves, L. II. Ch. 12. Cotes' Hydroſtat. p. 71. Helſh. Lect. XV.

out of the unexhauſted into the exhauſted receiver as ſoon as the cock is turned.

SCHOLIUM.

15. The limits of the condenſation and rarefaction of air, by human powers, cannot be aſcertained. The degrees of condenſation produced by Boyle, Halley and Hales, are reſpectively deſcribed by them to be 30, 60 and 38 times greater than the denſity of atmoſpheric air; but Dr. Hales condenſed air into 1551 times leſs ſpace than it naturally occupies, by water forced into an iron globe, and frozen‡ But there are, in nature, limits which cannot be tranſgreſſed. Air has never been diſcovered in ſuch a ſtate of condenſation, as to ſuffer a penetration of its parts and an aggregate portion of it become one continued ſubſtance. Muſſchenbroek rarefied air till it occupied a ſpace 4000 times leſs than in its natural ſtate *. Mr. Boyle obſerves, that air may be ſo dilated as to become 10,000 times rarer than in its natural ſtate; and in aſcending the atmoſphere it becomes continually rarer, but the rarefaction muſt ceaſe when the expanſive force of the particles is equal to their gravitation. A particle of the elaſtic fluid, ariſing from warm water, expanded itſelf into a ſpace 466560000000 times greater than itſelf †; and poſſibly the fluid, known by the name of air, may contain particles of different magnitudes and degrees of elaſticity, and many other properties yet unknown.

16. PROP. *The elaſtic force of air is equal to the force of compreſſion.*

EXP. If the air be exhauſted from an open tube, one of whoſe orifices is immerſed in mercury, contained in a veſſel ſo cloſed as to exclude the external air; the internal air in the veſſel will expand itſelf, and preſſing upon the ſurface of the mercury, make it riſe in the tube to the ſtandard altitude nearly. Q. E. D.

SCHO-

‡ Hæmaſtat. Appen. p. 348, &c. * Muſſchenbroek, Vol. II. p. 886. † Id.

SCHOLIUM.

17. When any portion of air, or of any other elaſtic fluid, is perfectly quieſcent, the reſiſtance and compreſſing force are evidently equal to each other; and preſuming that the particles do actually expand themſelves with forces equal to this reſiſtance, their elaſticity is, by many philoſophical writers, concluded to be equal to the force of compreſſion. But this concluſion appears to me to be derived ſatisfactorily, only from the above, and ſimilar experiments.

18. PROP. *The elaſticity of air, not very different from its natural ſtate, is inverſely as the ſpace occupied by it* *.

EXP. 1. A cylindrical tube AB, open at one end A, is filled with mercury to an altitude equal to AC before inverſion, and, after the immerſion of A in a baſon of mercury, the air, which before occupied a ſpace equal to BC, is dilated through BD, and, if AH be the ſtandard altitude, depreſſes the mercury from H to D. The elaſtic force of air in its natural ſtate, or occupying a ſpace equal to BC (E), is to the elaſtic force of air occupying the ſpace BD (e) as the columns of mercury which they are capable of ſupporting, or as $AH : DH$; but it is collected from experiments, that $AH : DH :: DB : BC$, therefore $DB : BC :: E : e$. FIG. III.

2. Let the bottom CD of the tube $ACDB$ be juſt covered with mercury, and the orifice B being ſtopped, and more mercury poured into the leg AC, to any altitude CL, the air which, in its natural ſtate, occupied the ſpace BD, will be compreſſed into the ſpace EB by the weight of the column of mercury LF, ſuppoſing F to be upon a level with E. The elaſticity of air in its natural ſtate, or occupying the ſpace BD (e), is to that of air occupying the ſpace BE (E) :: ſtandard altitude : ſtandard altitude $+ LF$ (16); but it appears from experiments, that $BD : BE$ :: ſtand. alt. $+ FL$: ſtand. alt.; and conſequently $e : E :: BE : BD$. Q. E. D. FIG. IV.

19. Cor.

* Muſſ. Vol. II. p. 858.

FIG. IV. 19. Cor. 1. Let h be the ſtandard altitude at the time of obſervation; and $BD : BE :: h + LF : h$, and, dividendo, $BE : DE :: h : LF$. If $h = 29$ inches, and $BE = 6, 5, 4$, &c. inches, and $BD = 7$ inches, LF is found from experiments to agree with the value aſcribed to it by the above analogy,

$$BE : DE :: h : LF = \frac{h \times DE}{BE}, \text{ or from experiments}$$

$6 : 1 :: 29 : \frac{29}{6} = 4\frac{5}{6}$ inches
$5 : 2 :: 29 : \frac{58}{5} = 11\frac{3}{5}$
$4 : 3 :: 29 : \frac{87}{4} = 21\frac{3}{4}$
$3\frac{1}{2} : 3\frac{1}{2} :: 29 : 29 = 29.$

20. Cor. 2. The denſity of the air being inverſely as the magnitude (Mechan. art. 87.) or ſpace occupied, is therefore as the elaſticity, or as the compreſſing force (16); or, putting D, C, E, S for the denſity, compreſſing force, elaſticity and ſpace reſpectively, C is as $\frac{1}{S}$, or as D, or as E.

SCHOLIUM.

21. When the air is compreſſed into a ſpace four times leſs than the ſpace occupied by it in its natural ſtate, or when BE is equal to about $\frac{1}{4}$th of BD, it does not vary inverſely as the force of compreſſion, the reſiſtance increaſing then much faſter*. The law muſt neceſſarily be limited and cannot always obtain, for when the particles are in actual contact, the maſs muſt then become abſolutely incompreſſible.

22. PROP. *The elaſticity of air is encreaſed by heat.*

EXP.

* Comment. Bonon. Vol. I. pag. 209.

*EXP. 1. Air, included in a thin glaſs hermetically ſealed and expoſed to a hot fire, will expand as it becomes warm, and at length break the glaſs. If air be included in a flaccid bladder expoſed to a fire, the bladder will be inflated by the rarefaction of the air and burſt.

2. Air included in any veſſel, expoſed to the heat of boiling water, is ſo much expanded, that the encreaſe of elaſticity is to the elaſticity of air in its natural ſtate, as 10:33, and ſometimes as 10: 35. Let *ABGDF* be a narrow tube communicating with a large bulb, whoſe content is very great compared with the tube; pour in mercury to any altitude, *DELG*, and, cloſing up the orifice at *F*, the elaſticity of the included air acting upon the ſurface *DE* is in equilibrio with the preſſure of the atmoſphere upon *LG*. Immerſe the bulb *DFE* in boiling water, and the dimenſions of the encloſed air will be encreaſed and elevate the mercury to an altitude, *LH*, of about 8.286 inches, when the ſtandard altitude is 29 inches †. FIG. V.

3. Let a globule of mercury *C* be put into a tube *AB*, cloſed at *B* and open at *A*, and prevent the air between *C* and *B* from eſcaping: immerſe the tube in boiling water, and the ſpaces occupied by the air then, and when freezing cold, are diſcovered to be to each other as 3 : 2. By methods ſimilar to this it is found, that the expanſion of air in the heat of ſummer, and at the point of freezing, are to each other as 7 : 6 ‡ in England. At Peterſburgh, the expanſion of air expoſed to boiling water, the extreme heat of ſummer and cold of winter, were obſerved to be as 6, 4, 3 reſpectively §. FIG. VI.

SCHOLIUM.

23. This expanſion of air, though expoſed to the ſame degree of heat, is not the ſame in experiments made at different times, owing to the difference of denſity, coldneſs, humidity, &c. If air be tolerably pure and be condenſed, its encreaſe of expanſion, when

* Amonton. l' Hiſtoire de l'Acad. Roy. 1708.
† Muſſ. Vol. II. pag. 883.
‡ Hawkſbee Phyſico Mechan. Exp. pag. 170.
§ Bernoullii Hydrodynamic. Sect. 16.

when expoſed to boiling water, is proportionable to its denſity; but if the air be moiſt, the encreaſe of expanſion does not obſerve this law *. The expanſive force of hot ſteam, may exceed the force of gunpowder more than 30 times, and indeed is irreſiſtible when the heat is intenſe †; and air therefore impregnated with water, may poſſeſs an expanſive power by heat, much greater than that of pure air. Whether the degree of expanſion in pure air be proportionable to that of heat, by which it is produced, is not known; but it is obvious, that the variation of ſpace, occupied by a portion of air expoſed to different degrees of heat, may be ſufficient to convey uſeful though imperfect ideas of the real quantity of heat. And upon this principle air thermometers are conſtructed, which are chiefly uſed to ſhew ſmall variations of heat.

24. PROP. *Water, mercury, &c. and gold, ſilver, braſs and other metals, are expanded by heat.*

EXP. 1. If water, mercury, or any fluid excepting air, be contained in a veſſel placed over a fire, they ſoon become as hot as they can be in the open air, are agitated, and emit bubbles or boil, and their dimenſions are found to be gradually augmented. From the point of freezing to that of ebullition, the encreaſe of magnitude of alcohol is $\frac{1}{9}$, of water $\frac{1}{85}$, of mercury $\frac{4}{51}$, part of the whole magnitude. If the following fluids, at the point of freezing, be placed in boiling water, their dimenſions are thus magnified, viz. mercury $\frac{14}{1000}$; water $\frac{37}{1000}$; ſpirit of wine $\frac{87}{1000}$; linſeed oil $\frac{75}{1000}$ parts of their whole bulk ‡.

2. If the heat be encreaſed, the contiguity of the parts is farther diminiſhed, and the fluid is diſſipated in elaſtic vapours.

3. A bar of iron, braſs, &c. is ſenſibly longer when hot than cold; and a cylindrical piece of iron, braſs, or other metal, when heated, cannot paſs into a ring though equal to their circumference when cold.

4. The

* Muſſ. Vol. II. p. 579. † Phil. Tranſ. No 454. pag. 162.
‡ Muſſ. Vol. II. pag. 625. Nollet's Leçon's Phyſ. Tom. IV. pag. 397.

4. The ratio of expanſion of metallic rods of the ſame diameter, placed in boiling water, are deſcribed to be, in braſs 94, in orichalcum 106, iron 73, lead 154, ſilver 81 †.

SCHOLIUM.

25. It does not appear that the expanſion of metals, expoſed to the ſame heat, follows any inverſe ratio of their weight, or hardneſs, or any ratio compounded of theſe. And it ſeldom happens that the iſſues of ſimilar experiments agree; becauſe the metals, though known by the ſame names, may not have the ſame qualities, not being equally pure from foreign mixtures; and this with different ſtates of the atmoſphere when the experiments are made, and other cauſes, may operate to the production of very different reſults. The expanſion of water, mercury, or of metal bars, may be applied as thermometers, for the menſuration of heat, in the ſame manner with air, and with more certainty, becauſe more homogeneous. It does not appear from this dilatation of fluids by heat that they are elaſtic.

26. PROP. *Water, mercury, &c. are compreſſible.*

EXP. 1. Take a glaſs tube, whoſe length is about 24 inches, with a glaſs ball at the end of $1\frac{1}{4}$ inch in diameter. Fill the ball and part of the tube with mercury, water, ſpirit of wine, &c. exhauſted of air, and heat it till the fluid riſe to the top of the tube, and ſeal it hermetically. When the tube and fluid are cool, obſerve the height of the fluid, and, breaking the top of the tube to admit the air, the fluid is obſerved to be depreſſed very ſenſibly.

2. In the laſt experiment, obſerve the height of the mercury or water, when heated to the temperature of 50° by Fahrenheit's thermometer; heat the fluid till it riſe to the top of the tube, which muſt then be hermetically ſealed to prevent the readmiſſion of the air; and when the fluid is brought to the ſame degree of heat

† Muſſ. Vol. II. pag. 612.

heat as before, its altitude above the ball is found to be greater. In mercury and water, the exceſs of altitude is reſpectively $\frac{31}{100}$ and $\frac{43}{100}$ parts of an inch. Q. E. D.

SCHOLIUM.

27. If the ratio of the tube to the ball be known, the quantity of compreſſion correſponding to any given weight may be known. Thus Mr. Canton filled the ball and part of the tube with water exhauſted of air, and, leaving the tube open, placed it under the receiver of an air-pump, and obſerved the expanſion of the water correſponding with any degree of rarefaction of the air; and then by a condenſer, he obſerved the quantity of compreſſion anſwering to any degree of condenſation. After repeated experiments, in a temperature of 50°, and the mercury ſtanding in the barometer at a mean altitude, the compreſſion of water by an additional atmoſphere, was diſcovered to be one part in 10870 of its whole bulk. By this proceſs, the compreſſions of the following fluids are found, when the thermometer is 50° and the barometer at $29\frac{1}{2}$.

Compreſſion of ſpirit of wine	66	mill. parts.
oil of olives	48	
rain-water	46	
ſea-water	40	
mercury	3.	

As theſe fluids expand when the preſſure is removed, and are contracted when it is incumbent upon them, they are elaſtic *.

* Philoſ. Tranſ. Vol. LII. 1762, Art. 103. Vol. LIV. 1764, Art. 475.

CHAP.

CHAP. IV.

OF THE RESISTANCE OF FLUIDS.

*28. THE reſiſtance of fluids is the force with which the motion of bodies, in fluid mediums, is retarded, and is generally meaſured by the quantity of motion deſtroyed in the moving body in a given time. In the impact of hard or ſoft bodies of ſenſible magnitudes, the whole motion loſt by the impinging body, is communicated to the body ſtricken, ſuppoſing the motion to be eſtimated by the quantity of matter multiplied into the velocity; and the ſame law would certainly obtain in the communication of motion to the particles of fluids, were the circumſtances ſimilar in every reſpect. If the particles of fluids were therefore perfectly diſunited, the reſiſtance would be meaſured, either by the quantity of motion loſt by the moving body, or that communicated to the fluid; but the conſtitution of all fluids is ſo much unknown, that this law ought not to be conſidered as a general principle, nor applied to ſynthetic reaſoning, without an appeal to experiments upon each particular fluid. The ſmall corpuſcles, of which ſome fluids are compoſed, though poſſeſſing the ſame inertia and mechanical affections with other bodies, are impreſſed with powers of acting upon each other, though not in actual contact; which are different in different fluids, and imperceptible in each till excited to action by the impulſe of ſome external force that changes their natural diſtances; and then motion communicated to one particle, will, by this connexion, be diffuſed laterally, in different directions, according to its influence upon the corpuſcles lying within the ſphere of its action. And beſides

* Robins' Tracts, Vol. I. p. 125. Newt. Princip. Vol. II. Sect. 6. Herman. Phoron. p. 336.

beſides this, the reſiſtance to a body moving in a fluid is dependant upon ſome other circumſtances not obtaining in the impact of bodies of a ſenſible bulk, and ariſes frequently from the inertia of the parts combined with the weight of the incumbent fluid, and will be different at different depths. Reſiſtance, as far as we have occaſion to conſider it, ariſes from the inertia and the tenacity, or adheſion of the conſtituent particles, and is much affected by the compreſſion of the fluid in which a body moves; and as the quantity of motion communicated to the fluid, and the force of coheſion, are not diſcoverable, it is generally meaſured by the quantity of motion loſt by the moving body.

29. I. When the foremoſt ſurface of a body, moving in a fluid, is perpendicular to the direction of its motion, or when the particles of a fluid are impelled by any ſurface, or impinge againſt it quieſcent, perpendicularly; let $\mathcal{Q}$ be the quantity of matter, or number of equal particles of the fluid, to which motion is communicated, V the velocity, and $\mathcal{Q} \times V$ is the quantity of motion loſt by the body and acquired by the fluid, which meaſures the reſiſtance (R) ariſing from inertia, in a given time. But $\mathcal{Q}$ evidently varies as $V \times D$ (ſuppoſing D to be the denſity) and conſequently R varies as $V^2 \times D$. If the firſt ſurface be oblique to the direction of motion, or the fluid be impelled obliquely, the whole force of the moving body is not employed in communicating motion, in the direction in which it moves, nor is the reſiſtance meaſured by the whole motion communicated, becauſe part of it is in the direction of a tangent to the ſurface at the point of impulſe, and occaſions no reſiſtance to the motion of the body; and the obliquity of the ſurface may conſequently be ſuch that the reſiſtance, ariſing from inertia only, will be evaneſcent. The concluſion is clearly the ſame whether a body move in a fluid, or the fluid impel the body, in an oppoſite direction with the ſame velocity.

30. II. In

30. II. In fluids uniformly tenacious, a determinate quantity of force is required to deſtroy the coheſion and ſeparate the parts, whatever be the velocity of the moving body; and conſequently the quantity of motion loſt, or the reſiſtance, is as the time, or conſtant. In a fluid therefore poſſeſſing any degree of tenacity, the whole reſiſtance ariſes from this tenacity and the inertia of the parts combined together, and is partly conſtant, and partly varies as the ſquare of the velocity. In air, water, and all other fluids that are not glutinous, the reſiſtance ariſing from tenacity is inconſiderable compared with that ariſing from inertia.

31. III. If a fluid be uncompreſſed, or the effect of compreſſion be ſo ſmall, compared with the velocity of the body, that the ſpaces deſerted by it, remain for ſome time empty; the reſiſtance upon the foremoſt ſurface will be encreaſed. Let a body P deſcend in water or air, with a velocity ſo great, that the compreſſing force does not impel the fluid into the ſpace immediately deſerted, before the body changes its place again; and the preſſure upwards, and conſequently reſiſtance, upon any point n of the firſt ſurface, beſides the inertia of the parts, is equal to the weight of a column of the fluid, whoſe depth is equal to that of n; for there is no weight upon the oppoſite point r to balance it, the preſſure upon the ſurface mPq being entirely removed. The reſiſtance is alſo encreaſed by an accumulation of the fluid upon the ſurface mnq; and ſtill more ſo if the fluid be elaſtic, becauſe the repulſive force of the particles is encreaſed by their nearer approach. The velocity of air ruſhing into empty ſpace, is about 1296 feet in 1″; and if the velocity of a body in air be greater than this, the ſpace laſt deſerted by it muſt for ſome time be without air, and the reſiſtance upon any point, in the foremoſt ſurface, conſequently be encreaſed by the incumbent atmoſphere, or the weight of a column of mercury whoſe height is about 30 inches. The diameter of a ball of 24 pounds, is 5.457 inches, and the area of a great circle of the ball is 23.385 inches; and FIG. VII.

allow-

allowing 15 pounds to be the preſſure upon one ſquare inch, the reſiſtance ariſing from the incumbent weight is 350.78 pounds. The encreaſe of reſiſtance depends upon the degree of compreſſion produced by the body, and encreaſes ſo faſt (21) as to render all attempts to augment the velocity of military projectiles, beyond a certain point, very inconvenient, or perhaps impracticable. The velocity may be ſo great as to produce a degree of compreſſion that deſtroys the body's motion, and it will then be reflected in an oppoſite direction. Thus ſmall ſhot are often ineffectual, or repelled, when fired with very great charges of powder.

32. IV. If the fluid be ſo much compreſſed, that the velocity generated by compreſſion is much greater than the velocity of the body, the fluid will be impelled, almoſt inſtantaneouſly, into the places deſerted by P: ſo that the preſſure, upon any two oppoſite points r and n, will be nearly in equilibrio; for it is evident, that the fluid is always accumulated upon the foremoſt ſurface of a body moving in it; which, from the compreſſion, will move towards r, where the fluid is rarer. And hence, the particles contiguous to the firſt ſurface, will circulate round the body with a retrograde motion; and for this reaſon, the quantity of fluid protruded, and reſiſtance ariſing from it, is leſs in this medium than in one where the parts are perfectly diſunited, the reſiſtance of a cylinder, moving in the direction of its axis, being only $\frac{1}{4}$th, and that of a ſphere $\frac{1}{2}$ of the reſiſtance ariſing from inertia. The quantity of reſiſtance in a medium much compreſſed, depends upon the greater or leſs facility with which the particles of fluid impelled by the firſt ſurface, can circulate backwards; and it is the ſame conſequently when the traverſe ſection is the ſame. Air is compreſſed by the incumbent weight of the atmoſphere, and, any point of a body immerſed in water, by the weight of the incumbent column; and theſe compreſſing forces will equally urge the firſt and laſt ſurfaces of bodies moving with velocities not very great; but the laws of reſiſtance are only diſcoverable from experiments;

riments; and from them it appears to be an eſtabliſhed principle, that, in motions within certain limits, the reſiſtances of air and water vary nearly as the ſquares of the velocities of the moving body.

33. LEMMA, *If a pendulous body,* P, *vibrate about* C *in a curve line* AVB, *in a reſiſting medium, by the action of a conſtant force,* G, *parallel and equal to* CV, *and deſcending from* P *aſcend to* F, *and* PVF *be ſtretched out into a right line, to which perpendiculars,* R, *are erected proportional to the reſiſtance in the correſponding points of the curve, the rectangle under* CV, *and the difference of the abſciſſes, or* CV × IK, *is equal to the area deſcribed by* R. FIG. IX.

Let $\mathcal{Q}m = G$ be reſolved into two forces, qm perpendicular, and $\mathcal{Q}q$ parallel to the tangent at $\mathcal{Q}$; and from ſimilar triangles, $\mathcal{Q}r : rn\,(ON) :: \mathcal{Q}m : \mathcal{Q}q :: G$: that part of G which accelerates the body, and which is equal to $\frac{G \times ON}{\mathcal{Q}r}$. The whole accelerating force $(A) = \frac{G \times ON}{\mathcal{Q}r} - R$, and taking $Z' = \mathcal{Q}r$, $A \times Z' = G \times ON - R \times Z' = v\dot{v}$ (ſuppoſing v to be the velocity at $\mathcal{Q}$); and taking the fluents, and ſuppoſing P to be at F, or $v = o$, $G \times IK =$ area PFv deſcribed by R. Q.E.D.

34. Cor. If AVB be a cycloid, and $G = CV = 2VD$; then the area PFv, deſcribed by R, $= 2VD \times \overline{VI - VK} = 2VG^2 - 2VH^2$ (ſim. triangles) $= \frac{1}{2}VP^2 - \frac{1}{2}VM^2 = \frac{1}{2} \times \overline{VP + VM} \times \overline{VP - VM} = \frac{1}{2}PVF \times FX$, the defect from a vibration in vacuo. This is Newton's expreſſion, Vol. II. Sect. 6. Prop. 30.

FIG. X. 35. LEMMA. *If* pf *be the arc of another vibration ſtretched out into a right line, and* xf *be the defect from that in vacuo, and* PF, pf *be divided into very ſmall correſponding parts* Z′ *and* z′, *which are to each other in the ſame ratio of* PF : pf, *and the ratio of* R : r *be given, then the defects* XF *and* xf *will be to each other in that ratio.*

The times of deſcribing *PF* and *pf*, and any correſponding parts *Z′* and *z′* are equal, and conſequently $V : v :: Z' : z'$, or as $PF : pf$. Conceive *XF* and *xf*, the whole defects, to be divided into the ſame number of parts, each of which is occaſioned by the reſiſtances at correſponding points, or *R* and *r*, and whoſe ratio is that of the decrements of the velocity, and, componendo, $XF : xf$ in the ſame ratio. But (33) $XF \times PF : xf \times pf ::$ ſum of all the $R \times Z' :$ ſum of the $r \times z' :: PF \times R : pf \times r$, becauſe $Z' : z' :: PF : pf$, and $R : r ::$ whole reſiſtance ariſing from *R* : whole reſiſtance from *r*. Q. E. D.

36. Cor. Since the defects *XF*, *xf* are as the reſiſtances at any correſponding points; if theſe be the middle points of the vibrations and the reſiſtances be as V, V^2, V^3, &c. ſuppoſing *V* to be the velocity at *V*, the defect *XF* will be as V, V^2, V^3, &c. or as PF, PF^2, PF^3, &c.

37. PROP. *The reſiſtance of air to ſpherical bodies moving in it, with velocities contained between certain limits, varies as the ſquare of the velocity.*

1. Let a ſpherical piece of wood, whoſe weight is $57\frac{7}{25}$ oz. and diameter $6\frac{7}{8}$ inches, be ſuſpended by a ſlender thread, whoſe length between the point of ſuſpenſion and center of oſcillation is $10\frac{1}{2}$ feet, perform different vibrations; and it appears from experiments that,

if

if the arc described in the first descent be	2,	4,	8,	16,	32,	64,	inches.
the arc of last ascent is	$1\frac{1}{4}$,	$3\frac{1}{2}$,	7,	14,	28,	56,	inches.
their differences are - -	$\frac{1}{4}$,	$\frac{1}{2}$,	1,	2,	4,	8.	
Numbers of oscillations are	164,	121,	69,	$35\frac{1}{2}$,	$18\frac{1}{2}$,	$9\frac{2}{3}$.	

Divide the differences by the number of oscillations, and the quotients $\frac{1}{656}$, $\frac{1}{242}$, $\frac{1}{69}$, $\frac{4}{71}$, $\frac{8}{37}$, $\frac{24}{29}$, will be the arcs described in one descent, and the subsequent ascent in a mean vibration *. But if the resistance be as the square of the velocity, these last differences will be as the squares of the whole arcs, and their square roots as the arcs themselves. The ratio of the arcs is that of 1 : 2, and the square roots of the differences $\frac{1}{656}$, $\frac{1}{242}$, $\frac{1}{69}$, $\frac{4}{71}$, $\frac{8}{37}$, $\frac{24}{29}$, or of numbers in the same ratio 1, 2.71, 9.5, 36.95, 141.8, 542.7, are nearly equal to 1, 1.6, 3, 6, 11.9, 23.3, whose ratio is nearly that of 1 : 2.

2. Let a spherical piece of wood vibrate till $\frac{1}{4}$th of its motion be destroyed by the resistance of the air, and the motion be reduced to that in a cycloid; and the velocities at the middle point of the vibrations are as the arcs, or as 1 : 2, and the resistances at those points as the differences of these arcs, whose square roots are found to be nearly as 1 : 2.

Arcs described in the first des.	2,	4,	8,	16,	32,	64,	in.
Ibid. in the last ascent	$1\frac{1}{2}$,	3,	6,	12,	24,	48.	
Number of vibrations	374,	272,	$162\frac{1}{2}$,	$83\frac{1}{3}$,	$41\frac{2}{3}$,	$22\frac{2}{3}$.	
Differences of the arcs	$\frac{1}{2}$,	1,	2,	4,	8,	16.	
Differences of the arcs described in one descent, and the subsequent ascent of a mean vibration, proportionable to the resistance.	$\frac{\frac{1}{2}}{374}$,	$\frac{1}{272}$,	$\frac{2}{162\frac{1}{2}}$,	$\frac{4}{83\frac{1}{3}}$,	$\frac{8}{41\frac{2}{3}}$,	$\frac{16}{22\frac{2}{3}}$.	

The

* The mean oscillation is an arithmetic mean between the greatest and least oscillation. In Exp. 1. the arc described in the first vibration is almost 4 inches, the arc described in the last is $3\frac{1}{2}$, and an arithmetic mean between them is $3\frac{3}{4}$ inches. The other mean arcs are $7\frac{1}{2}$, 15, 30, 60, 120; and the ratio of their length is that of 1 : 2.

The ſquare roots of theſe differences, or of numbers in the ſame ratio, viz. 1, 2.75, 9.21, 35.9, 143.6, 528, are 1, 1.65, 3, 5.99, 12, 23, and are nearly as 1 : 2.

3. If a ſphere of lead, whoſe diameter is 2 inches, and weight 26¼ oz. be ſuſpended by a ſlender thread whoſe length between the point of ſuſpenſion and the center of oſcillation is 18½ feet, vibrate as before, till ⅛th of its motion be loſt, the reſult is as follows:

Arcs deſcribed in the firſt deſ.	1,	2,	4,	8,	16,	32,	64, in.
Ibid. in the laſt aſcent	$\frac{7}{8}$,	$\frac{7}{4}$,	$3\frac{1}{2}$,	7,	14,	28,	56, in.
Numbers of vibrations	226,	228,	193,	140,	$90\frac{1}{2}$,	53,	30.
Diff. of the arcs deſcribed in one deſcent, and the ſubſequent aſcent	$\frac{1}{1808}$,	$\frac{1}{912}$,	$\frac{1}{380}$,	$\frac{1}{140}$,	$\frac{4}{181}$,	$\frac{4}{53}$,	$\frac{4}{15}$,

Theſe differences are nearly as 1 : 4, and their ſquare roots, or of the following numbers in the ſame ratio, as 1 : 2. Numbers in the ſame ratio with the differences are

1, 1.98, 4.86, 12.91, 39.96, 136.4, 482.1, whoſe ſquare roots are 1, 1.4, 2.2, 3.59, 6.32, 11.67, 21.95, which in the greater oſcillations are nearly as 1 : 2.

4. Let the ſame ſphere vibrate till it loſe ¼th of its motion, and

Arcs deſcribed in the firſt deſcent are	1,	2,	4,	8,	16,	32,	64, in.
Ibid. in the laſt aſcent are	¾,	$1\frac{1}{2}$,	3,	6,	12,	24,	48, in.
Numbers of vibrations are	510,	518,	420,	318,	204,	121,	70.
Diff. of the arcs deſcribed in one deſcent and the ſubſequent aſcent	$\frac{1}{2040}$,	$\frac{1}{1036}$,	$\frac{1}{420}$,	$\frac{1}{159}$,	$\frac{1}{51}$,	$\frac{8}{121}$,	$\frac{8}{35}$.

Numbers in the ſame ratio 1, 1.97, 4.86, 12.83, 40, 134.9, 466.3; whoſe ſquare roots are 1, 1.4, 2.2, 3.58, 6.32, 11.6, 21.6.

5. When a greater ſphere of lead, whoſe diameter was $3\frac{5}{8}$ inches, weight $166\frac{1}{2}$ oz. and diſtance between the point of ſuſpenſion and center of oſcillation 126 inches, vibrated till ¼th of its motion were loſt, the reſult was as follows:

Arcs

Arcs deſcribed in the firſt deſcent	8,	16,	64, inches.
Ibid. in the laſt aſcent	6,	12,	48, inches.
Numbers of vibrations	535,	287,	$85\frac{1}{2}$,
Difference of the arcs deſcribed in one deſcent, and the ſubſequent aſcent	$\frac{2}{535}$,	$\frac{4}{287}$,	$\frac{32}{171}$.
Numbers in the ſame ratio	1,	3.73,	50, whoſe
ſquare roots are nearly as the vel.	1,	2,	8.

The above ſets of experiments prove that in ſome caſes the velocity varies as the ſquare root of the reſiſtance nearly, and the deviation from this law, in other caſes, is probably owing to errors in the experiments.

6. *ABCD* is a cylindrical braſs barrel, whoſe diameter is 2.06 inches. *GH* is an horizontal arm, with a thin feathered edge, inſerted in the cylinder perpendicular to its axis, whoſe length is 49.5 inches. Upon *GH* at its extremity, is fixed a ſpherical ball, *P*, of paſte-board; and the cylinder and arm *GH* are made to revolve about the axis by a weight at *M*, connected with the cylinder by a ſtring tied to it and paſſing over the pulley *N*. FIG. VIII.

To find the reſiſtance of *P*, moving upon the horizontal arm *GH* as deſcribed above, with ſmall velocities, ſuppoſing them and the weight at *M* to be known: let the weight at $M = w$, and the velocity of *P*, after its motion becomes uniform, be *k* turns in *F*″. Subſtitute for *P* a thin plate of lead, equal in weight to *P*, and whoſe reſiſtance is evaneſcent; and let *b* be the weight of a body at *M* that will make *GH* perform *k* revolutions in *F*″, and *b* is evidently the reſiſtance of the arm *GH*, and $w - b$ the weight of a body at *M* equivalent to the reſiſtance at *P*. If *w* be equal to $3\frac{1}{4}$lb. *P* is found to revolve 20 times uniformly in 21.5″; and a plate of lead equal in weight to *P*, being ſubſtituted for it, a weight of 1 lb. at *M* makes *GH* revolve 20 times in leſs than 19″; and conſequently a weight of $2\frac{1}{4}$ lb. is not more than equivalent to the reſiſtance of *P*. Since the radius of the cylinder is nearly $\frac{1}{30}$th of the radius of the circle deſcribed by the center of *P*, the reſiſtance of *P* is, at leaſt, equivalent to $\frac{1}{30}$th of $2\frac{1}{4}$lb.

and

and not leſs than $\frac{1}{2}$oz. avoirdupoiſe, when its velocity is 20 revolutions in 21.5″, or 25 feet in 1″. The reſiſtance of air to *P*, moving with different velocities, being thus eſtimated in the following experiments; the concluſions are found to coincide with thoſe derived from vibrating bodies, and to confirm the truth of the prop.

EXP. 1. If $w = 1$ lb. *P* revolves 10 times in $27\frac{1}{4}''$.
2. If $w = 2$ lb. *P* revolves 20 times in $27\frac{1}{2}''$.
3. If $w = 4\frac{1}{2}$ lb. *P* revolves 30 times in $27\frac{1}{4}''$.
4. If $w = 8$ lb. *P* revolves 40 times in $27\frac{1}{2}''$.

The velocities of *P* are as 1, 2, 3, 4; and the reſiſtances are found, according to the preceding lemma, to be as the ſquares of the velocities, or as 1, 4, 9, 16. Q.E.D.

38. PROP. *The reſiſtance of water to bodies moving in it with velocities contained between certain limits, varies nearly as the ſquare of the velocity.*

1. A ſphere of lead vibrated in water till $\frac{1}{4}$th of its motion was loſt, and the reſult was this:

Arcs deſcribed in deſcending	16,	8,	4,	2,	1,	$\frac{1}{2}$,	$\frac{1}{4}$,	in.
Ibid. in aſcending	12,	6,	3,	$1\frac{1}{2}$,	$\frac{3}{4}$,	$\frac{3}{8}$,	$\frac{3}{16}$,	in.
Numbers of vibrations	$3\frac{3}{8}$,	$6\frac{1}{2}$,	$12\frac{1}{12}$,	$21\frac{1}{3}$,	34,	53,	$62\frac{1}{3}$,	
Diff. of the arcs deſcribed in theſe numbers of vibrations	4,	2,	1,	$\frac{1}{2}$,	$\frac{1}{4}$,	$\frac{1}{8}$,	$\frac{1}{16}$,	
Diff. of the arcs of aſc. and deſc. in the time of one vibrat. and which varies as the reſiſtance	$\frac{32}{27}$,	$\frac{4}{13}$,	$\frac{32}{145}$,	$\frac{5}{242}$,	$\frac{1}{136}$,	$\frac{2}{424}$,	$\frac{5}{4928}$,	

Numbers

Numbers in the same ratio	860.7, 306, 80.98, 23.47, 7.54, 2.3, 1.
whose square roots are nearly as 2 : 1, or as the velocities	29.27, 17.5, 8.99, 4.8, 2.7, 1.5, 1.

* 2. A hollow sphere of brass, whose weight was equal to that of an equal bulk of water, being loaded with different quantities of shot, descended from rest through 60 inches of water; and the results were these, viz.

	Weight included.	Time by exp.	Time by theory.	Diff.
EXP. 1.	11 grains,	14.6,	13.94,	.66,
2.	3 grains,	27.3,	26.66,	.64,
3.	1 grain,	44.33,	46.21,	—1.88.

Q. E. D.

†39. PROP. *The resistance of water and air, to spherical bodies moving in them with velocities contained between certain limits, varies as the square of the velocity, multiplied into the density nearly.*

A globe of wax, including different quantities of lead, descended from rest through 112 inches of water, and the results were these:

Space described by experiment is 112 inches in water.

	Weight of the globe in air and water. Grains.	Time.	Space described by theory. Inches.
EXP. 1.	$156\frac{1}{4}$ in air, 770 in water	4″,	112.08.
2.	76 in air, $5\frac{1}{16}$ in water	15″,	113.174.
3.	121 in air, 1 in water	48″,	112 in 40″.

In the following experiments, which seem to have been made very

* Atwood, on Rect. and Rot. Mot. p. 153. † See Nouvelles Experiences sur la Resistance des Fluides, par M. M. D'Alembert, le Marquis de Condorcet, and l'Abbè Bossut, where these propositions upon resistances are demonstrated very satisfactorily by a great many experiments upon floating bodies.

very accurately, equal balls of wax including, as before, different weights, deſcended through 182 inches of water; and the time was counted by a pendulum vibrating half-ſeconds.

Space deſcribed in water by experiments is 182 inches.

	Weight in air and water. Grains.	Mean number of oſcillations by experiment.	Number of oſcillations by theory.
EXP. 1.	$139\frac{1}{4}$ in air $7\frac{1}{8}$ in water	$49\frac{7}{10}$,	50
2.	$154\frac{3}{8}$ in air $21\frac{1}{2}$ in water	$30\frac{3}{7}$,	29
3.	$212\frac{3}{8}$ in air $79\frac{1}{2}$ in water	$16\frac{3}{10}$,	15
4.	$293\frac{1}{2}$ in air $35\frac{7}{8}$ in water	$31\frac{1}{10}$,	28
5.	139 in air $6\frac{1}{2}$ in water	51,	52
6.	$273\frac{1}{4}$ in air $140\frac{3}{4}$ in water	$12\frac{1}{2}$,	$11\frac{1}{3}$
7.	48 in air $3\frac{29}{32}$ in water	$44\frac{1}{2}$,	$46\frac{5}{9}$
8.	141 in air $4\frac{3}{8}$ in water	63,	$64\frac{1}{2}$.

The two following tables exhibit the reſult of experiments made by Sir I. Newton and Dr. Deſaguliers. In the firſt a ſphere of glaſs deſcends through 220 feet in air; and in the ſecond a ſpherical bladder, filled with air, deſcends in air through 272 feet.

Space deſcribed from reſt in air by experiment is 220 feet.

	Weight of the ſphere. Grains.	Diam. in inches.	Time.	Space by theory. Ft. In.	Difference. Ft. In.
EXP. 1.	510,	5.1,	$8''.12'''$,	226 . 11,	6 . 11
2.	642,	5.2,	7 .42,	230 . 9,	10 . 9
3.	599,	5.1,	7 .42,	227 . 10,	7 . 10
4.	515,	5,	7 .57,	225 . 5,	5 . 5
5.	480,	5,	8 .12,	225 . 5,	5 . 5
6.	641,	5,	7 .42,	230 . 7,	10 . 7

Space

Space deſcribed from reſt, in air, by experiment is 270 feet.

	Weight of the ſphere. Grains.	Diam. in inches.	Time.	Space by theory. Ft. In.	Difference. Ft. In.
Exp. 1.	128,	5.28,	19″,	271 . 11,	—0 . 1
2.	156,	5.19,	17,	272 . 0½,	+0 . 0½
3.	137½,	5. 3,	18½,	272 . 7,	+0 . 7
4.	97½,	5.26,	22,	277 . 4,	+5 . 4
5.	99½,	5,	21½,	282 . 0,	+10 . 0

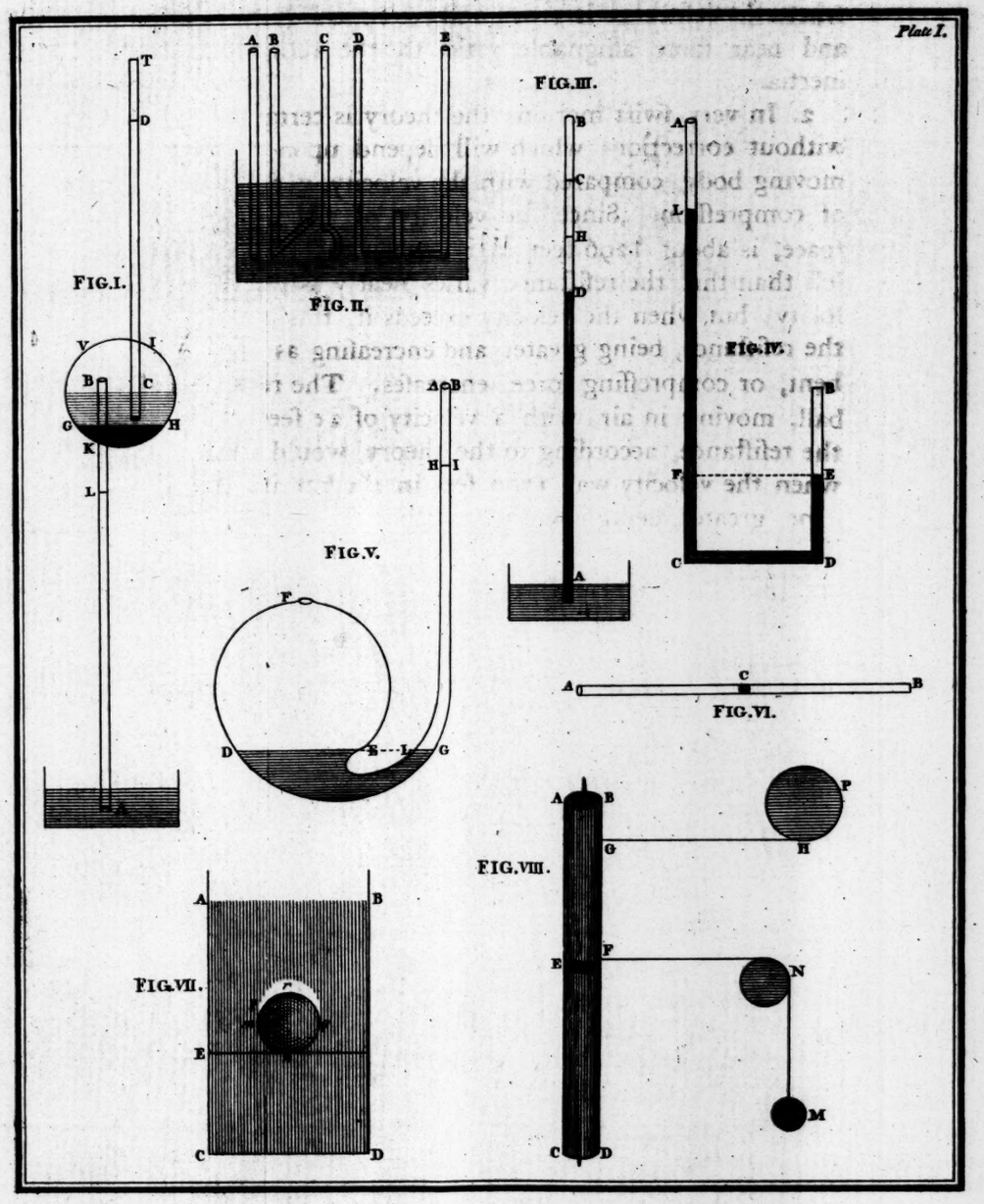
The reſults of theſe experiments, of bodies falling in water and air, do not differ very much from the theoretical concluſions; but the theory ſuppoſes the reſiſtance, cæteris paribus, to vary as the denſity multiplied into the ſquare of the velocity, which hypotheſis, from the coincidence of experiment with it, muſt conſequently be nearly true. Q.E.D.

SCHOLIUM.

40. The limits of the velocity, where this law of the reſiſtance begins and ceaſes to take place, is not aſcertained; but it appears from theſe and other ſimilar experiments, that it does not obtain in very ſlow, nor in very ſwift motions. 1. In ſlow motions, the deviation from theory may be owing chiefly to ſome errors in making the experiments, which, in eſtimating very minute quantities, are unavoidable. The leaſt protuberance and difference from a ſpherical figure, or improper ſituation of the center of gravity, when the body is liberated, will communicate a librating motion, and deflection from the vertical line, in which it is ſuppoſed to deſcend, or curve in which it is ſuppoſed to vibrate. Theſe cauſes are undoubtedly ſufficient to produce the want of coincidence between the theory and experiments; for though their effects be ſmall, yet they may be obſervable in the compariſon of very ſmall quantities. And in water and every other fluid, poſſeſſing any tenacity, the coheſion being conſtant, may be inſenſible in great velocities,

where the reſiſtance is great, and yet appear in very ſlow motions, and bear ſome aſſignable ratio to the reſiſtance ariſing from inertia.

2. In very ſwift motions the theory is certainly not applicable without correction; which will depend upon the velocity of the moving body, compared with the velocity, generated by the force of compreſſion. Since the velocity of air, moving into empty ſpace, is about 1296 feet in 1″; when the velocity of a body is leſs than this, the reſiſtance varies nearly as the ſquare of the velocity; but when the velocity exceeds it, this law cannot obtain, the reſiſtance, being greater and encreaſing as the weight incumbent, or compreſſing force, encreaſes. The reſiſtance to a 12 lb. ball, moving in air, with a velocity of 25 feet in 1″, is $\frac{1}{2}$ oz. and the reſiſtance, according to the theory, would be about $144\frac{3}{4}$ lb. when the velocity was 1700 feet in 1″; but it is in reality three times greater, being equal to $433\frac{1}{2}$ lb.


Plate I.
FIG.I.
T
D
V
I
B
C
G
H
K
L
A
FIG.II.
A B C D E
FIG.III.
B
C
H
D
A
FIG.IV.
A
L
B
F
E
C
D
FIG.V.
A B
H I
F
D
E
L
G
FIG.VI.
A
C
B
FIG.VII.
A
B
E
C
D
FIG.VIII.
A B
F
G
H
E
F
N
M
C
D


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CHAP. V.

OF THE ASCENT OF FLUIDS BY THE ATTRACTION OF COHESION*.

41. PROP. *WATER, ſpirits of wine, &c. in contact with glaſs, metals, ſtone, and moſt dry porous ſubſtances, will ſpontaneouſly move in a direction oppoſite to that of gravity, and remain ſuſpended above the level of that in the veſſel containing it.*

EXP. 1. If two poliſhed plates of glaſs, metal, ſtone, &c. holden contiguous, or glaſs tubes of ſmall diameters, be dipped in water, it is ſenſibly perceived to aſcend in a direction oppoſite to that of gravity, and then continue ſuſpended. If the diameter of a glaſs tube be $\frac{3}{25}$ths of an inch, the perpendicular altitude of the fluid is about $\frac{1}{4}$th of one inch. In capillary tubes, its altitude is much greater: the greateſt altitude, in the fineſt capillary, is not, according to Dr. Hook, above 21 inches.

2. Sugar, ſalt, ſponge, and moſt dry porous ſubſtances in contact with water, will make it riſe to conſiderable altitudes above the level. If a large tube of glaſs be filled with ſifted aſhes, well preſſed together, and one end be dipped in ſtagnant water, the fluid will aſcend ſlowly in the aſhes, and in ſome days arrive at an altitude of 30 or 40 inches above the ſtagnant water. The aſcent of each lamina of water, ſeems to ariſe from the attractive force of the lamina of aſhes, contiguous to its upper ſurface.

3. The aſcent of moiſture up the fine tubes of vegetables, and the motion of the various juices through the glands in animal bodies, according to their ſeveral natures, ſeem to reſult from this prin-

* Muſſchenbroek, 1066, &c. Graves, Lib. I. Ch. V. Helſham, Lect. I. II Cotes's Hydroſ. p.223. Newt. Opt. p.380. Hamilton, Lect. II. Deſagul. p. 10. Philoſ. Britan. p.17.

principle of motion. The ſmall tubes, compoſing their roots, take in different juices, according to their different conſtitutions, juſt as the ſame capillary tube of glaſs acts differently on different fluids; for water, though heavier than ſpirit of wine, is raiſed to nearly twice the altitude in the ſame tube, and tubes made of different ſorts of glaſs, are found to poſſeſs different attractive forces. Q.E.D.

42. PROP. *The altitudes to which water riſes in cylindrical glaſs tubes of ſmall diameters, or between contiguous glaſs plates immerſed in it, vary as the diameters of the tubes, and diſtances of the plates, inverſely.*

PLATE II. FIG. XI. EXP. 1. If the tubes *A, B, C, D, E*, &c. ſucceſſively decreaſing in diameter, be immerſed in water, it will ſpontaneouſly aſcend to the points *a, b, c, d, e*, &c. whoſe perpendicular altitudes above the ſurface, are inverſely as the diameters of the tubes.

2. If a drop of water be included in a ſmall conical glaſs tube, or between two glaſs plates, making a very ſmall angle, it is ſenſibly perceived to move with a velocity, perpetually encreaſing as the diſtance between them decreaſes.

3. If two poliſhed plates of glaſs be joined at one end, and ſeparated at the other about $\frac{1}{4}$th of a degree, and the loweſt be horizontal; then wetting the inſides with any thin oil, and placing a drop of it between them, ſo as to touch both ſurfaces, it will move towards their concourſe, and continue to move till, elevating the plates, the weight of the drop counterbalance the attraction. The figure of the drop, as it adheres equally to both plates, is nearly cylindrical, and its content equal to either ſurface of contact, multiplied into the diſtance between them, and conſequently the diſtance and ſurface are inverſely as each other. But from experiments it appears, that the elevations neceſſary to ſtop the motion of the drop, at the diſtances of 3 and 6 inches from the

the concourſe, are as 4 : 1; and ſince the ſurfaces of contact are as 2 : 1, the attractions of equal ſurfaces at the diſtances of 3 and 6 inches, muſt be as 2 : 1, or inverſely as the diſtances of the plates.

4. If two poliſhed glaſs plates, meeting at AB, and ſeparated towards C, at a diſtance equal to $\frac{1}{4}$th or $\frac{1}{5}$th of an inch, be immerſed in water, it will aſcend ſpontaneouſly to altitudes GD, HI, &c. inverſely as the diſtance between them, or inverſely as their diſtances from the concourſe. The ſurface of the fluid is therefore an hyperbola. FIG. XII.

43. Cor. The quantity of water raiſed in a cylindrical tube, is equal to the area of a ſection of the tube, perpendicular to the axis, multiplied into the altitude; if Q, H, D, repreſent the quantity, altitude and diameter, and be variable; Q will be as $D^2 \times H$, or, from this propoſition, as D, or as $\frac{1}{H}$. This concluſion can only be obtained, ſatisfactorily, from experiment.

SCHOLIUM.

44. This propoſition ſeems to be true, within certain unknown, and perhaps immeaſurable, limits; for that it is generally true in glaſs tubes, is not fully aſcertained. In thoſe experiments, which I have ſeen, water does riſe higher and apparently with greater velocity, as the diameter of the tube is diminiſhed; but it is impoſſible to make tubes, ſucceſſively decreaſing in diameter, and equally ſo perhaps, to aſcertain the diameters of very ſlender tubes with ſufficient accuracy.

45. PROP. *The altitudes of water, ſuſpended in compound tubes, vary inverſely as the diameters of the upper orifices nearly.*

EXP. 1. In open cylindrical tubes, whoſe diameters are equal to thoſe of the different parts AB and C of the compound tube ABC, FIG. XIII.

ABC, water riſes ſpontaneouſly to altitudes equal to *gr* and *gs*, which are inverſely as their diameters nearly. Whether the orifice *A* or *C* be immerſed in the fluid, it riſes to the altitude, *gr*; but if the tube be filled, ſo that the water may be in contact with the narrow part of the tube, *C*, the whole column will be ſuſpended, if its altitude does not exceed *gs*.

FIG. XIV. 2. Let *af* and *bg*, be the altitudes of water, raiſed ſpontaneouſly in the open cylindrical tubes *ED* and *DF* reſpectively, which are inverſely as their diameters. When the wider orifice is immerſed, and the tube is filled with water to an altitude leſs than the length of the wider part, it will ſubſide to a level with the point *g*: but if the water be admitted into the narrower tube, the whole column will be ſuſpended if its length do not exceed *af*.

3. If the tube be inverted, and the narrower orifice immerſed, and water raiſed into the tube *DF*, it immediately ſinks, if the altitude of the column exceed *bg*.

FIG. XV. 4. Let *adc* be a capillary ſyphon, in whoſe narrower and wider legs, if of ſufficient length, columns of water, whoſe altitudes are *ef* and *gh* reſpectively, and which are inverſely as their diameters, may be ſuſpended. Fill this ſyphon with water, and hold it in the poſition of fig. 15, and the fluid will not run out of the orifice *c*, unleſs the difference of the legs, *dc*, exceed *ef*.

5. If the narrower leg be longer than the wider, the water will flow out of the orifice *c* (fig. 16.) if the difference of the legs, *dc*, exceed *gh*.

SCHOLIUM I.

46. Becauſe the velocity of a fluid in a compound tube, always kept full, is inverſely as the area of the ſection, the moments in every ſection are the ſame, and hence it is concluded, that this propoſition is generally true, a concluſion not to be admitted without experimental proof. It may obtain when the diameter of the wider tube is not very great, but not otherwiſe.

SCHOLIUM

SCHOLIUM II.

47. It ſeems to follow from theſe experiments, and indeed hath been aſſerted (Cotes's Hydroſtatics, pag. 227.) that the ſuſpenſion of fluids is owing to the attraction of the annular ſurface, contiguous to their upper ſurface, which however is not allowed by Dr. Hamilton (Lect. 2d. pag. 47.) for theſe reaſons. 1. When the orifice of a ſmall glaſs tube, open at both ends, is dipped in water, the ſmall annular ſurface of glaſs, on the inſide of the tube contiguous to the orifice, will draw up the water lying immediately under it, and every plate of water elevated will raiſe up that lying above it, until the weight of the column raiſed counterbalance the force by which this annulus endeavours to draw up more water. The thin plate of water lying over the loweſt annulus, and every other plate of the elevated water, muſt be attracted upwards and downwards with equal forces, becauſe the attracting ſurfaces above and below it are equal; and therefore the whole column of water, lying above the loweſt annulus, being drawn equally in oppoſite directions, may be conſidered as unaffected by the attraction of the glaſs, and muſt preſs with its whole weight, on the water retained at the orifice of the tube by the attraction of that loweſt annulus, which has no other ſurface below it to counteract its force. 2. When the water has aſcended into a ſmall glaſs tube, open at both ends, take it up and wiping off the drop hanging at the end of it, invert the tube: the ſmall column of included water, being drawn equally by the glaſs in oppoſite directions, will deſcend by its own gravity, and ſtop only when it arrives at the loweſt orifice. Or, whilſt the included column is deſcending, let the loweſt orifice be dipped in water, and taken up, and a ſmall plate of water will be retained at its orifice, and then the included column will be quieſcent. Now this column muſt, by means of the intermediate air, preſs with its whole weight upon the plate of water at its orifice, which can be retained by no other force, than the attraction of the glaſs annulus, lying juſt within

within the orifice of the tube. From hence it appears, that this attraction alone is ſufficient to ſuſtain, and conſequently to raiſe, all the water that riſes in glaſs tubes. Theſe arguments appear to me to be very unſatisfactory, and do not eſtabliſh the concluſion intended. 1. Let a cylindrical glaſs tube be divided into very ſmall equal annuli, *a*, *b*, *c*, *d*, *e*, &c. and *a* being dipped in water, a thin plate of the fluid will be detached from the maſs, which being in contact now with the ſecond annular ſurface *b*, that poſſeſſes the ſame power of attraction with *a*, muſt, partly at leaſt, be raiſed by it, and ſo of the remaining annuli. And if the effects of the intermediate annuli ſhould be equal and oppoſite, which cannot be allowed, yet the ſuperior annulus at *g* would be unaffected, and its whole force remain entire, and it muſt contibute, at leaſt, towards the ſupport of the column of water. But that each annulus ſhould exert a force in elevating the lamina of fluid contiguous to it, and not contribute towards the ſupport of the fluid thus elevated, is unaccountable, the power of attraction being inherent in the glaſs, and inſeparable from it. 2. Let the orifice *F*, of the compound tube *EDF*, be dipped in water, and it will riſe to an altitude equal to *gb*; and if *DF* be filled ſo that the fluid be not in contact with the narrower part, it will ſubſide till its height be equal to *gb*. But let the tube be filled ſo that the fluid be admitted into the narrower part at *D*, the height of the column ſuſpended will be equal to *af*, though the loweſt annulus continues the ſame. When this tube is inverted, and the orifice *E* dipped in the fluid, it riſes to an altitude equal to *af*, if the narrower part of the tube be ſo long; and if *ED* be leſs than *bg*, it riſes into the wider tube till its height be equal to *bg*, the ſame altitude, as if the tube were cylindrical, and its diameter equal to that of the wider. If theſe experiments be true, it is clear that the water riſing ſpontaneouſly in glaſs tubes, is neither raiſed, nor ſupported, by the attraction of the loweſt annulus. 3. This influence of glaſs, and other ſubſtances, upon fluids, is ſcarcely perceptible beyond the immediate points of contact, and therefore extends to the central parts of a tube, only by the connexion of the

PLATE III. FIG. XVI.

PLATE II. FIG. XIV.

the particles of the fluid. Let water be kept ſuſpended in the cylindrical glaſs tube *ABCD*, and let the attraction of the glaſs extend to the lines *EG*, *FH*; and circles of the fluid, whoſe breadths are *m n*, *r s*, &c. will adhere to the ſurface, and certainly be ſupported by the immediate attraction of the glaſs, which is ſo great, that water ſuſpended in a capillary tube will not evaporate, though expoſed to the air for ſix or ſeven months: but the intermediate column *EGHF* is only ſupported by the adheſion of the particles of the fluid to theſe circles, and to the higheſt lamina *GH*, the attraction of any intermediate lamina *LM* being the ſame both upwards and downwards; and when the weight of this intermediate column exceeds their accumulated force, it muſt deſcend. This ſeems to be certain, that the ſhell of particles *CEFD* is ſuſtained by the immediate action of the glaſs, which is diffuſed to the intermediate lamina *n p*, *s t*, &c. by the coheſion of the particles themſelves; and in this manner the forces of the oppoſite points *m*, *q*, and *r*, *v*, conſpire, which does not obtain in the forces of the particles upon the outſide of the tube; and this ſeems to be the reaſon why a column of the fluid is not elevated upon the outſide, as it is within. Not having leiſure to make experiments, I do not ſpeak with confidence, and do not affirm that the height of water ſuſpended in compound tubes, is, in general, inverſely as the diameter of the higher orifice. It may be ſo when the diameters of the parts are not very different.

PLATE III. FIG. XVIII.

48. DEF. *Solution of a body in a fluid is ſuch an intimate union of their component parts, that the appearance of the fluid is not changed, but, if tranſparent, its tranſparency remains.*

49. PROP. *Solutions are the effect of this attraction, or of the mutual action of bodies, in contact, upon each other.*

EXP. Sugar, ſalt, and other ſubſtances ſoluble in water, juſt dipped in the fluid, imbibe it and make it move in a direction oppoſite to that of gravity, and they become united and form one maſs. And in a ſimilar manner, if ſugar, ſalt, &c. be placed at the bottom of a veſſel of water, particles are ſenſibly detached from them, attracted upwards, and incorporate with the water, and, if its quantity be ſufficient, a ſolution is formed and the fluid is tranſparent.

SCHOLIUM.

50. Many ſimilar experiments might be deſcribed of the ſolutions of metals, but the phenomena of all ſolutions ſeem to have ſuch marks of reſemblance, that the truth of the propoſition will not be queſtioned. When the component parts of a body are leſs ſtrongly attracted by each other than by the fluid ſurrounding it, they muſt be disjoined and attach themſelves to the particles of the fluid, in which they remain ſuſpended. The ſame power that is capable of diſuniting the particles of bodies, which in ſome caſes adhere together with great force, muſt alſo be capable of producing a leſs effect, or of overcoming their gravity and making them aſcend in fluids much lighter than themſelves. When the component parts of a ſubſtance attract each other more ſtrongly than they are attracted by another body, they are not affected by it. The particles of mercury do not riſe at the ſides of glaſs plates, or into glaſs tubes, immerſed in it; and particular metals are only diſſolved by particular menſtrua.

51. PROP. *Air and water mutually act upon, and attract, each other.*

EXP.

EXP. 1. Let an oil flask, filled with water deprived of air as much as possible, be inverted and its neck immersed in water, and a bubble of air will ascend to the higher part of the flask, which, after standing some days, will be entirely absorbed by the water, and the flask will be full.

2. Dry salt of tartar, exposed to air apparently pure and homogeneous, will extract moisture from it so as to become fluid. Q. E. D.

52. PROP. *Evaporation of water is a solution of water in air.*

EXP. 1. If a lump of sugar or salt be immersed in a fluid that dissolves them, the surrounding fluid is quickly impregnated with particles detached from it, and appears to be muddy and loaded, and, if the fluid be quiescent, the solution is carried on very slowly, but, if stirred about, it is soon effected, By a procedure similar to this, dry air, contiguous to water, quickly raises a quantity of the fluid and covers its surface with vapour; and, when the air is stagnant, evaporation is carried on very slowly, but it is promoted by a current of air, which removes the vapour and brings fresh particles of air into contact with the water.

2. If a lump of salt be thrown into a glass of clear cold water, which, after standing a little time, is gently agitated, the fluid contiguous to the salt, being soon saturated, will ascend in curled wreaths, and objects, seen through it, will appear to have a tremulous motion, which will continue till the transparency be restored by an uniform diffusion of the salt. Smoke, steam, or vapour arising from moist ground in hot weather, seem to be phenomena similar to this, as they ascend in long stria, render the air opake, and by their encreased refractive power, occasion the same undulating motion to quiescent objects, which is observable in solution, or in the mixing of two fluids, previous to their being entirely incorporated.

3. Salt is diſſolved ſooner, and in greater quantities, in hot, than cold, water; and, if hot water be ſaturated with ſalt, part of it will fall to the bottom, or ſhoot into chryſtals, as the water becomes cold. In the ſame manner, evaporation from a given ſurface of water is quicker and more copious in warm, than cold, weather; aqueous vapours, ſuſpended in the air during a hot day, are precipitated at night, when the air is colder, into drops of dew, or, if the night be very cold, are chryſtallized, in a hoar froſt; a bottle filled with cold liquor, and expoſed to the air of a very hot day, will ſoon collect a dew upon its outſide, the air being cooled by it, and the water, incorporated with the air, precipitated by the cold; a dew is formed on the inſide of the windows of a warm room or cloſe chaiſe, their outſide being expoſed to a cold air; moiſture, breathed from the lungs of animals, becomes viſible in very cold air, which cannot keep that moiſture in a ſtate of ſolution; and hence the air is more pure and clearer in cold, than in hot, weather.

4. The quantity of a body diſſolved, and of a fluid evaporated, in a given time, depends, cæteris paribus, upon the quantity of ſurface; for a body reduced to powder is ſooner diſſolved, than in a concrete form, and ſmoke or ſteam (which is water reduced to very ſmall particles by heat) is ſooner incorporated with air, than water in its uſual form; and the moiſture adhering to a cold poliſhed plate of glaſs, or metal, when breathed upon, is inſtantly dried up by the air.

5. Cold is produced in the immediate act of ſolution of ſea ſalt, ſal ammoniac, or nitre in water, or of eſſential oils in ſpirit of wine, and the cold is greater as the ſolution is quicker. Cold is alſo produced in the immediate act of evaporation: ſailors cool their liquors, in hot countries, by wrapping the veſſels in wet cloths and expoſing them to the wind, and continuing to wet them when become dry; the thermometer will ſink if the ball be lightly rubbed with ſpirit of wine or with æther, &c. and it will ſink faſter as the evaporation is quicker; the hand appears to be colder when rubbed

Plate II.

FIG. IX.

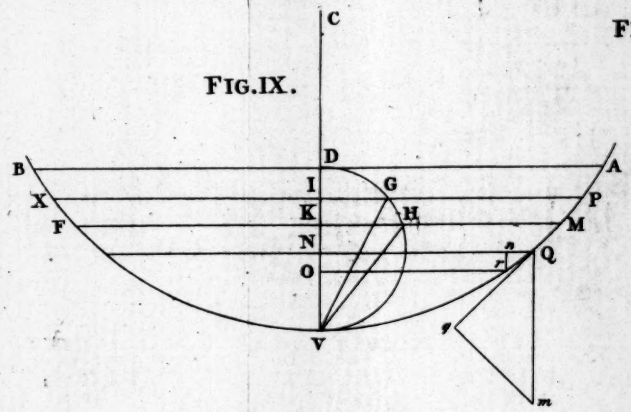
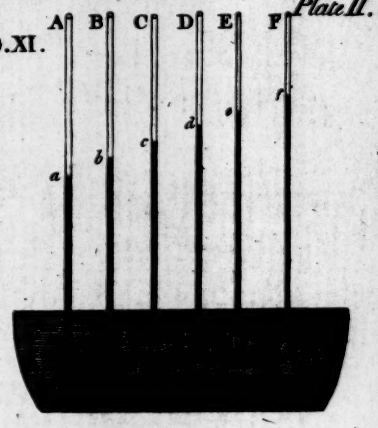


FIG. XI.



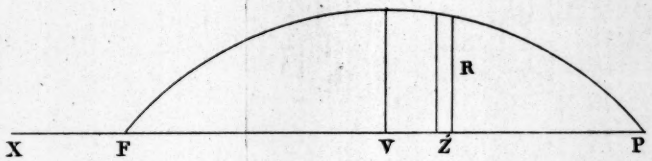


FIG. XII.

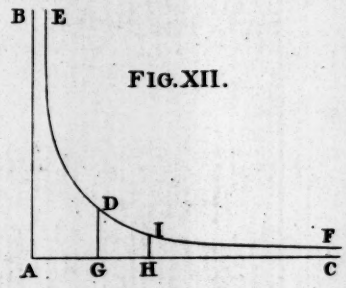
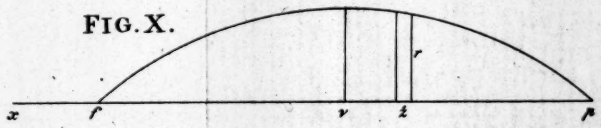


FIG. X.



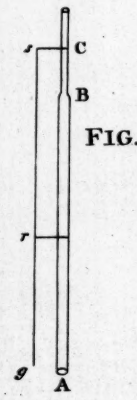


FIG. XIII.

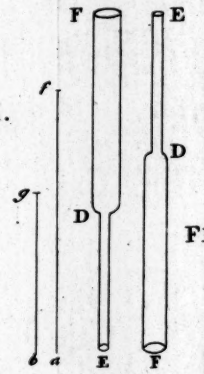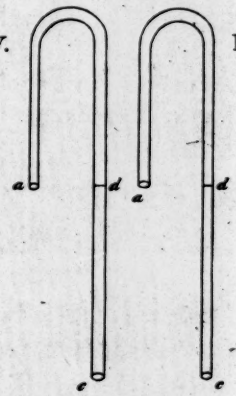


FIG. XIV.

FIG. XV.



FIG. XVI.




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rubbed with Hungary, than with common, water; and ſtill colder when rubbed with æther; but Hungary water evaporates faſter than common water, and æther ſtill faſter than this.

6. Rectified ſpirits of wine, purged of air, will imbibe a large bubble of air ſooner than water will, and there is therefore a greater attraction between this ſpirit and air, than between water and air; and ſpirit of wine evaporates alſo much faſter than water.

7. If a body be diſſolved in any fluid, and another body be preſented to it, with which it has a greater affinity, the laſt will be diſſolved, and the firſt, returning from a ſtate of ſolution, will fall to the bottom; and this alſo obtains with fluids incorporated with air. If copper filings be thrown into a ſolution of ſilver in aqua fortis, the aqua fortis will diſſolve the copper and quit the ſilver, which will be precipitated in the form of a white powder; if iron filings be then thrown into the ſolution of copper, they will be diſſolved, and the copper will fall to the bottom; and if ſalt of tartar, or any other alkaline ſalt, be thrown in, the aqua fortis will unite with it, and diſmiſs the iron. Aqua fortis therefore has a greater affinity, or a ſtronger attraction, to the alkaline ſalts than to the metals, and it attracts iron, copper, ſilver with different forces; which muſt be very great, as they diſunite the particles of ſuch hard metals. And in a manner ſimilar to this, if to well rectified ſpirit of wine, an equal quantity of clear rain water be added, theſe fluids will incorporate, and diſmiſs a great part of the air contained in them, which will riſe to the upper ſurface or adhere to the ſides of the veſſel.

By the preceding experiments, ſuch marks of ſimilitude appear between evaporation and ſolution, that they may be concluded to be phenomena of the ſame kind, and therefore owing to the ſame cauſe. Q. E. D.

CHAP.

OF THE PRESSURE, SPECIFIC GRAVITIES, MOTION, AND OTHER PHENOMENA, OF FLUIDS, ARISING FROM GRAVITY.

CHAP. VI.

OF THE PRESSURE OF FLUIDS*.

53. PROP. *THE ſurfaces of all quieſcent fluids are perpendicular to the direction of gravity.*

FIG. XIX. DEM. Let the ſurface, *SR*, of a quieſcent fluid be enclined, if poſſible, to the horizontal line *SN*, *P* any particle of the fluid, *PL*, perpendicular to *SN*, its gravity. Reſolve *PL* into two forces, *LM* perpendicular, and *PM* parallel, to *SR*, or to the tangent at *P*; and it is clear (12) that *PM* will communicate motion to *P*, and it cannot be quieſcent till *PM* vaniſh, or the ſurface be horizontal. Q.E.D.

Otherwiſe:

The preſſure upon any particle *D*, which is clearly equal to the column *DE*, is diffuſed laterally (11), and, ſince the fluid is quieſcent, in equilibrio with the greater column *RG* incumbent upon *G*, or

* Cotes, pag. 22. Muſſchen. Ch. XXI.

G, or the leſs column *SH* incumbent upon *H*, which is abſurd. Q. E. D.

54. Cor. 1. If the force of gravity paſs through any point *C*, *CP* must always be perpendicular to the ſurface, and conſequently any portions of the ſurface of a fluid are perpendicular to lines paſſing through the center of the earth, becauſe the direction of gravity paſſes through that center, and ſurfaces are only horizontal when ſmall. FIG. XX.

55. Cor. 2. If the ſurface of a fluid be equally compreſſed by every point of a heavy ſurface, or another incumbent fluid, it will be horizontal, and the horizontality of the ſurface of fluids is not therefore affected by the preſſure of the atmoſphere. If the ſurface of a quieſcent fluid be not horizontal, the preſſure of the atmoſphere upon it cannot be every where the ſame: and if two fluids contained in the ſame veſſel be quieſcent, the ſurfaces of both of them are horizontal.

56. Cor. 3. In a quieſcent fluid, the preſſure upon any particle is the ſame in all directions; for were it unequal, motion would enſue. If an imaginary plane *GH* paſs through the fluid, parallel to the ſurface, the preſſure upon every part of it is the ſame; and if it be the ſame, the fluid muſt be quieſcent. The preſſure upon any particle, ariſing from the column of particles perpendicularly incumbent, being equal to its weight, and diffuſed laterally in every direction (11), the preſſure upon every ſide of a particle is alſo equal to its weight.

57. PROP. *To find the perpendicular preſſure of an homogeneous fluid upon the ſurface of the veſſel containing it.*

1. If

FIG.XXI. 1. If the bottom of the veſſel be parallel, and its ſides perpendicular, to the horizon, the preſſure upon any part of it, C, is clearly equal to the weight of the column perpendicularly incumbent upon C, whoſe content is equal to the area of $C \times CA$; and, for the ſame reaſon, the preſſure upon every part, or upon the whole bottom BM, is the weight of a magnitude of the fluid equal to $BM \times CA$.

FIG.XXII. 2. If the ſides of the veſſel be enclined to the horizon, and the area of the bottom greater or leſs than the area of the ſurface; draw from any point P, PI perpendicular to the horizon, IH parallel to it, and HS perpendicular to it, and proceed till the laſt perpendicular HS meet the ſurface. The preſſure upon H, and conſequently upon I (11), is clearly equal to the weight of the column SH, and the preſſure upon P is equal to this preſſure upon I, together with the weight of the column IP, or to the weight of a column whoſe altitude is Sp. And as this is true of every point in the bottom BM, the whole preſſure upon it is equal to the weight of a column of fluid whoſe magnitude is $BM \times Sp$.

FIG.XXIII. 3. If the bottom be an irregular ſurface, the preſſures upon any indefinitely ſmall portions K, L, M, &c. are the weights of columns, whoſe baſes are theſe portions, and heights their perpendicular depths, or whoſe contents are $K \times Kk + L \times Ll$, &c.; conſequently the whole preſſure upon the bottom is equal to the ſum of the weights of all the ſmall columns, whoſe magnitudes are $K \times Kk + L \times Ll + M \times Mm + N \times Nn$, &c. or of one column, whoſe magnitude is the whole ſurface multiplied into Gg, the perpendicular depth of its center of gravity (Mechan. 6.). Q. E. I.

58. Cor. 1. If the bottom be horizontal, and the veſſel cylindrical, or its ſides be perpendicular to the horizon, the preſſure upon the bottom is equal to the weight of the fluid contained in it; but in other caſes this preſſure may be greater, or leſs, than the weight, in any aſſignable ratio. The preſſure therefore upon the

the ſides or bottom of a veſſel does not at all depend upon the quantity of fluid contained in it.

59. Cor. 2. If the area of the ſurface preſſed be given, the preſſure is as its perpendicular depth, if it be plane and horizontal, and if irregular or inclined to the horizon, as the perpendicular depth of its center of gravity.

60. Cor. 3. If the fluid contained in a crooked tube, veſſel or channel, however irregular, be quieſcent, every point of its ſurface SR and sr is upon the ſame level: for, ſuppoſing AB to be any ſection of the veſſel, the preſſures on each ſide of it are as the area multiplied into the perpendicular diſtances of its center of gravity, G, from SR and sr, which products are equal to each other, becauſe the fluid is quieſcent, and conſequently the diſtances of G from SR and sr are equal. FIG. XXIV.

61. Cor. 4. If different fluids F and f be contained in the ſame tube, the perpendicular diſtances of the center of gravity of the ſection, AB, where they meet, from their ſurfaces, are inverſely as their ſpecific gravities. Let the plane AB ſeparate the fluids, which will be horizontal (55); and the preſſures upon it will be equal upon each ſide, becauſe the whole is quieſcent; but theſe preſſures are the weights of columns of different fluids, whoſe baſes are AB, and altitudes the perpendicular depths of the center of gravity, G, below the ſurfaces SR and sr, for that part of the fluid contained in AFD is in equilibrio; therefore the ſurface $AB \times$ S.g. of $F \times SA = AB \times$ S.g. of $f \times sC$, and S.g. of $F \times SA =$ S.g. of $f \times sC$, and S.g. of F: S.g. of f:: sC: SA.* FIG. XXV.

* The reading of this corollary may be poſtponed till the next chapter be read.

SCHOLIUM.

62. That the preſſure of a fluid, upon any given ſurface, ſhould be proportional to the area of the ſurface multiplied into its perpendicular depth, and therefore exceed its weight in any given proportion; and that the fluid, contained in the legs of an irregular tube, ſhould be upon the ſame level, are ſometimes thought paradoxical, and therefore proved by the following and ſimilar experiments. The machine, called the hydroſtatical bellows, conſiſts of two oval boards, 16 inches broad, and 18 inches long, covered with leather to open and ſhut like common bellows, but without valves; an hollow cylindrical pipe, whoſe length is three feet, is ſcrewed into one of the boards, and, pouring water into the bellows through this tube, the boards will be ſeparated; and if weights of 200 or 300 pounds be placed upon the higher board, it, together with the weights, will be raiſed, and the boards ſeparated as far as the leather permits, by filling the tube with water. The weight of water in the tube, about $\frac{1}{4}$th of a pound, is in equilibrio with 300 pound placed on the top of the bellows.

FIG. XXVI.

63. PROP. *The preſſure of a fluid upon the ſurface of a veſſel containing it, is equal to the weight of a body, equally heavy with the fluid, which is formed by raiſing perpendiculars, upon every indefinitely ſmall part of the ſurface, equal to the perpendicular depth of that part.*

Let the ſurface preſſed be compoſed of lines indefinitely near to each other, and AC be one of them, and drawing perpendiculars Ll, Mm, Nn, Pp, &c. to every point L, M, N, P, &c. equal to their perpendicular depths, and ſuppoſing them to be columns equally heavy with the fluid, the preſſure upon AC will be equal to the weight of all the columns, or of the area ACD. Let the ſame be done for every other line, and the whole preſſure is equal to the weight of all the areas, or of a body formed by drawing perpendiculars

FIG. XVII.

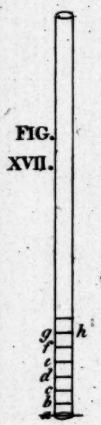


FIG. XVIII.



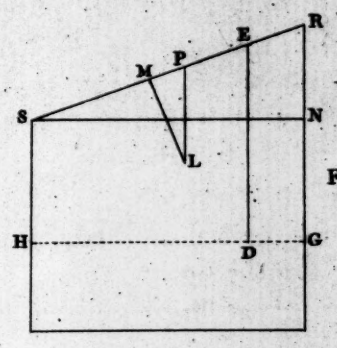


FIG. XIX.

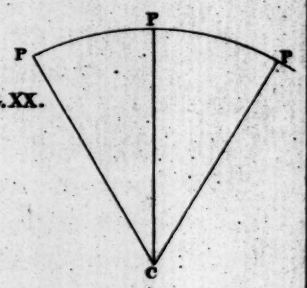


FIG. XX.

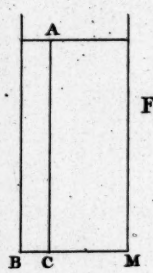


FIG. XXI.

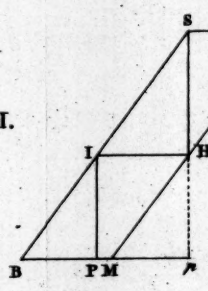
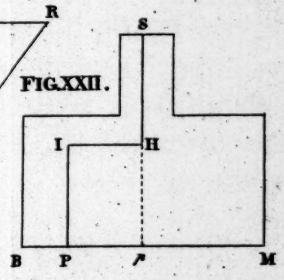


FIG. XXII.



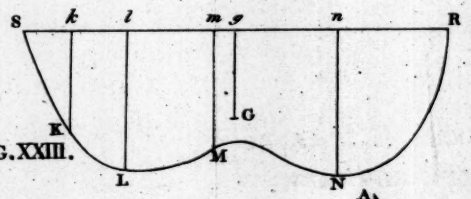


FIG. XXIII.



FIG. XXIV.

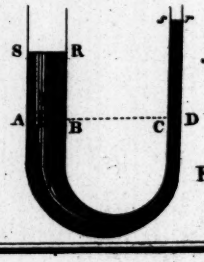


FIG. XXV.

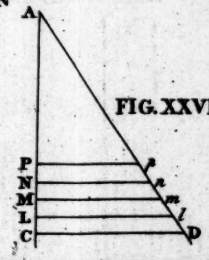


FIG. XXVI.


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APR

pendiculars to every point of the ſurface preſſed, equal to their perpendicular depths. Q.E.D.

64. Cor. If *ABCD* be a ſide of a cubical veſſel, or of a parallelopipedon, perpendicular to the ſurface of the fluid, *AB*, the preſſure upon a vertical line *AD* is equal to the triangular area *ABD*, and that on the whole ſurface is equal to the weight of a triangular priſm of the fluid, *ABDEGF*. If the ſides of a veſſel, filled with a fluid, be ſix equal ſquares, the preſſure upon each is a triangular priſm of the fluid; whoſe baſe is one of the ſquares and altitude one of its ſides; and conſequently the whole preſſure upon the ſides is the ſum of theſe, or of a triangular priſm of the fluid of the ſame height, whoſe baſe is a rectangle, contained by two ſides, one of which is equal to the periphery of the veſſel, and the other a ſide of one of the ſquares. FIG. XXVII.

65. DEF. *The center of preſſure of any ſurface is a point, to which a force, equal to the whole preſſure, being applied and acting againſt the preſſure, is in equilibrio with it.*

66. PROP. *The center of preſſure*, P, *of any plane ſurface* BA, *is the ſame with the center of percuſſion of the plane, moving round an axis, coincident with the interſection of the plane and the ſurface of the fluid.* FIG. XXVIII.

Let *Aa*, *Cc*, *Bb*, &c. be drawn in the ſame plane, perpendicular to the ſurface of the fluid, and meeting the plane in *A*, *C*, *B*, &c. and let *BA*, produced, meet the ſurface in *S*. The preſſures upon *A*, *C*, *B*, &c. are as *Aa*, *Cc*, *Bb*, &c. or as the diſtances of *A*, *C*, *B*, &c. from *S* (ſim. triang.), and conſequently *P* is the center of percuſſion of the plane revolving round *S* as a center of ſuſpenſion (Mechan. 488). Q.E.D.

SCHOLIUM.

67. The preſſures upon all ſurfaces, whoſe areas and centers of gravity are diſcoverable, may from hence be eaſily aſcertained; but as theſe cannot be found without a long proceſs, a few ſimple caſes only are here adduced.

1. The preſſure upon the bottom of a cubical veſſel, when horizontal, is the whole weight, or the weight of a magnitude of the fluid, which is found by multiplying the area of the bottom into the perpendicular altitude. The preſſure when any one ſide is equal to half of that upon the bottom, being equal to the weight of a bulk of the fluid, which is found by multiplying the area of the ſide into the perpendicular depth of its center of gravity, or into half the perpendicular height; and conſequently the whole preſſure upon the bottom and ſides of a cubical veſſel is equal to three times the weight of the fluid contained in it. 2. If two ſpheres, or two circular ſurfaces, be juſt immerſed in any fluid, the preſſures upon them are as the ſurfaces multiplied into the depths of the centers of gravity, or as the cubes of the radii. 3. The preſſures upon the ſame triangular ſurface, when its baſe coincides with and is parallel to the ſurface of a fluid in which it is juſt immerſed, are as the area of the triangle multiplied into $\frac{1}{3}$ and $\frac{2}{3}$ of the perpendicular depth of the center of gravity reſpectively, or as 1 : 2. 4. The preſſures upon a ſquare ſurface, when one ſide, and an angle, coincide with the ſurface of a fluid in which it is juſt immerſed, are reſpectively as the area of the ſurface multiplied by half of a ſide, and half of the diagonal, or as $1 : \sqrt{2}$. 5. The preſſure upon the bottom of a conical veſſel, whoſe axis is vertical, juſt immerſed in, or filled with a fluid, is equal to the weight of a cylinder of the fluid of the ſame baſe and altitude with the cone; and conſequently the reaction of the ſides of the cone is equal to the weight of $\frac{2}{3}$ of this cylinder, or to twice the weight of the fluid contained in the conical veſſel.

68. PROP.

68. PROP. *To find a point* P *of a rectangular ſurface* SBTR, *juſt immerſed in any fluid, through which if a line be drawn parallel to the ſurface of the fluid or ſide* SR, *the preſſures upon* SE *and* DT *may be to each other as* m : n. FIG. XXIX.

Let LM (drawn perpendicular to the ſurface SR) $=a$; $LP=x$; $SR=b$; s and r be the centers of gravity of the ſurfaces SE and DT; and $Ls=\frac{x}{2}$; $Pr=\frac{a-x}{2}$ and $Lr=a-Mr=a-\frac{a+x}{2}$ $=\frac{a+x}{2}$. But the preſſure upon SE and DT are to each other as $SE\times Ls : DT\times Lr$, or as $b\times x\times\frac{x}{2} : b\times\overline{a-x}\times\frac{a+x}{2}$, or as $x^2 : a^2-x^2$, or, from the ſuppoſition, as $m : n$, and $nx^2=ma^2-mx^2$, and $x=a\times\sqrt{\frac{m}{m+n}}$.

69. Cor. If the preſſures upon SE and DT be equal, or $m=n$, $x=a\times\sqrt{\frac{1}{2}}$.

SCHOLIUM.

70. By the preſſure upon any plane, or curvilineal ſurface, is always underſtood the aggregate of all the preſſures upon every point of thoſe ſurfaces, eſtimated in directions perpendicular to them at each point, no part being loſt by obliquity of direction.

CHAP.

CHAP. VII.

OF SPECIFIC GRAVITIES*.

71. DEF. *THE ſpecific gravity of a body is its weight compared with that of another body of the ſame magnitude.*

SCHOLIUM I.

72. The magnitude of a body is expreſſed by a number, denoting its relation to ſome criterion generally uſed and ſimilar to itſelf, as a cubical inch, foot, &c.; and, if the criteria be different, as in different kingdoms, the magnitudes of bodies are to each other as the criteria multiplied into the numbers expreſſing theſe magnitudes. Abſolute and relative weight uſually ſignify weight in vacuo and in a reſiſting medium, air, water, &c.; but the abſolute weight of a body is relative, being expreſſed by a number, denoting its relation to ſome arbitrary and conventional ſtandard, as 1 lb., 1 ounce, of which it is a multiple or aliquot part; and in the ſame ſort of matter, ſuppoſed to be homogeneous, it depends upon and varies as the magnitude. The ſpecific weight or gravity of the ſame ſpecies of matter, whether its magnitude be great or ſmall, as of A, $2A$, $3A$, &c. is the ſame, being, according to the definition, the weight of a given bulk; and the object therefore of ſpecific gravities is to diſtinguiſh different ſpecies of matter from each other, in one of their moſt obvious qualities, weight of matter contained in a given ſpace. The weight of any portion of matter is eaſily aſcertained; but it is not always eaſy to meaſure the ſpace occupied by a body, or its magnitude, and, in ſome inſtances, this cannot well be effected without artificial methods. It is

* Nicholſon, Vol. II. Ch. V. Martin, Vol. I. pag. 272. Cotes, p. 37. Roh. P. I. Ch. XVI. a. 3. 4. not. ad. a. 8. Helſham, Lect. XIII. Muſſchen. Chap. XXIV. p. 474.

is expedient to employ, as a criterion, ſome pure and homogeneous ſubſtance, viz. diſtilled water, whoſe ſpecific gravity, or weight of a given bulk, is nearly at all times the ſame; and by comparing this with other ſubſtances, the ratio of their ſpecific gravities may be diſcovered; and denoting the ſpecific gravity of water by any number taken at pleaſure, the numbers expreſſing the ſpecific gravities of other bodies are hence given. Since the weight of one cubical foot of pure rain-water is equal to 1000 ounces avoirdupois, if its ſpecific gravity be denoted by 1, or 1000, the weight of one cubical foot, or other meaſure, of other ſubſtances, are hence found, and tables of the ſpecific gravities of bodies conſtructed. One ounce avoirdupoiſe = 437.5 grains, and one ounce troy = 480 grains, and conſequently 1 avoirdupoſe oz.: troy oz. :: 437.5 : 480; and 1 avoirdupoiſe lb. : 1 troy lb. :: 437.5 × 16 : 480 × 12 :: 1750 : 1440. A cubic foot of water = 1000 ounces avoirdupoiſe = 62.5 lb. avoirdupoiſe; but 62.5 lb. avoir. : x (or number of troy lbs. contained in a cubic foot of water) :: $\frac{1}{1750}$: $\frac{1}{1440}$ (or inverſely as the number of grains contained in each) :: 1440 : 1750, and $x = \frac{62.5 \times 175}{144}$ lb. = 75.95 lb. troy. One cubic inch of water = $\frac{1000 \times 437.5}{1728}$ grains = 253.18 grains = $\frac{253.18}{437.5}$, or .57869 parts of one avoirdupoiſe ounce = $\frac{253.18}{480}$ or .5274 parts of one troy ounce.

SCHOLIUM II.

73. The ſpecific gravities of water and any other fluid may be compared together by the following proceſs: weigh very accurately an ounce, or other weight, of diſtilled water, in a cylindrical glaſs phial, and mark preciſely the ſpace occupied by it; then pour in any other fluid till it fill exactly the ſame ſpace with the water, and, weighing it, we ſhall know the weights of equal magnitudes of

of the water and the other fluid, or their ſpecific gravities. 2. The magnitude of a body, however irregular, may be found by immerſing it in a cylindrical veſſel of water, and marking how far the fluid riſes; for the ſpace contained between the ſurfaces of the water, before, and after, the immerſion of the body, is equal to its magnitude, and this, together with its weight, being known, its ſpecific gravity is alſo known. In this manner may the ſpecific gravities of all ſubſtances be aſcertained, though not with the ſame preciſion as by other ſubſequent methods. 3. The capacity of any irregular veſſel may be known by filling it with water; for the water being weighed, its magnitude, or the number of cubical inches contained in it, and conſequently in the veſſel, will be found.

74. PROP. *The ſpecific gravities of different bodies,* A *and* B, *vary as their weights directly and magnitudes inverſely.*

Let *S. g.* repreſent ſpecific gravity, *wt.* weight, and *A* and *B* the magnitudes of theſe bodies, and, taking equal portions of *A* and *B*, viz. *P* and $\mathcal{Q}$;

S. g. of *A* : *S. g.* of *B* :: *wt.* of *P* : *wt.* of $\mathcal{Q}$ (69); but

wt. of *P* : *wt.* of *A* :: *P* : *A* (the body being homogeneous)

and *wt.* of $P = \frac{\textit{wt. of } A \times P}{A}$. By the ſame proceſs,

the *wt.* of $\mathcal{Q} = \frac{\textit{wt. of } B \times \mathcal{Q}}{B}$; and conſequently

S. g. of *A* : *S. g.* of *B* :: $\frac{\textit{wt. of } A \times P}{A} : \frac{\textit{wt. of } B \times \mathcal{Q}}{B} :: \frac{\textit{wt. of } A}{A} : \frac{\textit{wt. of } B}{B}$, *P* and $\mathcal{Q}$ being equal. Q. E. D.

75. Cor. 1. If *M* and *W* repreſent the magnitude and weight of

of a body, and be variable, *S. g.* varies as $\frac{W}{M}$; M as $\frac{W}{S.g.}$; and W as $M \times S.g.$

76. Cor. 2. Becauſe the denſity (D) of a body varies as $\frac{W}{M}$, it will alſo vary as the ſpecific gravity.

77. Cor. 3. Let the accelerating force of gravity (A) be variable, and the quantity of matter be equal to Q; and W is as $Q \times A$, or as $A \times D \times M$, or as $M \times S.g.$; and conſequently $S.g.$ will vary as $A \times D$; D as $\frac{S.g.}{A}$; and A as $\frac{S.g.}{D}$.

78. Cor. 4. Let rain-water be the ſtandard with which the ſpecific gravities of other bodies are compared, and a cubic foot of this being found uniformly equal to 1000 avoirdupoiſe ounces, the actual weight or magnitude of any other ſubſtance, B, may be diſcovered by finding its ſpecific gravity. *S. g.* of water : *S. g.* of B :: *wt.* of a cubic inch of water : *wt.* of a cubic inch of the ſubſtance B (w) :: 253.18 : w (72), and $w = 253.18 \times S.g.$ of B grains, ſuppoſing the *S. g.* of water $= 1$; but the *wt.* of $B = w \times M$ (ſuppoſing M to be the number of cubic inches in B) $= 253.18 \times S.g.$ of $B \times M$ grains $= .57869$ *S. g.* of $B \times M$ avoirdup. ounces $= .52746 \times S.g.$ of $B \times M$ troy ounces. The *S. g.* of $B = \frac{wt.\text{ of } B}{253.18 \times M}$ grains; and the number of cubic inches, or M, $= \frac{wt.\text{ of } B}{253.18 \times S.g.\text{ of } B}$.

79. Cor. 5. If the weight of a body $B = W$, be known in avoirdupoiſe ounces, its weight in troy ounces will be found by multi-

plying W into $\frac{.52746}{.57869}$ or .91145; and if W be known in troy ounces, it will be found in avoirdupoiſe ounces by multiplying W into $\frac{.52746}{.57869}$ or into 1.0971. For the weight of B in avoirdupoiſe ounces multiplied into $\frac{.52746}{.57869} = .57869 \times S.g.$ of $B \times M \times \frac{.52746}{.57869}$ (.91145) $= .52746 \times S.g.$ of $B \times M =$ *wt.* of B in troy ounces: and the weight of B in troy ounces $\times \frac{.57869}{.52746}$ (1.0971) $= .52746 \times S.g.$ of $B \times M \times \frac{.57869}{.52746} = .57869 \times S.g.$ of $B \times M =$ the weight of B in avoirdupoiſe ounces. To exemplify theſe rules, ſuppoſe an area of 3000 ſquare feet were to be covered with lead, whoſe thickneſs is uniformly $\frac{1}{100}$th part of a foot, and whoſe ſpecific gravity is 11.325, that of water being unity. The magnitude of lead $= 30000 \times \frac{1}{100} = 300$ cubic feet; and its weight $= 300 \times 11.325 \times 1000 = 3397500$ avoirdupoiſe ounces $= 3397500 \times 1.0971$ troy ounces.

2. The ſpecific gravity of a body is equal to its weight divided by its magnitude: let a piece of marble be 4 cubic feet, and its weight 675 lb. or 108000 avoirdupoiſe ounces, and $\frac{10800}{4 \times 1000} = 2.700 =$ the ſpecific gravity of the marble, that of water being $= 1$. If the ſpecific gravity of water be 1000, then that of the marble $= 2700$.

3. Let the weight of ſeveral fragments of coral be 7 ounces, and their ſpecific gravity 2.69, then their magnitude $= \frac{7}{2.69 \times 1000} = .0026$ parts of a cubic foot $= .0026 \times 1728 = 4.5$ inches nearly.

80. PROP.

80. PROP. *If a body,* B, *float upon a fluid,* F, *the magnitude of the part immerſed, or* P, *is to the magnitude of the whole body, as the ſpecific gravity of the body to that of the fluid.* FIG. XXX.

When the preſſure upwards is equal to the weight of *B*, it will ceaſe to deſcend, and the ſurface of the fluid to riſe, and both will be quieſcent; but if a portion of the fluid equal to *P* were to occupy its place, the fluid would be quieſcent, and conſequently the weight of this portion of the fluid is equal to that of the body.

Or, Suppoſe *B* to be ſupported by an imaginary plane *NE*, parallel to the ſurface of the fluid, and, when the fluid is quieſcent, the preſſure upon every part of this plane is the ſame; but it would continue to be the ſame, were a portion of the fluid, equal to *P*, ſubſtituted in its place, and conſequently the weight of *B* and of a portion of the fluid equal to *P*, muſt be equal; therefore $P \times S.g.$ of $F = B \times S.g.$ of B, and $P : B :: S.g.$ of $B : S.g.$ of F. Q. E. D.

81. Cor. 1. Becauſe $S.g.$ of $B = S.g.$ of $F \times \frac{P}{B}$, if two bodies float upon the ſame fluid, their ſpecific gravities will be as $\frac{P}{B}$, or directly as the parts immerſed in the fluid, and inverſely as their whole magnitudes.

82. Cor. 2. Since $S.g.$ of $F = \frac{B \times S.g.\text{ of } B}{P}$, if different bodies float upon different fluids, and the parts immerſed be equal, the ſpecific gravities of the fluids will be directly as the magnitudes of the bodies multiplied into their ſpecific gravities, or directly as their weights.

83. Cor. 3. If the ſame body float upon different fluids, their ſpecific gravities will be to each other inverſely as the parts.

84. Cor. 4. Since $P = \frac{B \times S.g. \text{ of } B}{S.g. \text{ of } F} = \frac{wt. \text{ of } B}{S.g. \text{ of } F}$; if two bodies of the ſame weight, but of different ſpecific gravities, float upon the ſame fluid, the magnitudes of the parts immerſed are equal.

85. Cor. 5. If inſtead of B a fluid ſpecifically lighter than F be poured into the veſſel, it will float upon F, and, when they are both quieſcent, their ſurfaces will be parallel. And every body, ſpecifically lighter than F, muſt float unleſs it touch the bottom CD ſo intimately that the preſſure upwards upon the bottom be entirely removed, and then the weight neceſſary to raiſe the body is equal to its weight and that of the incumbent column.

SCHOLIUM.

FIG. XXXI. 86. Hence may be underſtood the conſtruction of the *Hydrometer*, a machine contrived to aſcertain expeditiouſly the ſpecific gravities of different fluids. AB is a hollow cylindrical tube of glaſs, ivory, copper, &c. 5 or 6 inches in length, annexed to a hollow ſphere of copper, D, to the bottom of which is united a ſmaller ſphere, E, containing ſome mercury, or ſhot, ſufficient to make the machine ſink vertically in a fluid. The ſtem AB is graduated in ſuch a manner, that the figures upon it exhibit the magnitudes of the parts below, and conſequently, ſpecific gravities of the fluids in which it deſcends to thoſe figures. If the parts immerſed in water and ſpirit of wine be to each other as 10 : $11\frac{1}{10}$, then (83) *S.g.* of water : *S.g.* of the ſpirit :: $11\frac{1}{10}$: 10. This machine not being formed to diſcover ſpecific gravities, widely differing from each other, one more convenient is conſtructed by joining a ſmall plate, G, to the top of the tube, and loading it with weights as is expedient.

FIG. XXXII.

dient. Let this hydrometer deſcend in rectified ſpirit of wine to any point *L*, and a weight muſt be placed upon *G* to make it ſink to the ſame point in any heavier fluid, as water: if the weight of the inſtrument only be 10 dwts. and the additional weight, when immerſed in water, $1\frac{6}{10}$ dwts.; then equal magnitudes of water and rectified ſpirits of wine are $11\frac{6}{10}$ and 10 dwts. reſpectively; and conſequently (71) *S. g.* of water : *S. g.* of the ſpirits :: $11\frac{6}{10}$: 10. By a ſimilar proceſs may the ſpecific gravities of all fluids be diſcovered expeditiouſly and with tolerable accuracy.

87. PROP. *The weight loſt by a body*, B, *totally immerſed in a fluid*, F, *is equal to the weight of a portion of the fluid, whoſe magnitude is equal to that of the body.* FIG. XXXIII.

If the ſpecific gravity of *B* were equal to that of the fluid, its weight would be equal to that of an equal magnitude of *F*, and it would be quieſcent at every depth and loſe all its weight; and if the ſpecific gravity of *B* were greater or leſs than that of *F*, it would evidently deſcend, or aſcend, with a force equal to the difference between its own weight and that of an equal bulk of the fluid; for the preſſure upon the imaginary plane *s r*, would clearly be greater or leſs than that upon the reſt of the plane by this difference. Otherwiſe: Becauſe the preſſure upwards or upon the lower ſurface of a priſmatic body *B*, which is parallel to the ſurface of the fluid, is the weight of a column of the fluid, whoſe height is *A r*, and the preſſure downwards equal to the weight of a column, whoſe height is *A m*, and the lateral preſſures are equal and oppoſite; the preſſure upwards is greater than that downwards by the weight of a column whoſe altitude is *r m*, or of one equal to the body. But every body, however irregular its figure, may be conceived to be formed of ſuch priſms indefinitely ſmall, and conſequently the loſs of weight ſuſtained by any body immerſed

merſed in a fluid, is equal to the weight of a portion of the fluid, which is equal to the body. Q. E. D*.

88. Cor. 1. Every body, immerſed in a fluid ſpecifically heavier or lighter than itſelf, aſcends or deſcends by the action of a conſtant force, equal to the difference between their ſpecific gravities, ſuppoſing the reſiſtance to be nothing.

89. Cor. 2. If a body B be weighed in vacuo, and in a fluid, F, ſpecifically lighter than itſelf, the difference of theſe weights or weight loſt by B, is to the weight of B as the ſpecific gravity of the fluid to that of the body: for the loſs of weight is equal to the weight of a portion of F equal to B, and the magnitudes being given, the $S.g.$ of B : $S.g.$ of F :: *wt.* of B : *wt.* loſt by B; and $S.g.$ of $B = S.g.$ of $F \times \frac{wt.\text{ of } B}{wt.\text{ loſt by } B} =$ (if F be water and its ſpecific gravity unity) $\frac{wt.\text{ of } B}{wt.\text{ loſt by } B}$. A piece of tin is 300 grains in air, and in water 259½ grains, which being ſubtracted from 300, leaves 40½ grains for the loſs of weight, and conſequently $S.g.$ of tin $= \frac{300}{40\frac{1}{2}} = 7.4$.

90. Cor.

* This propoſition may be demonſtrated more particularly by the following proceſs:

FIG. XXXV. Divide the body B into lamina $abed$, by drawing the vertical lines cd, me, infinitely near and parallel to each other; draw bP perpendicular to ab and equal to bm, and Pr, bo, ek, perpendicular to me. Since bm, or bP is the preſſure upon every part of ab, perpendicular to it, and br the preſſure perpendicular to the horizon ($= P$); $P : p$ ($=$ preſſure upon ob) :: $ab \times rb : mb$ or $bP \times bo$; but the triangles bPr and obm are ſimilar, and $Pb : rb$ :: $ab : bo$ and $Pb \times bo = rb \times ab$, and the preſſure upon $ab =$ preſſure upon ob. In the ſame manner the preſſures upon de and ke are equal, and conſequently the difference of the preſſures upon ab and $ed =$ the difference of preſſure upon ob and $ke =$ the difference between the weight of columns of fluid whoſe magnitudes are mo and mk, whoſe magnitude is $abed$. The ſame may be proved of the whole body.

90. Cor. 3. Becauſe the *S. g.* of $B = \frac{wt.\text{ of } B \times S.g.\text{ of the fluid}}{wt.\text{ loſt by } B}$, the ſpecific gravities of the bodies may be found by weighing them in vacuo and in the ſame fluid; for, from the above equation, they are to each other as their weight in vacuo directly, and loſſes of weight in the fluid inverſely, and, if their abſolute weights be equal, inverſely as the loſſes of weight. Let two pieces of lead and tin, each of whoſe weight is 400 grains, and the loſs of weight ſuſtained by the lead is $35\frac{1}{2}$ grains, and that ſuſtained by the tin is 54 grains; and conſequently *S. g.* of lead : *S. g.* of tin :: 54 : $35\frac{1}{2}$.

91. Cor. 4. Becauſe the ſpecific gravity of the fluid is equal to the $\frac{S.g.\text{ of } B \times wt.\text{ loſt by } B}{wt.\text{ of } B}$, the ſpecific gravities of two fluids *F* and *f* may be found by weighing the ſame body *B* in them, and finding the weight loſt in each; for, from the above equation, they are to each other directly as the loſſes of weight. A glaſs bubble, whoſe weight in air is 1727 grains, being weighed in water and ſpirit of wine, ſuſtains a loſs of weight, equal reſpectively to 641 and 558 grains, and conſequently the ſpecific gravity of the ſpirit $= \frac{558}{641} = .86$. The loſs of weight ſuſtained by a glaſs ball in water and milk, is reſpectively 803 and 831 grains and the *S. g.* of the milk $= \frac{1 \times 831}{803.} = 1.031$ nearly.

92. Cor. 5. When a body (*B*), whoſe ſpecific gravity is deſired, is ſoluble in water, let it be weighed in ſome other fluid, *F*, in which it is not ſoluble, and comparing the ſpecific gravity of this fluid, *F*, with that of the body and of water *(f)*, the ſpecific gravities of the two laſt may be compared together by this proceſs:

S. g. of *B* : *S. g.* of *F* :: *m* : *n*,

S. g. of *F* : *S. g.* of *f* :: *p* : *q*,

and conſequently *S. g.* of *B* : *S. g.* of *f* or water :: $m \times p : n \times q$.

Let

Let the weight of a piece of Roman vitriol, in air, be 67 grains, and in ſpirit of wine 41 grains, and, the loſs of weight being 26 grains,
S. g. of vitriol : *S. g.* of wine :: 67 : 26,
S. g. of wine : *S. g.* of water :: .86 : 1, and conſequently
S. g. of vitriol : *S. g.* of water :: 67 × .86 : 26 :: 222 : 100 nearly.

93. Cor. 6. If inſtead of *B*, a fluid ſpecifically heavier than *F* be poured into the veſſel, it will ſink to the bottom, and their ſurfaces will be parallel when they are quieſcent. If the open end of a tube filled with mercury be inſerted in water, it will deſcend, and the water will aſcend into its place; and if this tube of water be inſerted in any lighter fluid, as red wine, it will deſcend, and the wine will aſcend and occupy its place.

FIG. XXXIV. 94. Cor. 7. If a body *B* be immerſed in a veſſel containing two fluids *F* and *f*, the ſpecific gravity of *F* being greater, and that of *f* leſs, than the ſpecific gravity of the body; it will deſcend through the lighter fluid and float upon the heavier, and the parts immerſed in *F* and *f*, or *P* and *p*, will be to each other as the difference between the ſpecific gravities of the body and lighter fluid, to the difference between the ſpecific gravities of the heavier fluid and that of the body. For if, inſtead of *P* and *p*, portions of *F* and *f*, reſpectively equal to them, were ſubſtituted, both fluids would be quieſcent, and conſequently the weights of theſe portions muſt be equal to the weight of the body, or
$P \times$ *S. g.* of $F + p \times$ *S. g.* of $f = \overline{P + p} \times$ *S. g.* of *B*, and
$P \times$ *S. g.* of $F - P \times$ *S. g.* of $B = p \times$ *S. g.* of $B - p \times$ *S. g.* of *f*;
$P : p ::$ *S. g.* of $\overline{B - f}$: *S. g.* of $\overline{F - B}$, and comp.
$P : P + p\,(B) ::$ *S. g.* of $\overline{B - f}$: *S. g.* of $\overline{F - f}$; and
$p : P + p\,(B) ::$ *S. g.* of $\overline{F - B}$: *S. g.* of $F - f$.

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95. The weight of a body, in any fluid, is the difference between its abſolute weight, or weight in vacuo, and the weight of an equal bulk of the fluid; and, conſequently, the weight of a body in air, is leſs than its real weight; but the difference of weight in vacuo and the open air is inconſiderable, unleſs the body be very large. The *S. g.* of water : *S. g.* of air :: 850 : 1 nearly, or as 1000 oz. : the weight of a cubical foot of air, which is therefore equal to $\frac{1000}{850}$ or $= 1.176$ avoird. oz.; and a cubical inch of air $= \frac{1.176}{1728} = .00117$ avoird. oz.; and the weight of a body in air is leſs than its real weight by a number of avoirdupoiſe ounces equal to 1.176 × number of cubical feet, or .00117 × number of cubical inches, in the body, which is very ſmall in metals and all heavy bodies, and in general may be neglected. This diminution of weight is different in different ſtates of the atmoſphere, and varies about $\frac{1}{10}$th part of the greateſt diminution.

96. PROP. *To find the ſpecific gravity of a body*, B, *by immerſing it in a fluid*, F, *ſpecifically lighter than itſelf.*

Unite another body *H* to *B*, which will make it ſink in the fluid, and the weight loſt by $\overline{B+H}$ — weight loſt by *H* alone, is equal to the weight loſt by *B*, and equal to the weight of a portion of the fluid whoſe magnitude is *B*. But (89) *S. g.* of *B* : *S. g.* of *F* :: *wt.* of *B* : *wt.* of $\overline{B+H}$ — *wt.* of *H*, ſuppoſing them to be weighed in the fluid. Otherwiſe, let *B* be detained in the fluid, and kept in equilibrio by a weight, *w*, placed in the oppoſite ſcale of a compound balance, and *w* is the difference between the weights of *B*, and an equal portion of the fluid, and conſequently the weight of

a portion of F, equal to B, is equal to $w +$ *wt.* of B and *S. g.* of F; *S. g.* of B :: $w +$ *wt.* of B : *wt.* of B. Q. E. I.

97. Cor. 1. If the fluid be water and its ſpecific gravity $= 1$, then *S. g.* of $B = \frac{\textit{wt. of } B}{\textit{wt. of } B + H - \textit{wt. of } H}$. Let a piece of wood, ſpecifically lighter than water, be 220 grains, and let it be annexed to a piece of tin whoſe weight is 160 grains, and loſs of weight in water 17 grains, and the compound will loſe in water 334 grains, and conſequently the *S. g.* of the wood is equal to $\frac{220}{334 - 17} = \frac{220}{317} = .694$. Or, by the ſecond method, let the weight of a piece of dry elm in air be 36 grains, and the weight required to detain it under water be 24 grains, and conſequently *S. g.* of the elm : *S. g.* of water :: 36 : 60, and *S. g.* of the elm $= \frac{36}{60} = .6$.

98. Cor. 2. If B be a powder, or a ſubſtance ſoluble in water, let it be covered with wax, whoſe magnitude is H and its loſs of weight previouſly found; and the ſpecific gravity of B will be known by the analogy in this propoſition.

99. Prop. *The ſpecific gravities of two bodies* B *and* b, *and of the ſubſtance compounded of them,* C, *being given, to find the ratio of the magnitudes of* B *and* b, *ſuppoſing the magnitudes of* B *and* b *to be equal to that of* C.

The weight of B, b, and C, being reſpectively equal to their magnitudes multiplied into their ſpecific gravities (75), and the weight of B and b being equal to that of C;

$B \times$

$B \times$ S.g. of $B + b \times$ S.g. of $b = C \times$ S.g. of $C = \overline{B+b} \times$ S.g. of C,
and $B \times$ S.g. of $\overline{B-C} = b \times$ S.g. of $\overline{C-b}$,
and $B : b ::$ S.g. of $\overline{C-b}$: S.g. of $\overline{B-C}$. Q.E.I.

100. Cor. 1. Since $B : b ::$ S.g. of $\overline{C-b}$: S.g. of $\overline{B-C}$,
$B : B + b\,(C) ::$ S.g. of $\overline{C-b}$: S.g. of $\overline{B-b}$, and
$b : B + b\,(C) ::$ S.g. of $\overline{B-C}$: S.g. of $\overline{B-b}$. Q.E.I.

101. Cor. 2. Let the ſpecific gravity of gold be 19, that of ſilver 11, and that of their mixture 16; then
magn. of gold : magn. of ſilver :: 16—11 : 19—16 :: 5 : 3, and
wt. of gold : wt. of ſilver :: $5 \times 19 : 3 \times 11 :: 95 : 33$.

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102. Let it be remembered, that this propoſition, and the analogies derived from it, are only applicable when the magnitudes of the ingredients are equal to that of the ſubſtances compounded of them, which is ſeldom true. But the ſpecific gravities of other compounds may be found by the chemical experiments of Kirwan, or by other methods.

103. PROP. *The weight of a body* P *and its ſpecific gravity* p, *together with* q, *the ſpecific gravity of another body* Q, *being given; to find what weight of* Q, *united with* P, *will make their ſpecific gravity equal to that of any fluid* W.

The $\frac{wt.\text{ of } P}{p} + \frac{wt.\text{ of } Q}{q} = P + Q =$ the magnitude of a portion of the fluid whoſe weight is equal to that of both bodies. Therefore the *wt.* of $\overline{P+Q} = \overline{P+Q} \times w$ (the ſpecific gravity of W) =

W) = *wt.* of $P \times qw + $ *wt.* of $Q \times pw$, and *wt.* of $P \times \overline{qw - pq} =$ *wt.* of $Q \times \overline{pq - pw}$, and *wt.* of $Q = \frac{\text{wt. of } P \times \overline{qw - pq}}{pq - pw}$.

104. Cor. If the ſpecific gravities of P, w, and Q be reſpectively $1\frac{1}{9}$, 1, .25, and $P = 158$ lb., then the weight of $Q = \frac{\text{wt. of } P \times \overline{qw - pq}}{pq - pw} = \frac{150 \times \overline{.25 - \frac{10}{9} \times .25}}{\frac{10}{9} \times .25 - \frac{10}{9}} = 5$ lb.

TABLE OF SPECIFIC GRAVITIES.

Refined gold	19.640
Engliſh guinea	18.888
Mercury	14.019
Lead	11.344
Refined ſilver	11.091
Biſmuth	9.700
Copper from Japan	9.000
Copper from Sweden	8.843
Hammered braſs	8.349
Caſt braſs	8.100
Turbeth mineral	8.235
Cinnabar, factitious	8.200
Cinnabar, natural	7.300
Elaſtic ſteel	7.820
Soft ſteel	7.738
Iron	7.645
Pure tin	7.471
Glaſs of antimony	5.280
A pſeudo topaz	4.270
A diamond	3.400
Chryſtal glaſs	3.150
Iſland chryſtal	2.720
Rock chryſtal	2.650

Common

Common glaſs	2.620
Fine marble	2.704
Stone of a mean gravity	2.500
Selenitis	2.252
Sal gemmæ	2.143
Brick	2.000
Nitre	1.900
Alabaſter	1.875
Dry ivory	1.825
Brimſtone	1.800
Dantzick vitriol	1.715
Allum	1.714
Borax	1.714
Calculus humanus	1.700
Oil of vitriol	1.700
Oil of tartar	1.550
Bezoar	1.500
Honey	1.450
Gum arabic	1.375
Spirit of nitre	1.315
Aqua fortis	1.300
Pitch	1.150
Spirit of ſalt	1.130
Craſſamen of the human blood	1.126
Spirit of urine	1.120
Human blood	1.054
Amber	1.040
Serum of human blood	1.030
Milk	1.030
Urine	1.030
Dry box-wood	1.030
Sea-water	1.030
Common water	1.000
Camphire	0.996
Bees-wax	0.955
Linſeed oil	0.932

Dry oak	0.925
Oil olive	0.913
Spirit of turpentine	0.874
Rectified ſpirit of wine	0.866
Dry aſh	0.800
Dry maple	0.755
Dry elm	0.600
Dry fir	0.550
Cork	0.240
Air	0.001$\frac{5}{8}$

This table exhibits the ſpecific weights of the various ſubſtances contained in it, diſcovered by ſome of the methods already deſcribed, and the abſolute weight of a cubic foot of each body is aſcertained in avoirdupoiſe ounces by multiplying the number oppoſite to it into 1000; as, for example, *S. g.* of water : *S. g.* of mercury :: 1 : 14.019 :: 1000 oz. : *wt.* of a cubical foot of mercury, which is therefore equal to 1000 × 14.109 avoirdupoiſe ounces. There are ſome uncertainties in this ſubject, for ſubſtances of the ſame kind, though denominated by the ſame name, may not be preciſely ſimilar, and ſome ſmall errors may perhaps be inevitable in phyſical experiments: but they will be inconſiderable, if the ſcales be nicely adjuſted, and the experiments cautiouſly conducted, ſo that the body weighed do not touch the bottom or ſides of the veſſel, nor riſe above the ſurface of the fluid, nor bubbles of air adhere to its ſurface. There is another cauſe of uncertainty: for moſt ſubſtances are dilated by heat and contracted by cold, and the dimenſions of the ſame body, and conſequently its ſpecific gravity, are different according to the different temperatures of the ambient air; and the altitude of the thermometer ought to be conſidered in conſtructing a table of ſpecific gravities. The different expanſion of bodies in ſummer and winter, and conſequently their different ſpecific gravities, appear from the experiments of Homberg, and Eiſenſchmedites in his Diſquiſitio nova de Ponderibus, &c.; from the latter of whom the following

lowing table, exhibiting the weight of a cubical inch, Paris meaſure, of different ſubſtances, is taken *.

A TABLE OF THE SPECIFIC GRAVITIES OF SEVERAL FLUIDS.

A cubic inch Paris meaſure.	In ſummer. oz.	dr.	gr.	In winter. oz.	dr.	gr.
Of Mercury -	7	1	66	7	2	14
Oil of vitriol -		7	59		7	71
Spirit of vitriol		5	33		5	38
Spirit of nitre		6	24		6	44
Spirit of ſalt -		5	49		5	55
Aqua fortis -		6	23		6	35
Vinegar -		5	15		5	21
Diſtilled vinegar		5	11		5	15
Burgundy wine		4	67		4	75
Spirit of wine		4	32		4	42
Pale ale -		5	1		5	9
Brown ale -		5	2		5	7
Cow's milk -		5	20		5	25
Goat's milk -		5	24		5	28
Urine -		5	14		5	19
Spirit of urine		5	45		5	53
Spirit of tartar		7	27		7	43
Oil of olives -		4	53	frozen in winter.		
Oil of turpentine		4	39		4	46
Sea water -		6	12		6	18
River water -		5	10		5	13
Spring water -		5	11		5	14
Diſtilled water		5	8		5	11

It appears from this table, that the expanſions of different fluids are

* Dr. Richard Davies hath collected experiments made by the moſt accurate authors, ſhewing their diſagreement and the cauſes of their difference, and alſo the ſpecific gravities of ſeveral ſubſtances by his own obſervations, for which ſee Phil. Tranſ. No. 488. See alſo Muſſchenbroek Introd. ad Phil. Nat. Vol. I. pag. 290. &c.

are different, in the ſame changes of the temperature of the air; and it appears from obſervation, that ſubſtances, not fluid, are alſo, in ſimilar circumſtances, differently dilated; but the weight of given magnitudes, both of fluid and firm bodies, being diminiſhed by heat and encreaſed by cold, the variation of their ſpecific gravities is leſs, than if the dimenſions of one of them only had been variable.

105. PROP. *When a floating body is quieſcent, a right line, paſſing through the centers of gravity of the part extant and part immerſed, is perpendicular to the horizon.*

If the body be conceived to be divided into two parts by the ſurface of the fluid, the part extant endeavours to deſcend, and the part immerſed to aſcend, with equal forces, otherwiſe the body would be more or leſs immerſed. But the part extant endeavours to deſcend in a vertical line, paſſing through the center of gravity, and the part immerſed to aſcend in a vertical line, paſſing through its center of gravity; and theſe forces are not only equal but oppoſite, or theſe right lines muſt coincide, when the body is quieſcent. Q. E. D.

CHAP.

Plate IV.

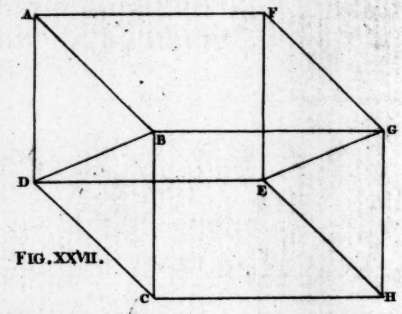


FIG. XXVII.



FIG. XXVIII.

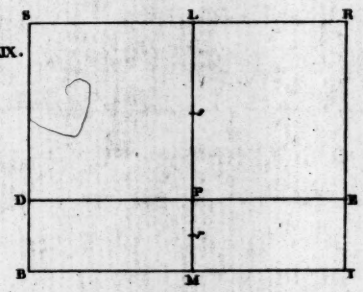


FIG. XXIX.

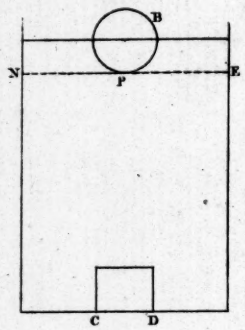


FIG. XXX.

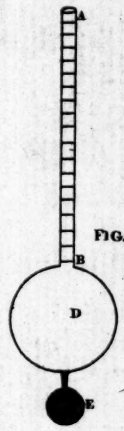


FIG. XXXI.



FIG. XXXII.

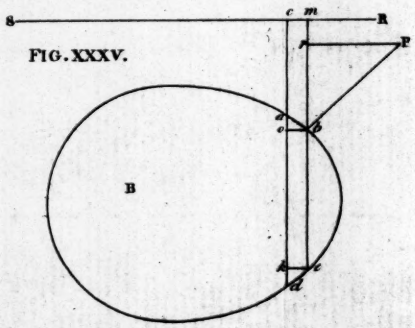


FIG. XXXV.

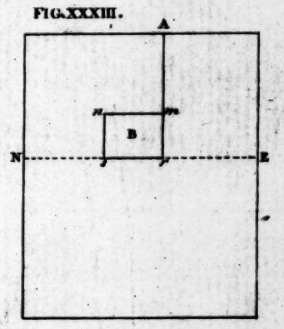


FIG. XXXIII.

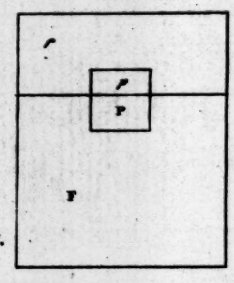


FIG. XXXIV.

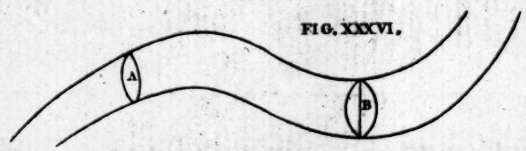


FIG. XXXVI.


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CHAP. VIII.

OF THE MOTION OF FLUIDS ARISING FROM THEIR GRAVITY*.

106. LEMMA. *IF a fluid flows through any tube and always fills it, or through any channel, and always is at the ſame height in the ſame place, the velocity in any part will vary inverſely as the area, in that part, of a ſection of the tube, or channel, perpendicular to the ſtream.*

DEM. Since the tube is always full, and the height of the fluid, in the channel, always the ſame, the quantities flowing through any ſections, perpendicular to the ſtream, or the area of any ſection (*A*) multiplied into the velocity in that ſection (*V*), muſt be the ſame, or $V \times A$ is as 1, and they are inverſely as each other. Q. E. D.

107. Cor. If the diſtances of the particles of the fluid be equal, the quantities of matter, or laminæ, of the fluid in different ſections (Q), are evidently as the areas of theſe ſections, or inverſely as the velocity, that Q is as $\frac{1}{V}$, and $Q \times V$, or the moment of the fluid, is a given quantity.

108. PROP. *The velocity* (V) *of a fluid iſſuing through a ſmall orifice* (O) *in the bottom of the veſſel* SRBM, *generated by the preſſure of the incumbent column, is equal to that which would be acquired by a body falling, from reſt, through a ſpace equal to a half of the perpendicular depth of the orifice.* PLATE V. FIG. XXXVII.

* Graves. L. II. Ch. VII. Muſſchen. Ch. XXIV. Atwood's Analyſis, p. 105. Emerſon's Mechan. Helſham, Lect. XIV. Newt. Princip. Prop. 36. Ferguſon's Lect. p. 70.

DEM. Let a column of the fluid, whose base is equal to the orifice, O, and height equal to AL, its perpendicular depth, descend from rest by its gravity for any time T'', and in the same time let the column ADC flow through the orifice with an uniform velocity equal to V; and, because equal moving forces generate equal quantities of motion in equal times, $O \times AL \times 32\,T''$ (or the quantity of motion generated in the descending column) $= O \times AD \times V$, or because AD varies as $V \times T''$ (V being uniform) $AL \times 32\,T'' = V^2 \times T''$ or $64 \times \frac{AL}{2} = V^2$, and the space, through which a body must descend to acquire a velocity equal to V, is equal to $\frac{AL}{2}$ (Mechan. 537.). Q. E. D.

109. Cor. 1. The velocity of the effluent fluid, through an aperture in the side of a vessel, is equal to that through one in the bottom, at the same perpendicular depth, because the generating forces are the same.

110. Cor. 2. The velocity of efflux, through apertures at different perpendicular depths, varies as the square roots of these depths.

FIG. XXXVIII.

111. Cor. 3. If the fluid be uniform, and its surface, SR, always at the same height, equal quantities will issue, in equal times, through any sections EN, AB, CD, GF, &c. perpendicular to the stream (106), and if their areas be known, and the quantity of fluid issuing out of the vessel in a given time, or $\mathcal{Q}$, be measured, the velocity may be discovered; being equal to $\frac{\mathcal{Q}}{EN} \cdot \frac{\mathcal{Q}}{AB} \cdot \frac{\mathcal{Q}}{CD} \cdot \frac{\mathcal{Q}}{O}$; for the quantity of fluid flowing out in a given time, or $\mathcal{Q}$, divided by the area of any section, is evidently equal to a right line, described uniformly in that time, by the velocity of the fluid in that

that ſection. The velocity of efflux, at the orifice, is found, by this method, to be nearly, though not preciſely in all caſes, equal to that acquired in falling, from reſt, through half of its perpendicular depth.

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112. Different theoretical demonſtrations have been given of this propoſition, and different concluſions apparently eſtabliſhed by them, ariſing from the admiſſion of data, whoſe exiſtence in fact is doubtful, or at leaſt not aſcertained with certainty, or whoſe efficacy cannot be clearly inveſtigated; and, although I have given a theoretic demonſtration, experiments, only, ſeem to me to furniſh ſatisfactory concluſions. Suppoſing the column of fluid, perpendicularly incumbent upon the orifice, to be divided into indefinitely thin laminæ, parallel to the horizon, it is evident, that the preſſure upon each lamina is different, and proportional to the number of laminæ above it, and the preſſure upon the lamina, contiguous to the orifice, equal to the weight of the whole column; but it is difficult to determine the preciſe effect of the preſſure, and how much the velocity communicated by it, is leſs than that communicated by gravity, in deſcending from reſt through the ſame perpendicular height, for the following reaſons. 1. The preſſure of the incumbent column upon the lamina of fluid at the orifice, is not tranſmitted inſtantaneouſly nor its effect always the ſame, but this effect ſeems to be greater or leſs, as the area of the orifice is leſs or greater; for if the bottom of a veſſel were ſuddenly removed, the whole body of fluid contained in it would deſcend with the ſame velocity, and the preſſure upon the loweſt lamina would produce no effect; and it ſeems to follow, that in eſtimating the velocity, communicated by the incumbent preſſure, the magnitude of the aperture ought to be conſidered; and, however ſmall it may be, the velocity of efflux will always be leſs than that acquired in falling through the whole altitude. 2. The particles of many fluids adhere together, and as the initial velocity of the loweſt lamina is greater than that of the next contiguous to it, the preſ-

ſure upon it being greater, it will be impeded by its adheſion, and the motion of this will alſo be retarded by the adheſion and ſlower motion of the next, &c.; and this retardation will evidently encreaſe with the force of coheſion. 3. The fluid does not deſcend in a vertical column; for, let EN be a ſection of the fluid parallel to the ſurface, and each particle in it, being equally preſſed downwards, will deſcend with the ſame velocity, in a direction perpendicular to the horizon; but when arrived near to the bottom of the veſſel, it is found to deviate from its vertical deſcent, as is deſcribed in the figure, towards the center of the column, and meeting the particles deſcending perpendicularly, their motion muſt be affected by this colliſion. By this alſo, the dimenſions of the column are diminiſhed, and are the leaſt at about the diſtance of a diameter of the orifice below it, where a ſection of it is called the *vena contracta*.

113. PROP. *The velocity of the effluent fluid, in the vena contracta, is equal to that acquired by a body in falling, from reſt, through a ſpace equal to the perpendicular depth of the orifice* (H).

It is collected from experiments, that the areas of the vena contracta and orifice are to each other as $1 : \sqrt{2}$; and ſince the velocities are inverſely as theſe areas, and that in the orifice equal to that acquired in falling through $\frac{H}{2}$, the velocity in the vena contracta is equal to that acquired in falling, from reſt, through H. Otherwiſe: if a bent tube, whoſe mouth is turned upwards, be inſerted in the ſide of a veſſel, the fluid is obſerved to riſe to the level of the ſurface of the fluid. Or, ſuppoſing AB to be the ſide of a veſſel perpendicular to the horizontal line, BC; AO the perpendicular depth of the orifice, and BC the diſtance to which the fluid ſpouts; becauſe each particle deſcribes a parabola (Mechanics) and BC is found, by experiment, to be equal to $\sqrt{4AO \times BO}$; AO is equal to the ſpace through which a body muſt fall, to acquire

FIG. XXXIX.

quire a velocity that will carry it to C, or equal to that of efflux. Q.E.D.

114. Cor. 1. The velocities of fluids iſſuing out of orifices, at different depths, both in the orifice and vena contracta, vary as each other and as the ſquare roots of the perpendicular depths of the orifices.

115. Cor. 2. Suppoſing O, V, T, Q to repreſent the area of the orifice, velocity of efflux, time, and quantity flowing out in that time, reſpectively; Q will vary as $O \times V \times T$ or as $O \times T \times \sqrt{H}$, and, when T is given, as $O \times \sqrt{H}$.

116. Cor. 3. Suppoſing a body to fall from reſt through s feet in 1″; the time of falling through $\frac{H}{2}$ feet $= \sqrt{\frac{H''}{2s}}$; and, in this time, a column of fluid iſſues through O, equal to $O \times H$; and $Q : O \times H :: T'' : \sqrt{\frac{H''}{2s}}$, and $Q = \frac{O \times H \times T''}{\sqrt{\frac{H}{2s}}} = O \times T'' \times \sqrt{2sH}$;

$T'' = \frac{Q}{O \times \sqrt{2sH}}$: when all the fluid, contained at firſt in the veſſel, is diſcharged, and the veſſel is priſmatic or cylindrical, $T'' = \frac{B \times H}{O \times \sqrt{2sH}} = \frac{B}{O} \times \sqrt{\frac{H}{2s}}$; and $O = \frac{B}{T''} \times \sqrt{\frac{H}{2s}}$, ſuppoſing B to be the area of the bottom.

117. PROP. *When a fluid ſpouts horizontally, through an aperture,* O, *in the ſide of a veſſel, the amplitude is equal to twice the right ſine of a circular arc, whoſe diameter is the perpendicular diſtance of the ſurface from the horizon, and verſed ſine its diſtance from the orifice.* FIG. XXXIX.

DEM.

FIG. XXXIX. DEM. Let AB be the ſide of the veſſel, A the ſurface of the fluid, O the aperture, and BC the horizontal diſtance or amplitude: and BC^2 (ex conicis) = the product of the parameter and abſciſſa $= 4AO \times BO$ and $BC = \sqrt{4AO \times BO} = 2OD$. Q.E.D.

118. Cor. 1. BC is the greateſt poſſible, when O is in the biſection of AB and it is then equal to AB.

119. Cor. 2. If the ſide of the jet be enclined, in any angle, to the horizon, and the direction, and velocity of the effluent fluid be known, the amplitude, altitude, and time of flight may be diſcovered by the rules inveſtigated in projectiles.

FIG. XL. 120. PROB. *The velocity of efflux, through a ſmall orifice, O, in a cylindric or priſmatic veſſel, is uniformly retarded.*

DEM. Let a body be projected perpendicularly upwards, from O, with a velocity equal to that acquired in falling through a ſpace PO, equal to the perpendicular depth of O, and the velocities, deſtroyed by gravity when the body arrives at any points D and E, vary as the ſquare roots of DO and EO, or as the velocities of efflux, when the ſurface is at D and E (114); but the velocities, deſtroyed by gravity in the aſcending body, encreaſe uniformly, and conſequently the velocity of efflux decreaſes uniformly. Q.E.D.

Otherwiſe: The decrement of velocity of efflux, or $\dot{V}$, varies as the force which generates it (F) multiplied into the time ($\dot{T}$), or $V \times \dot{V}$ is as $F \times EO$, and that $\dot{V}^2$ may be as EO, F muſt be conſtant, and conſequently $\dot{V}$ uniformly retarded. Or (from 116) $T'' = \frac{B}{O} \times \sqrt{\frac{H}{25}}$, and if T be taken in arithmetic progreſſion, $\sqrt{H}$, or

$\sqrt{H}$, or the velocity (114) will be in arithmetic progreſſion, the other quantity $\frac{B}{O} \times \frac{1}{\sqrt{2g}}$ being conſtant.

121. Cor. 1. The velocity of efflux, is to the velocity with which the ſurface deſcends, as the area of the ſurface to O; and conſequently the ſurface deſcends with an uniformly retarded motion, and the ſpaces, deſcribed by it in equal times, are as the odd numbers 1, 3, 5, &c. taken backwards.

122. Cor. 2. If therefore the ſides of a cylindric, or priſmatic, veſſel, be divided into portions MN, $N2$, $2T$, &c. continued to the ſurface of the fluid, which are as the odd numbers 1, 3, 5, 7, &c. a clepſydra would be formed; for the ſurface SR would deſcend through theſe diviſions in equal times.

123. Cor. 3. The quantities of fluid diſcharged in equal times, being as O multiplied into the velocity at O, or as the velocity of the ſurface (cor. 1.) are uniformly diminiſhed.

124. Prop. *The quantity of fluid, diſcharged from a ſmall orifice,* O, *in a cylindrical or priſmatic veſſel, that empties itſelf, is equal to half of the quantity, which would be diſcharged in the ſame time, were the altitude of the fluid to continue the ſame.*

Dem. Let the time, T, be divided into n ſmall equal parts, each of which is unity, and the quantity diſcharged in the time, T, when the firſt velocity, V, continues the ſame, is equal to $O \times V \times T$ (115) or to $O \times V \times n$. But when the veſſel empties itſelf, the velocities, and quantities diſcharged in times, each of which $= 1$, decreaſe uniformly to nothing; and, from the nature of an arithmetic ſeries, the whole quantity is equal to the firſt term multiplied

plied into half of the number of terms, or equal to $O \times V \times \frac{n}{2}$.

Otherwiſe: The quantities diſcharged in equal, ſmall particles of time, when the velocities are uniform and uniformly retarded, are to each other as the velocities (the orifice being the ſame), or as the ſpaces deſcribed uniformly by theſe velocities; and the whole quantities are as the ſpaces deſcribed in the ſame time by a velocity equal to V, and velocities beginning at V, and uniformly retarded till they vaniſh, or as 2 : 1. Q.E.D.

125. Cor. 1. The time, therefore, in which a cylindric or priſmatic veſſel is emptied through a given orifice, is equal to twice the time, in which the ſame quantity of fluid would be diſcharged with the firſt velocity continued uniformly.

FIG. XLI. 126. Cor. 2. The time, in which the fluid, contained in the veſſel $SBMR$, would be diſcharged, if the firſt velocity were continued uniformly, is equal to $\frac{B}{O} \times \sqrt{\frac{2PO''}{2s}}$; and twice this time, or the time in which the veſſel would be emptied, is conſequently equal to $\frac{B}{O} \times \sqrt{\frac{2PO''}{s}}$. But the time of falling down $2PO = \sqrt{\frac{2PO''}{s}}$; and the time therefore in which the veſſel is emptied, is to the time in which a body falls down $2PO :: B : O$.

127. Cor. 3. The times in which SM, and DM, are emptied, being equal to $\frac{B}{O} \times \sqrt{\frac{2PO''}{s}}$, and $\frac{B}{O} \times \sqrt{\frac{2QO''}{s}}$ reſpectively, the time in which the ſurface SR deſcends to any other poſition, DE is equal to $\frac{B}{O} \times \sqrt{\frac{2PO}{s}} - \sqrt{\frac{2QO}{s}}$, and conſequently varies as $\sqrt{PO} -$

$\sqrt{PO} - \sqrt{QO}$. If therefore the ſide of the veſſel be divided in E, F, G, &c. ſo that $\sqrt{MR} - \sqrt{ME} = \sqrt{ME} - \sqrt{MF} = \sqrt{MF} - \sqrt{MG}$, &c., a clepſydra or hour glaſs would be formed; for the ſurface would deſcend from R to E, from E to F, from F to G, &c. in equal times.

128. PROP. *To find the time,* T, *which elapſes, whilſt the ſurface of a fluid,* SR, *contained in any irregular veſſel, and flowing through a ſmall orifice,* O, *deſcends to any poſition,* DE.

Let $H =$ the greateſt altitude or PO; $x =$ any other altitude QO; and $A =$ the area of a ſection of the fluid, parallel to the ſurface. The velocity of efflux, when the ſurface is at AB or $\sqrt{2sx}$: velocity of the ſurface $AB :: A : O$, and the velocity of $AB = \sqrt{2sx} \times \frac{O}{A}$; but $\dot{T}$ is equal to $\frac{-\dot{x}}{\text{velocity}} = \frac{A}{O} \times \frac{-\dot{x}}{\sqrt{2sx}}$, and if A be given in terms of x, the time, or T, may be found. Q. E. I.

129. Cor. 1. If the veſſel SOR be cylindrical or priſmatic, A is conſtant; and ſince $\dot{T} = \frac{A}{O \times \sqrt{2s}} \times -\dot{x}\, x^{-\frac{1}{2}}$, $T = C$, the correction, $+ \frac{A}{O \times \sqrt{2s}} \times -2x^{\frac{1}{2}} =$ (if H be equal to PO) $\frac{A}{O \times \sqrt{s}} \times \sqrt{2H} - \sqrt{2x}$. Let the fluid run till the veſſel be empty, and, $\dot{x}$ being equal to nothing, $T = \frac{A}{O} \times \sqrt{\frac{2H''}{s}}$; but the time of falling, from reſt, through $2H$ is equal to $\sqrt{\frac{2H''}{s}}$, and conſequently theſe times are to each other as $A : O$, the ſame analogy as in (art. 126). If the firſt velocity be continued uniform, the quantity of fluid diſcharged in the time, T, is equal to $T \times O \times \sqrt{2sH}$ (116) = (ſubſtituting T's value in this cor.) $\frac{A \times O \times \sqrt{4sH^2}}{O \times \sqrt{s}} = A \times 2H$, the ſame as in art. 124.

130. Cor. 2. If the veſſel be a paraboloid, whoſe axis is vertical, and the orifice be in the vertex of the axis, which is the loweſt point; A is circular and equal to $x \times$ parameter belonging to the axis $= x \times p$, and $\dot{T} = \frac{p\dot{x}}{O \times \sqrt{2s}} \times -\dot{x}x^{-\frac{1}{2}}$ and $T = \frac{p}{O \times \sqrt{2s}} \times \overline{\frac{2}{3}H^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}} =$ (when the veſſel is empty) $\frac{p}{O \times \sqrt{2s}} \times \frac{2}{3}H^{\frac{3}{2}}$.

131. Cor. 3. If the paraboloid be inverted, and the aperture be in the middle of the baſe, which is horizontal; $\dot{T} = \frac{p}{O \times \sqrt{2s}} \times \frac{\overline{H-x} \times \dot{x}}{\sqrt{x}} = \frac{p}{O \times \sqrt{2s}} \times \overline{H\dot{x}x^{-\frac{1}{2}} - \dot{x}x^{\frac{1}{2}}}$, and $T = \frac{p}{O \times \sqrt{2s}} \times \overline{2Hx^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}}}$. If the fluid run till the veſſel be empty, or till x be equal to H, $T = 2H^{\frac{3}{2}} - \frac{2}{3}H^{\frac{3}{2}}$; and the times in this, and the laſt, cor. are to each other as $\overline{2 - \frac{2}{3}} \times H^{\frac{3}{2}} : \frac{2}{3}H^{\frac{3}{2}} :: 2 : 1$. By a ſimilar proceſs may be computed the times, in which cones and other figures, formed by revolving about an axis, are emptied.

SCHOLIUM.

132. The preceding propoſitions are only applicable to the efflux of fluids, through very ſmall apertures, and if they were preciſely coincident with experiments, the laws of efflux through larger apertures could not be derived from them; for the coheſive force, and lateral motion, of fluids flowing through different orifices, are not the ſame; and it is not therefore allowable to conſider a large orifice, as compoſed of many ſmall ones, whoſe ſum is equal to its area. The deviation of theory from facts, appears from the following experiments*, which ſeem to have been made with great accuracy; but I would obſerve, that a law or principle in nature, and theoretic concluſions derived from it, cannot be inferred

* Theſe experiments were made by Mr. Vince, who had the goodneſs to communicate them, and permit their publication.

ferred to be falſe, becauſe they do not correſpond with facts; for theſe, ſometimes involving the menſuration of minute quantities, cannot be aſcertained with perfect truth, and any error will produce a proportionable diſagreement with the law, though its exiſtence and properties be inveſtigated by theſe and other ſimilar facts. The following experiments agree with the theory as nearly as could be expected.

Let the depth of a cylindrical veſſel be a, $n =$ the area of the orifice, $z =$ the area of the bottom, $m = 16\frac{1}{12}$ feet, $x =$ any ſpace through which the ſurface has deſcended from the commencement of the motion, and conſequently $a - x =$ the ſpace which it has to deſcribe, and let the velocity at that depth be ſuch as a body would acquire in falling down $a - x$ multiplied into ſome quantity s, which quantity will manifeſtly always remain the ſame provided the ſurface be uniformly retarded. Now if $t =$ the time correſponding to the deſcent of the ſurface through the ſpace x, we have by calculation $s = \frac{\overline{a^{\frac{1}{2}} - \overline{a - x}^{\frac{1}{2}}}^{2}}{m} \times \frac{z^2}{t^2 n^2}$, if the ſurface be uniformly retarded. Now the diameter of the cylinder uſed in all the experiments was 7.252 inches, and the depth of the fluid at the commencement of the motion was 8.1 inches, and the times of the deſcent of the ſurface through the obſerved ſpaces were meaſured by a well regulated pendulum.

EXP. 1. The diameter of the orifice was .1274 inches.

$x =$	$t =$	$s =$
1 in.	59″.14	.5123
2	120.43	.5289
3	186.43	.5407
4	259.71	.5439
5	344.57	.5397
6	443.16	.5405

EXP. 2. The diameter of the orifice was .3811 inches.

$x =$	$t =$	$s =$
1 in.	7″	.4567
2	14.33	.4663
3	22.25	.4743
4	30.71	.4856
5	40.79	.4808
6	52.25	.4854

EXP. 3. The diameter of the orifice was .5081 inches.

$x =$	$t =$	$s =$
2 in.	8″.75	.396
4	18.55	.4213
6	31.35	.4265

EXP. 4. The diameter of the orifice was .87 inches.

$x =$	$t =$	$s =$
3 in.	5″.35	.3016
6	11.55	.3658

From theſe and ſeveral other experiments of the ſame kind, it appears, that the velocity of the effluent water is in no *conſtant* ratio of any part of the depth, the value of s in all the experiments increaſing till the ſurface got within 3 or 4 inches of the bottom, when the motion began to be very irregular, owing probably to a circular motion which the fluid always acquired by that time; and when the hole was ſmall the fluid would ſtop running when its depth was not leſs than $\frac{1}{3}$th of an inch. Theſe reſults tend to ſhow how very imperfect all theory reſpecting theſe matters muſt be, and that the only way to arrive at any thing ſatisfactory muſt be from experiments. From thoſe which I have already made, I have not been able to deduce any general concluſion reſpecting the the law of velocity in different depths. In making the above experiments, I wiſhed alſo to try the effect of taking away a conſiderable

derable part of the lateral motion of the water at the orifice; for which purpoſe I took a hollow cylinder of the diameter of the orifice, leaving a narrow ſlit on one ſide, at the bottom, for the water to enter, when I found the time of emptying to be exactly the ſame as when there was no cylinder; the motion of the water therefore through the orifice, is not at all affected by the lateral motion of the water flowing in on all ſides in oppoſite directions, and at the ſame time it proves, that the horizontal is exactly equal to the perpendicular preſſure; for the preſſure of the water immediately over the orifice was taken off by ſtopping the cylinder above the ſlit †.

133. PROP. *Let a fluid aſcend in a bended cylindrical tube* PQRS, *whoſe legs* PQ *and* RS *are vertical and parallel, to find the time* (T) *of an oſcillation* *. FIG. XLIII.

Let the ſurface AB and CD be upon the ſame level, or the fluid be quieſcent; and if AB deſcend to EF, CD will be raiſed through an altitude DH equal to AE. Take IK upon the ſame level with EF and the fluid $EFQRK$ will be in equilibrio, and conſequently the column GK, being unſupported, will deſcend and protrude the quieſcent fluid. Let A = accelerating force, $HD = a$, P = periphery of a circle, whoſe radius is a, L = the length of the fluid's axis, O = a ſection of the fluid perpendicular to the axis. And becauſe A, or the increment of the velocity generated in a given time, is as the weight of GK directly, and inverſely as the

† For $\sqrt{m} : 2m :: \sqrt{s \times a-x}$: velocity at the orifice $= \sqrt{4ms \times a-x}$; hence $2 : n :: \sqrt{4ms \times a-x}$: velocity at the ſurface $= \frac{n \times \sqrt{4ms \times a-x}}{2}$; therefore $\dot{t} = \frac{2\dot{x}}{n\sqrt{4ms \times a-x}}$ $= \frac{2}{n\sqrt{4ms}} \times \overline{a-x}^{-\frac{1}{2}}\dot{x}$, whoſe fluent is $\frac{-2z}{n\sqrt{4ms}} \times \overline{a-x}^{\frac{1}{2}}$, but when $t=o$, $x=o$, therefore the correct fluent is $t = \frac{-2z}{n\sqrt{ms}} \times \overline{a-x}^{\frac{1}{2}} - a^{\frac{1}{2}}$, and conſequently $s = \frac{\overline{a^{\frac{1}{2}} - \overline{a-x}^{\frac{1}{2}}}^2}{m} \times \frac{z^2}{t^2 n^2}$.

* Newton Princip. Vol. II, Prop. XLII.

the quantity of matter to be moved, A is as $\frac{O \times HK}{O \times L}$, or as $\frac{2a}{L}$, and in any other position of the surface lm, A is as $\frac{mD}{L}$, or as the distance from CD. Let a pendulum, whose length is $\frac{L}{2}$, vibrate in a cycloid, and since the accelerating forces in the tube and cycloid, at distances from the surface CD, and the vertex, equal to $\frac{L}{2}$, are equal to the whole force of gravity, and in other points vary as their distances, they are equal at all equal distances from the surface CD and the vertex of the cycloid. The time in which the surface arrives at IK is equal to the time of a vibration in the cycloid, or (Mechan.611.) to $\frac{P}{a} \times \sqrt{\frac{\frac{L}{4}}{4s}}$, and is given. Q.E.I.

134. Cor. 1. If p be the periphery of a circle, whose diameter is 1, or $p = 3.14159$, then $P : p :: 2a : 1$, and $\frac{P}{a} = 2p$, and the time $T = p \times \sqrt{\frac{L''}{4s}}$.

135. Cor. 2. The time elapsed whilst the surface returns to its original situation, or two oscillations, $= 2p \times \sqrt{\frac{L}{4s}} = p \times \sqrt{\frac{L}{s}}$ = time in which a pendulum, whose length is $2L$, performs one vibration.

136. Cor. 3. When L, or the length of the axis of the fluid, varies; the time varies as $\sqrt{L}$.

137. Cor.

137. Cor. 4. The velocity of the ſurface GH is perpetually accelerated till it arrives at CD, where it is the greateſt, and is then retarded till it arrives at IK, where it vaniſhes. In the ſame tube the greateſt velocity is $\sqrt{\frac{4s}{\frac{L}{4}}} \times \frac{a}{2}$ (Mechan. 611.), and therefore varies as a; and in different tubes it varies as $\frac{a}{\sqrt{L}}$.

138. Cor. 5. If the length of PQ and RS be diminiſhed, and the whole length L continue the ſame, the horizontal part QR being encreaſed, T ſtill varies as $\sqrt{L}$, and V as $\frac{a}{\sqrt{L}}$. When the agitation is ſmall, the time in which a particle deſcends from the ſummit of one wave and arrives at the ſummit of the next, varies as the ſquare root of the diſtance of the ſummits, ſuppoſing the altitudes to be very ſmall compared with this diſtance.

139. DEF. A *river* is a ſtream, or current of water, deſcending by its own weight, from the more elevated to the lower parts of of the earth, in an open bed or channel. 2. It is ſaid to be in a permanent ſtate, or to flow uniformly, when the depth, in any place, always remains the ſame. 3. A ſection of a river is an imaginary plane, perpendicular to the bottom, or direction of the ſtream.

140. Cor. If the ſides of a river be plane ſurfaces, perpendicular to the bottom, and parallel to each other, and the bottom be alſo a plane ſurface, either horizontal or inclined to the horizon; a ſection will be a right angled parallelogram, and if the river be permanent, or flow uniformly, the ſame quantity of water will flow through every ſection, in equal times; and if V, B, D be the velocity,

velocity, breadth and depth respectively, $V \times B \times D$ will be a given quantity, and V will vary as $\frac{1}{B \times D}$.

FIG. XLIV. 141. PROP. *The velocity of any particle of water, in a river, is equal to that, which a body would acquire in falling, from rest, from the perpendicular distance of the surface of the water in the reservoir.*

Let BM be the bottom of the river, AP the path of a particle P, parallel to it, and since each particle descends by its own gravity, and is supposed to move freely and without obstruction, its velocity, at any point P, is equal to that which a body would acquire, in falling from rest through Pp, the perpendicular depth of the surface AE (Mech. 541). Q. E. D.

142. Cor. The velocity of the higher, is greater than the velocity of the lower, parts of a river, for the velocities of P and Q are to each other as $\sqrt{Pp} : \sqrt{Qp}$. If the breadth be the same, the depth perpetually decreases, and the velocities of the higher and lower parts approach to equality.

SCHOLIUM.

143. Rivers take their rise in mountains, or elevated grounds, and, descending from thence by their gravity, are received in beds or channels, often nearly horizontal, and their current is then only maintained by the velocity acquired in their descent, and their encreased depths. When the bed is become nearly horizontal, the velocity, acquired in descending, is perpetually diminished by obstacles and unevenes at the bottom, and irregular windings and turnings of the sides, against which the stream is impelled and its direction changed. And hence the velocity is greater

greater in the middle of the ſtream than near the ſides, and at ſome ſmall diſtance from the bottom, then contiguous to it. The velocity of the lower parts is encreaſed with the depth, or compreſſing force, which is communicated by coheſion to thoſe above; and hence one foot of declivity, to about one mile of horizontal diſtance, will preſerve a ſenſible current in deep, but not in ſhallow water *.

144. DEF. A *ſyphon* is a bended tube, as *ABC*, open at both ends, one of whoſe legs, *BC*, is uſually longer than the other *AB*. Immerſe the ſhorter leg in a fluid, and, ſucking out the air at *C*, it will aſcend up *AB* by the preſſure of the atmoſphere upon the ſurface *SR*, and fill the tube; or fill the tube with the fluid, and ſtopping the orifices till it be inverted and *A* immerſed, as before. The preſſure of the atmoſphere upon the orifices *A* and *C* is the ſame, and, ſuppoſing *OB* to be leſs than the ſtandard altitude, and *H* to be upon a level with *O*, the columns *OB* and *BH*, are ſupported by the preſſure of the atmoſphere, and in equilibrio; but *HC*, being unſupported, muſt deſcend by its own weight, and leaving a vacuum behind it, the preſſure of the atmoſphere at *A* will not be counteracted by any preſſure at *H*, and conſequently the column *BH* will deſcend with *HC*, and this motion be continued till the reſervoir *SOR* be empty. PLATE V. FIG. XLV.

145. Cor. 1. If the legs of the ſyphon be of the ſame length, it is clear that there will be no motion, the columns *OB* and *HB* being both ſupported by the weight of the atmoſphere.

146. Cor. 2. The phenomenon of *intermitting ſprings*, may be explained by this property of the ſyphon: Let *AEB* be a reſervoir, ſupplied with water by a ſtream at *B*; *E* the orifice of any covered channel whoſe higheſt point is *H*. The ſurface of the fluid *AB* FIG. XLVI.

* Leg. Guglielmini della Natura de Fiumi.

AB being upon the ſame level in the reſervoir and duct EH, there will be no efflux; but when A riſes higher in the reſervoir than H, the fluid will deſcend through HI, and ſuppoſing the efflux at I to be greater than the influx at B, it will continue to flow till the reſervoir be empty; and the motion will then be intermitted, till the reſervoir be again filled above the level of H, and then be emptied as before.

FIG. XLVII.

147. Cor. 3. Reciprocating ſprings may be explained from the ſame principle: Let the reſervoir FGH be ſupplied by a ſtream at F and by a ſyphon EBL, communicating with another reſervoir ACD. From the conſtant influx at F, a conſtant ſtream runs into the duct at G; and ſuppoſing the reſervoir ACD to be full of water, and to flow through the ſyphon EBL, the preſſure at G, and velocity of the ſtream in GI, will perpetually encreaſe till ACD be empty, and then decreaſe till it be full again, and flow through the ſyphon EBL, as before.

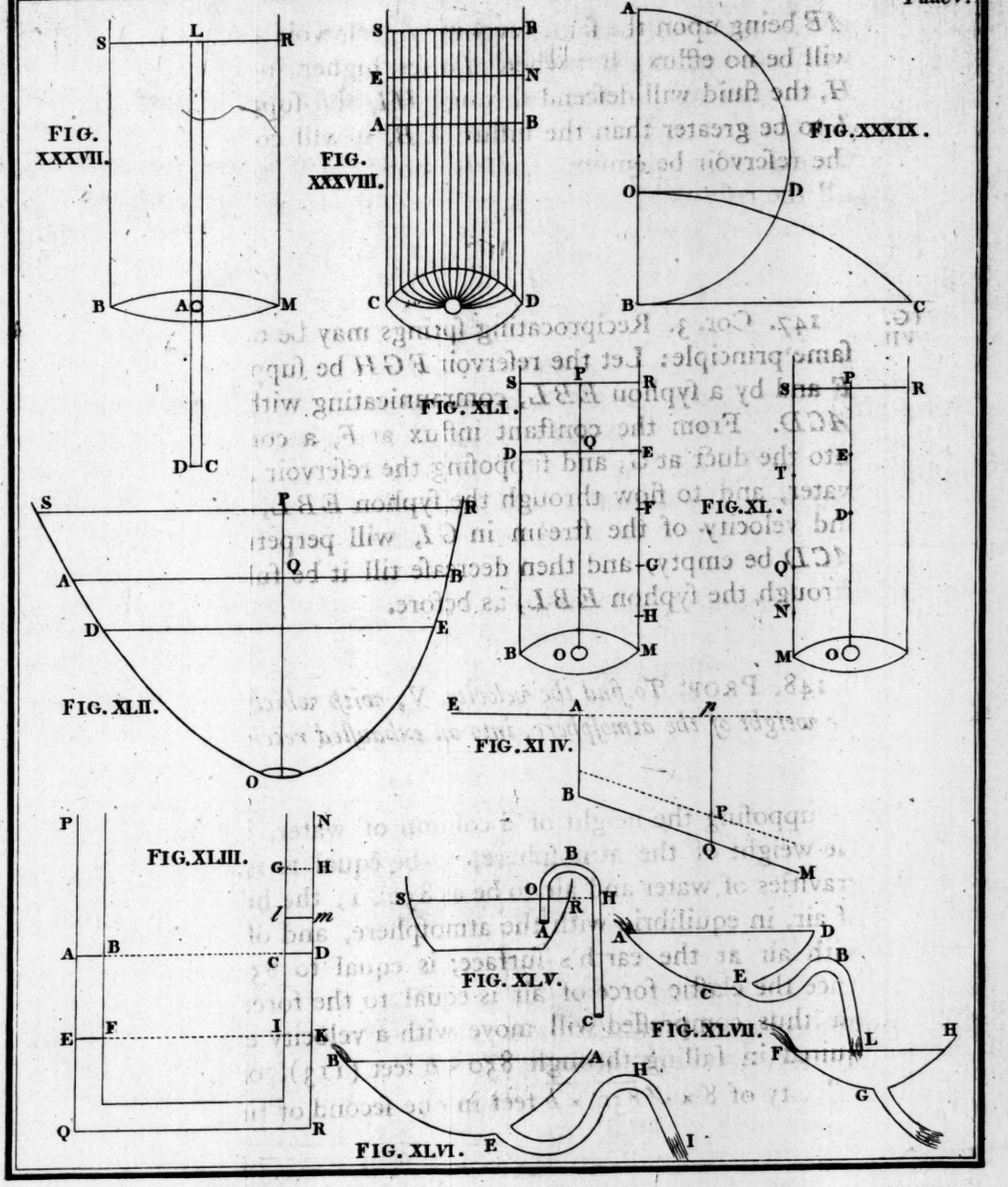
148. PROP. *To find the velocity, V, with which air is impelled, by the weight of the atmoſphere, into an exhauſted receiver.*

Suppoſing the height of a column of water, in equilibrio with the weight of the atmoſphere, to be equal to b, and the ſpecific gravities of water and air to be as 850 : 1; the height of a column of air, in equilibrio with the atmoſphere, and of the ſame denſity with air at the earth's ſurface, is equal to $850 \times b$ feet. But, ſince the elaſtic force of air is equal to the force of compreſſion, air thus compreſſed will move with a velocity equal to that acquired in falling through $850 \times b$ feet (113), or to an uniform velocity of $8 \times \sqrt{850 \times b}$ feet in one ſecond of time. Q. E. I.

149. Cor. 1. If $b = 32$ feet, the altitude of this homogeneous column of air is equal to $32 \times 850 = 27200$, and conſequently

$V = 8$

Plate V.


FIG. XXXVII.
S L R B A M D C
FIG. XXXVIII.
S B E N A B C D
FIG. XXXIX.
A O D B C
FIG. XLI.
S P R Q D E F G H B O M
FIG. XL.
S P R E T D Q N M O
FIG. XLII.
S P R A Q B D E O
FIG. XLIII.
P N G H l m A B C D E F I K Q R
FIG. XLIV.
E A n B P Q M
FIG. XLV.
S B O H R A
FIG. XLVI.
B A H E I
FIG. XLVII.
A D B E C F L H G


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$V = 8 \times \sqrt{27200} = 8 \times 164.9 = 1319.2$ feet. If a body therefore move with a greater velocity than 1319.2 feet in 1″, the ſpace deſerted by it, muſt for ſome time be a vacuum.

CHAP. IX.

OF PUMPS, BAROMETERS*, &c.

150. DEF. *A COMMON pump is a hollow cylindrical tube or barrel,* AB, *open at both ends, to whoſe cavity a piſton,* P, *having a valve that opens upwards, and moveable by means of a rod fixed to it, is ſo exactly adapted, that neither air nor water can paſs between it and the ſides of the barrel. If* C *be the loweſt point to which the piſton cau be depreſſed, and* D *the higheſt to which it can be raiſed,* DC *is called the ſtroke of the piſton. In the forcing pump the piſton is an entire cylinder without any perforation or valve, and a pipe,* EF, *is inſerted in the ſide, and communicates with the barrel by a valve opening towards* F. FIG. XLVIII. FIG. XLIX.

151. Cor. 1. If one end of a pump barrel be immerſed in a reſervoir of water, the ſurface of the water in the reſervoir and barrel will be upon the ſame level. Let a cylindrical plug, *L*, having a hole in it covered with a valve that opens upwards, be exactly adapted to the cavity of the barrel, and placed a little above *SR*; and raiſing the piſton to *D*, the air below will be expanded and rarefied (13), and the water will riſe by the external preſſure of the atmoſphere upon the ſurface *SR*, till its weight and the elaſticity of

* Cotes, pag. 71. Roh. P. I. Ch. XII. a. 17. 23. Comp. Syſt. P. II. pag. 85. Wells Mechan. pag. 72. Muſſ. Ch. XXXVI. Philoſ. Britan. pag. 43—55 & 123.

of the air above it be equal to the weight of the atmoſphere. If P be now ſufficiently depreſſed, the valve at L will be cloſed, and the air between P and L condenſed, and, when its elaſticity is greater than that of the atmoſphere, ruſh through the valve at P, which will be immediately ſhut by the weight of the external air. And, raiſing P again to D, the air below it will be farther rarefied, and aſcend through the valve at L; and by repeated ſtrokes of the piſton, the air below it will be exhauſted, and the water aſcend through the valve in P, and be diſcharged from an orifice in the ſide of the barrel, a little above D. It is evident, that, in the loweſt depreſſion of the piſton, if the elaſticity of the air below it be not greater than that of the external air, the valve at P will not be opened, nor conſequently can the water be raiſed any higher, but will be the greateſt when theſe elaſtic forces are equal.

152. Cor. 2. In the forcing pump, the air between P and L is expelled by depreſſing the piſton, through the valve at E; and when, in the loweſt depreſſion of the piſton, the elaſticity of the air below it is not greater than that of the external air, the valve at E will continue cloſed, and the water can be raiſed no higher. But when the water is raiſed above E, it is forced, by depreſſing the piſton, into the tube EF, through the valve at E, which is immediately cloſed, by the weight of the atmoſphere and water admitted.

153. Cor. 3. The greateſt altitude to which water can be raiſed, in the common pump, by a ſingle piſton, is when the column is in equilibrio with the weight of the atmoſphere; or, in this kingdom, when its altitude is between 32 and 36 feet.

154. PROP. *The greateſt altitude* (g) *and leaſt* (l) *of the piſton above the ſurface of the water in the reſervoir, being given; to find the aſcent of the water* (S s or y) *by one ſtroke.*

Sup-

Suppoſing the air below the piſton to be in its natural ſtate, when it is at the loweſt depreſſion, C, and when elevated to the higheſt point, D, the water to riſe to sr; the air which occupied the ſpace CR, will be uniformly expanded into the ſpace Dr, and then its elaſtic force (e) : elaſtic force of the external air (E) :: $Cr : Dr :: l : g-y$ :: a column of water whoſe altitude is $h-y$: a column whoſe altitude is h (ſuppoſing it to be equivalent to the weight of the atmoſphere upon the ſame baſe) :: $h-y : h$; and conſequently $h \times l = \overline{g-y} \times \overline{h-y} = gh - \overline{g-h} \times y + y^2$, and $y = \mp \sqrt{\left|\frac{g+h}{2}\right|^2 + \overline{l-g} \times h} + \frac{g+h}{2}$.

155. Cor. 1. Let $g = 16$ feet; $l = 12$; $g - l = 4$; $h = 32$; and $y = \mp \sqrt{\left|\frac{48}{2}\right|^2 - 128} + 24 = 2.834$ feet.

156. Cor. 2. If the piſton be depreſſed to C again, and the air below it be more elaſtic than natural air, the valve in the piſton will be opened, and the ſpace below filled with natural air; and being raiſed to D, the air will be rarefied as the water aſcends to ſome higher altitude GH. Let $SG = x$; E and e be the elaſticity of common air and of air expanded through DH, and $E : e :: DH : Cr :: g-x : l-y\,(L) :: h : h-x$, and $L \times h = \overline{g-x} \times \overline{h-x}$, and $x = \mp \sqrt{\left|\frac{g+h}{2}\right|^2 + \overline{L-g} \times h} + \frac{g+h}{2}$. Let $g = 16$; $l = 12$ and $L = 12 - 2.834 = 9.166$; $x = 5.1$, and the aſcent of the water from the ſecond ſtroke $= x - y = 5.1 - 2.834 = 2.566$. By a ſimilar proceſs may the aſcent be found from the 3d, 4th, &c. ſtroke of the piſton.

157. PROP. *Let* z *be the higheſt aſcent of the water,* g *and* l *the greateſt and leaſt heights of the piſton reſpectively; to find the value of* z.

When

When the piſton is at the loweſt depreſſion, the ſpace below it, which is filled with natural air, is equal to $l-z$, and when at the higheſt elevation, the ſpace is equal to $g-z$; but the elaſticity of air in its natural ſtate (E) : its elaſticity when thus expanded :: $g-z : l-z :: h : h-z$; and $\overline{g-z} \times \overline{h-z} = \overline{l-z} \times h$, and $z^2 - gz = \overline{l-g} \times h$ and $z = \pm\sqrt{\frac{g^2}{4} + \overline{l-g} \times h} + \frac{g}{2}$.

158. Cor. When $\overline{l-g} \times h = \frac{g^2}{4}$, $z = \frac{g}{2}$; and when $l-g$ is greater than $\frac{g^2}{128}$, the values of z are impoſſible, or no pump can raiſe water, unleſs the length of a ſtroke be greater than the ſquare of the greateſt altitude of the piſton divided by 128.

SCHOLIUM.

159. The phenomena of ſyringes, cupping-glaſſes, ſuction, reſpiration, &c. are explicable from the preſſure of the atmoſphere. The fluid into which the lower orifice of a ſyringe is inſerted, muſt riſe, from the preſſure upon the external ſurface, into the vacuum produced by drawing up the piſton. Suction by the mouth, placed under the ſurface of a fluid, or applied to a tube, is effected by an exertion of the muſcles uſed in reſpiration, and enlarging the capacity of the cheſt; for the air within, being expanded into a larger ſpace, becomes leſs elaſtic, and the external preſſure forces in the fluid to reſtore the equilibrium. In the ſame manner, when the cheſt is dilated, the equilibrium between the external and internal air is deſtroyed, and the firſt is conſequently forced in by the preſſure of the atmoſphere.

BAROMETERS.

160. DEF. *The common mercurial barometer, or weather glaſs, is a cylindrical glaſs tube, whoſe diameter is generally about* $\frac{1}{3}d$ *or* $\frac{1}{4}$*th of an inch in diameter, and length* 34 *inches, filled with prepared mercury; one end of the tube,* A, *is hermetically ſealed, and the open end,* B, *inſerted into a baſon of mercury. The tube and baſon are fixed to a frame of wood, and ſuſpended in a vertical ſituation. The height of the mercury in the tube above the ſurface of the mercury in the baſon, is called the ſtandard altitude; and the difference, between the greateſt and leaſt altitudes, is called the limit, or ſcale, of variation.* FIG. L.

161. Cor. 1. The mercury in the barometer tube will ſubſide, till the column be equivalent to the weight of the external air, upon the ſurface of the mercury in the baſon, and it is therefore a criterion to meaſure that weight, and chiefly directed to that purpoſe. In this kingdom, the ſtandard altitude fluctuates between 28 and 31 inches, and from hence it is juſtly inferred, that the greateſt, leaſt, and intermediate, weights of the atmoſphere upon a given baſe, are reſpectively equal to the weights of columns of mercury upon the the ſame baſe, whoſe vertical altitudes are 28, 31, inches, and ſome altitude contained between them.

162. Cor. 2. The ſtandard altitude ought to be the ſame, whatever be the diameter of the barometer tube; but when this diameter is very ſmall, the attraction of coheſion between the mercury and glaſs prevents a variation of altitude, which ought to be, and in larger tubes is, ſenſible from ſmall differences in the weight of the atmoſphere.

163. LEMMA.

163. LEMMA. If a given line L be divided into n equal parts, and $L \times \frac{n+m}{n}$ be alſo divided into n equal parts, each diviſion of L will be leſs than that of $L \times \frac{n+m}{n}$ by $L \times \frac{m}{n^2}$.

DEM. When L is divided into n equal parts, each part is equal to $L \times \frac{1}{n}$ or $L \times \frac{n}{n^2}$; and each part of $L \times \frac{n+m}{n}$, thus divided, is equal to $L \times \frac{\overline{n+m}}{n^2}$, which is greater than the former by $L \times \frac{m}{n^2}$. Q.E.D.

FIG. LI. 164. Cor. 1. If each inch of the ſcale of variation, AD, of a barometer tube, be divided into ten equal parts, marked with 1, 2, 3, &c. encreaſing upwards, and a Vernier or Nonius, LM, whoſe length is $\frac{11}{10}$ths of an inch, be divided into ten equal parts, marked with 1, 2, 3, &c. encreaſing downwards, and ſo placed as to ſlide along the graduated ſcale of the barometer, the altitude of the mercury in the tube, above the ſurface of that in the baſon, may be found, in inches and hundredth parts of an inch, by this proceſs. If the ſurface of the mercury in the tube do not coincide with a diviſion in the ſcale of variation, place the index of the Vernier M even with this ſurface, and obſerving where a diviſion of the vernier coincides with one in the ſcale; the figure in the Vernier will ſhew what hundredth parts of an inch are to be added to the tenths immediately below the index. Let, for inſtance, the ſurface of the mercury be between 7 and 8 tenths above 30 inches, and the index of the Vernier being placed even with it, and the figure 5 upon the Vernier being obſerved to coincide with a diviſion upon the ſcale, the altitude of the barometer will be 30 inches $\frac{7}{10}$ and $\frac{5}{100}$ of an inch: for each diviſion of the Vernier being greater than that of the ſcale by $\frac{1}{100}$ of an inch (lemma), and there being

being five diviſions, the whole muſt be $\frac{5}{100}$ of an inch above the number 7 in the ſcale; and the height of the mercury is therefore 30.75 inches.

165. Cor. 2. Whatever be the number of diviſions in the ſcale of variation, and in the Vernier, the height of the mercury in the barometer is eaſily diſcovered by a proceſs ſimilar to that in the laſt corollary.

SCHOLIUM.

166. There are ſeveral other kinds of barometers, of which it will be ſufficient to give a ſhort deſcription. 1. In the *portable* barometer, the lower part of the tube is bent upwards, and wider than the reſt of the tube; and in this recurvated part, the mercury is expoſed to the preſſure of the atmoſphere: or the mercury in the baſon, is contained in a flexible leathern bag, expoſed to the ſame preſſure. In this laſt, the mercury is forced into the tube ſo as to fill it, by a ſcrew fixed in the bottom of a wooden box containing the bag, leſt the motion of the mercury ſhould break the tube. 2. In the *diagonal* barometer, the ſcale of variation is FIG. LII. bent into the direction DR, making an obtuſe angle with the vertical part BD. The ſcale of variation is, by this barometer, encreaſed in the ratio of $DR : DA$; but this encreaſe does not compenſate for the friction and attraction of coheſion upon the lower ſide of DR. And when the angle RDA is greater than 45°, the inſtrument is rendered uſeleſs by the ſeparation of globules of mercury from the column. 3. The *wheel* barometer is a compound tube $SERBD$ open at D and cloſed at E, the diameter of the FIG. LIII. higheſt part, SER, being much greater than that of the reſt, and filled with mercury from D to SR, and above that vacuous. Upon the ſurface of the mercury in the recurved leg, there is an iron ball in equilibrio with another H, by a ſtring paſſing over a pully P. As the ball at D riſes and falls with the mercury, the ſtring turns the pully, and an index IN fixed to it, which points

to different parts of a graduated circle. It is clear, that by encreaſing the diameter of the circle, this contrivance will ſhew the minuteſt variations of the air, provided the friction be inconſi-
FIG. LIV. derable, which is ſeldom true. 4. The *pendent* barometer is compoſed of a tube of a very ſmall bore, a little conical or tapering, cloſed at the ſmaller orifice, *A*, and filled with prepared mercury from *A* to *B*, whoſe diſtance is equal to the greateſt altitude, or about 31 inches. Let the tube be ſuſpended vertically, and the mercury will ſubſide, and be quieſcent in that part, whoſe length is equal to the ſtandard altitude at that time; and ſuppoſing that to be the leaſt, it will occupy a ſpace *FE* equal to 28 inches; and conſequently *AF* is the ſcale of variation. If *AE* = 60 inches, then *AF* = 32, when in the common barometer it is only 3 inches. The diameter of this barometer tube is very ſmall, and conſequently the attraction of cohefion conſiderable, which prevents the freedom of motion neceſſary to aſcertain minute variations of the air's preſ-
FIG. LV. ſure. 5. In the *horizontal rectangular* barometer, the higheſt part of the tube, oppoſite to the ſcale of variation, is wider than the reſt of the tube; and the mercury, deſcending 3 inches from *A* to *D*, will deſcribe a much longer ſpace in the horizontal leg *FG*, theſe ſpaces being to each other inverſely as the ſquares of the diameters of the tubes, and, that of *FG* being very ſmall, its motion will be extremely ſenſible. But the free motion of the mercury in *FG* is impeded by friction, and the attraction of coheſion, which from the ſmallneſs of the tube is conſiderable; and beſides this, globules of mercury are apt to be ſeparated from *M*, and flow out at *G*. By the above, and other expedients, as uſing water, or water and mercury, the ſcale of variation is enlarged; but the common barometer is the beſt, being ſubject to the feweſt inconveniences. In the conſtruction and uſe of it, the following particulars are to be obſerved: 1. The diameter of the tube ſhould be $\frac{1}{3}$d or $\frac{1}{4}$th of an inch, to prevent the effects of the attraction of coheſion; the length of the tube 33 or 34 inches with a bulb upon the top, into which the air may be diffuſed, ſhould any remain in the mercury. 2. The diameter of the ciſtern containing the mercury, ſhould be large

(at

(at leaſt ten times greater than that of the tube) that the addition or ſubtraction of the mercury, contained between the greateſt and leaſt altitudes, may not ſenſibly affect its depth; for the numbers, marked upon the ſide of the tube, ſhew their diſtance from a fixed point, and cannot ſhew the height of the column above the mercury in the ciſtern, unleſs its ſurface coincide with this point, and be immoveable. 3. The mercury ſhould be free from any mixture of other metals, and purged of air by being boiled in a glazed earthen veſſel, cloſely covered, and poured, when hot, through a glaſs funnel, with a long capillary, into the barometer tube waſhed with a rectified ſpirit, and cleaned with a piſton of ſhammy leather, if both ends were not hermetically ſealed when it was made, and heated and rendered electrical by rubbing. 4. Unleſs the temperature of the air remain the ſame, the dimenſions of a given quantity of mercury will be variable, and the altitude of the mercury is an uncertain meaſure of the weight of the atmoſphere, becauſe it is dilated by heat, and contracted by cold, when perhaps the weight of the atmoſphere is unaltered. If very great exactneſs be therefore required, the difference of temperature, at the different times of obſervation, and the depreſſion or elevation of the mercury produced by it, muſt be aſcertained, before the height of the column, raiſed by the weight of the atmoſphere, can be diſcovered.

CHAP. X.

ELASTICITY OF FLUIDS*.

167. PROP. *IF an elaſtic fluid be quieſcent, and compoſed of particles, equally repulſive at equal diſtances, and, at unequal diſtances, repelling each other according to any law of the diſtance, its denſity will be uniform.*

DEM. If the diſtances of any two particles, from an intermediate particle, be unequal, their repulſive forces muſt be unequal and motion enſue, which is contrary to the hypotheſis; and conſequently the fluid muſt be uniformly denſe. Q. E. D.

168. Cor. If any portion of an elaſtic fluid be uniformly denſe, and equally compreſſed on all ſides, it muſt be quieſcent.

FIG. LVI. 169. PROP. *If the component particles of an uniform cubical maſs of a fluid, repel each other with forces varying according to any inverſe, or direct, leſs than the direct duplicate, ratio of their diſtances, the fluid will be elaſtic.*

DEM. The whole repulſive force of any ſurface of the fluid, DE, is as the number of particles in DE and force of each, or as the number in DC, multiplied into the number in DG, multiplied into the force of each particle; or, ſubſtituting R for the whole repulſive force of the ſurface DE, I for the diſtance of two contiguous particles, F for the force with which they repel each other; R will

* Muſſc. Art. 2163. Newt. Princ. Vol. II. Sect. V. Helſham, Lect. XV. Philoſ. Britan. Cotes, Lect. XIV. Cyclop. Art. Air.

R will vary as $\frac{F}{I^2}$. If therefore F vary in any inverſe, or any direct, leſs than the direct duplicate, ratio of I, R will vary in ſome inverſe ratio of I, and the fluid will be elaſtic. Q. E. D.

170. PROP. *If the particles of an elaſtic fluid repel each other with forces, that are equal at equal diſtances, and, at unequal diſtances, vary inverſely as the* n^{th} *power of the diſtance; and the compreſſing force* (C) *upon any ſurface* DE, *be equal to its whole force of repulſion;* C *will vary as that power of the denſity whoſe exponent is* $\frac{n+2}{3}$.

Let D be the denſity of the fluid; I the diſtance of any two contiguous particles; and D varies as the magnitude inverſely, or as $\frac{1}{DG^3}$, or, ſince DG varies as I, the number of particles being given, as $\frac{1}{I^3}$ and $D^{\frac{1}{3}}$ is as $\frac{1}{I}$. But the whole repulſive force of the ſurface, DE, varies as $\frac{F}{I^2}$, or as $\frac{1}{I^{n+2}}$, or as C, from the hypotheſis, and $C^{\frac{1}{n+2}}$ is as $\frac{1}{I}$, or as $D^{\frac{1}{3}}$, and $D^{\frac{n+2}{3}}$ as C. Q. E. D.

171. Cor. 1. If $D^{\frac{n+2}{3}}$ be as C, the repulſive force of each particle, or F, varies as $\frac{1}{I^n}$: for the quantity of matter being given, D is as $\frac{1}{DG^3}$, or as $\frac{1}{I^3}$, and $D^{\frac{n+2}{3}}$ is as $\frac{1}{I^{n+2}}$; but F varies as C divided by the number of particles in DE, or as $C \times I^2$, or as $D^{\frac{n+2}{3}} \times I^2$, or as $\frac{I^2}{I^{n+2}}$, or $\frac{1}{I^n}$.

172. Cor.

172. Cor. 2. If the elaſtic fluid be air, the force of compreſſion is as the denſity (20), and conſequently $\frac{n+2}{3} = 1$ and $n = 1$; or F, the force of each particle, is as $\frac{1}{I}$, or inverſely as the diſtance.

173. Cor. 3. The denſity of the air being as the compreſſing force, and that perpetually decreaſing in aſcending the atmoſphere, the denſity and elaſticity of the air alſo perpetually decreaſe.

SCHOLIUM.

174. The ſpecific gravity of water being 850 times greater than that of air (9), and the ſpecific gravity of mercury 14 times greater than that of water; the ſpecific gravity of mercury is 11900 times greater than that of air. If therefore the atmoſphere were of the ſame denſity with air at the earth's ſurface and homogeneal, and the height of the mercury in the barometer, at the ſurface of the earth, were 30 inches, this would be equivalent to a column of air, whoſe height was 11900×30, or 357000 inches, or 5.6 miles nearly. In this ſuppoſition, the mercury in a barometer would deſcend $\frac{1}{10}$th of an inch, in an aſcent, from the earth's ſurface, of 99.1 feet nearly; and the height of any place of obſervation might be eaſily diſcovered. But the atmoſphere is not homogeneal, and the concluſions, derived from this ſuppoſition, deviate from fact, even in ſmall altitudes, as appears from the following obſervations, made by Dr. Nettleton.

	Perpendicular altitudes.	Loweſt ſtation.	Higheſt ſtation.	Diff.	for $\frac{1}{10}$th.
Town of Hallifax	102	29.78	29.66	0.12	.85
Coal-mine	140	29.48	29.32	0.16	.87
Another ditto	236	29.50	29.23	0.27	.87
A ſmall hill	312	29.81	29.45	0.36	.86
Hallifax hill	507	30.00	29.45	0.55	.91

175. PROP.

175. PROP. *The altitude of mercury in a barometer being equal to* h, *to find the altitude of an homogeneal atmoſphere* (H) *whoſe compreſſing force is the ſame with that of the natural atmoſphere, and denſity equal to that of the air immediately incumbent upon the mercury in the baſon.*

Let m, a, be the ſpecific gravities of mercury, and air incumbent upon the mercury in the baſon, and (61) $H : b :: m : a$, and $H = \frac{b \times m}{a}$. Q. E. I.

176. Cor. If m be conſtant, $\frac{b \times m}{a}$ is a given quantity; for the ſpecific gravity of air, or a, is as its denſity, or as the height of the mercury, and conſequently $\frac{b}{a}$ does not vary, and $\frac{b \times m}{a}$ is always the ſame, or the height, H, of this homogeneal atmoſphere is invariable, whatever be the height of the mercury in the barometer.

177. PROP. *Suppoſing the force of gravity to be conſtant, and vertical diſtances from the earth's ſurface be taken in arithmetic progreſſion, the denſities of the air, at thoſe diſtances, will be in geometric progreſſion.* FIG. LVII.

DEM. Let S be the center, and A the ſurface, of the earth, and AZ a cylindrical column of air, perpendicularly incumbent upon A, divided into equal cylindrical columns, each of whoſe magnitudes is equal to m, and whoſe altitudes AB, BC, CD, &c. are equal and ſo ſmall, that the denſity of each is uniform, and reſpectively equal to a, b, c, &c. and their weights therefore reſpectively equal to $a \times m$, $b \times m$, $c \times m$, &c. Since the denſity is as the force of compreſſion, or weight perpendicularly incumbent, $a : b :: bm + cm$, &c. : $cm + dm$, &c. or $a : b :: b : c :: c : d$, &c. Q. E. D.

178. Cor.

178. Cor. 1. Since the elevations above the earth's ſurface, AB, AC, AD, &c. are in arithmetic progreſſion, and the denſities, at thoſe elevations, in geometric progreſſion, decreaſing; the logarithm of the denſity will be inverſely as the elevation. Let the denſities at any known altitudes AP, AQ, be p and q, and log. of p : log. of q :: AQ : AP. But the altitudes of the mercury in the barometer are as the denſities of the air at the places of obſervation, and conſequently in geometric proportion, and their logarithms as the elevations above the ſurface of the earth inverſely.

179. Cor. 2. If the ſtandard altitude be 30 inches, and 85 feet the vertical elevation requiſite to make the mercury deſcend $\frac{1}{10}$th of an inch, the elevation which will make it deſcend $\frac{1}{n}$, or any other part, of an inch, may be found by this analogy: log. of $\frac{1}{29.9}$: log. of $\frac{1}{29.5}$ :: 85 : elevation which will make it deſcend $\frac{1}{n}$ of an inch. And, whatever be the deſcent of mercury, the elevation neceſſary to produce it, is diſcovered by a ſimilar proceſs.

FIG. LVIII. 180. PROP. *If perpendiculars*, AB, CD, EF, *&c. be drawn from different points of any vertical line*, AX, *proportional to the denſities of the air at thoſe points, and the force of gravity be conſtant, the line paſſing through their extremities* B, D, F, *&c. is a logarithmic curve, whoſe aſymptote is* AX.

DEM. For if diſtances AC, AE, &c. from the earth's ſurface, be taken in arithmetic progreſſion, the ordinates AB, CD, EF, &c. repreſenting the denſities, will be in geometric progreſſion, (177), a property peculiar to the logarithmic curve. Q. E. D.

181. Cor. 1. If AX be divided into infinitely ſmall equal parts, Al, lm, &c. and ordinates be drawn, proportional to the denſities, at

at the points of division; the whole pressure upon A is equal to $AB \times Al + lm \times ln$, &c. or to the whole area above A, or to $AB \times$ the subtangent AT. But the pressure upon A is equal to the weight of a homogeneal atmosphere, whose density is AB and altitude H (175), or $AB \times H$; and consequently H is equal to the subtangent at AT.

182. Cor. 2. The vertical distance, AE, is as the logarithm of the ratio of AB to EF, or as the logarithm of the ratio of the heights of the mercury in the barometer at A and E. And hence the density of the air, at any given altitude, AP, may be found; for the subtangent, or H : .43429448 (the modulus of Brigg's system) :: AP : Brigg's log. of the density at P.

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183. This curve, exhibiting the relation between the density of the air at any given point, and elevation of that point, is called the *atmospherical logarithmic*.

184. PROP. *Supposing the force of gravity* (F) *to vary inversely as that power of the distance from the center of the earth, whose exponent is* n, *to find the variation of density at different elevations.* FIG. LIX.

Let S be the center of the earth, UR its surface, $SP = x$, $C =$ the force of compression, $S =$ the specific gravity of air, $D =$ its density; and S is as $F \times D$, and $-\dot{C}$, being as $S \times \dot{x}$, is as $F \times D \times \dot{x}$, or as $D \times \frac{\dot{x}}{x^n}$.

Let C vary as D^m, and $-\dot{C}$ will be as $-\frac{1}{m} \times D^{\frac{1-m}{m}} \times \dot{D}$, which therefore will be as $D \times \frac{\dot{x}}{x^n}$; and $-\frac{1}{m} \times D^{\frac{1-m}{m}} \times \dot{D}$ is as $\frac{\dot{x}}{x^n}$, and, taking the fluents, $-\frac{1}{1-m} \times D^{\frac{1-m}{m}}$ is as $\frac{1}{1-n} \times x^{1-n}$. Q. E. I.

185. Cor. 1. Let $m = 1$, or C be as D; and $-\frac{\dot{D}}{D}$ will be as $\frac{\dot{x}}{x^n}$, and the hyperbolic logarithm of D as $\frac{1}{1-n} \times x^{1-n}$. If therefore $\frac{1}{x^{n-1}}$ be taken in arithmetic progreſſion, or its reciprocal x^{1-n} in muſical progreſſion, the hyperbolic logarithm of D will be as this progreſſion, and conſequently D will decreaſe in geometric progreſſion.

186. Cor. 2. If, in the ſame hypotheſis, $n = 0$, or F be conſtant, $-\frac{\dot{D}}{D}$ will be as $\dot{x}$, and if x be taken in arithmetic progreſſion, or $\dot{x}$ be conſtant; $-\frac{\dot{D}}{D}$ will be invariable, and D therefore decreaſes in geometric progreſſion.

187. Cor. 3. If, in the ſame ſuppoſition of m, $n = 1$, or F vary as the diſtance inverſely; $-\frac{\dot{D}}{D}$ will be as $\frac{\dot{x}}{x}$, and if x be taken in geometric progreſſion, D will alſo be in geometric progreſſion.

188. Cor. 4. If F vary inverſely as the ſquare of x, and C be as $D^{\frac{4}{3}}$, $D^{\frac{3}{2}}$, D^{2}, then D will vary as $\frac{1}{x^3}$, $\frac{1}{x^{\frac{3}{2}}}$, $\frac{1}{x^2}$ reſpectively.

189. Cor. 5. If F vary as $\frac{1}{x^2}$, and a third proportional, SL, to SP and SA be taken, wherever the point P be, and ordinates AB, LM, be drawn proportional to the denſities at A and P; the line paſſing through BM is the logarithmic curve. Since $SP : SA :: SA : SL$; SL varies as $\frac{1}{x}$, becauſe SA is given; the fluxion of the area

area BL, that is Lm, is equal to $LM \times L\mathcal{L}$, and varies as $D \times \frac{\dot{x}}{x^2}$, or as $-\dot{D}$ (becauſe $-\frac{\dot{D}}{D}$ is as $\frac{\dot{x}}{x^2}$ (Cor. 1.) or as $\dot{C}$; and, taking the fluents, the whole force of compreſſion, or C, is as the ſum of the areas Lm; that is, the ordinate LM, being as the denſity, or as C, varies as the area below it, and BM is the ſame logarithmic curve with that in (180) but inverted. For in that prop. AB was the denſity of air at the earth's ſurface, and $P\mathcal{Q}$ and LM, being as the denſity at the ſame point P, muſt be equal to each other, and the curves conſequently equal.

190. Cor. 6. The force varying as $\frac{1}{x^2}$, AL will be the logarithm of the ratio of the denſity at A and P; or the logarithm of the ratio of the altitudes of the mercury in a barometer at A and P.

191. PROP. *If the higheſt of two ſtations,* AP *and* AQ, *be very ſmall, compared with the ſemidiameter of the earth,* SA, *and the force of gravity vary inverſely as the ſquare of the diſtance* (x) *from the earth's center, the difference of the elevations, or* PQ, *is nearly the logarithm of the denſities of the air at* P *and* Q. FIG. LX.

Take $SP : SA :: SA : Sp$; and $S\mathcal{Q} : SA :: SA : Sq$; and conſequently $SP : S\mathcal{Q} :: Sq : Sp$; and $SP : Sq :: P\mathcal{Q} : pq$. But when AP is very ſmall, compared with SA, $SP = SA = Sq$, and $P\mathcal{Q} = pq =$ the log. of the ratio of the denſities of the air at P and $\mathcal{Q}$. Q. E. D.

192. Cor. 1. If the altitudes of mercury in the barometer at any ſmall elevations, above the earth's ſurface, AP, $A\mathcal{Q}$, be p and q, and be ſuppoſed proportional to the denſity of the air at thoſe places, and M be the modulus of Brigg's ſyſtem of logarithms, or

.434294481 9; and H the ſubtangent of the atmoſpheric logiſtic, then $PQ = \text{log. of } \frac{q}{p} \times \frac{H}{M}$, nearly. If AP were four miles, PQ would not be one part in 500 greater than pq. In great elevations, pq muſt be firſt diſcovered, and then PQ may be found.

193. Cor. 2. When p and q are obſerved at any known elevations above the earth's ſurface, AP and AQ, the length of the ſubtangent of the atmoſpheric logiſtic will be known, being equal to $\frac{M}{\text{log. of } \frac{q}{p}}$ multiplied into pq, or into PQ, if the elevation be ſmall. This ſubtangent is proved from experiments, and from theory it ought, to be different; for it is equal to $\frac{b \times m}{a}$ (175) and m is variable.

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194. If therefore the force of gravity be conſtant, or vary inverſely as the ſquare of the diſtance from the earth's center, and the elevation AP be ſmall; $AP = \frac{H}{M} \times \text{log. of } \frac{q}{p}$; and when AP is conſiderable, and the force variable as above, it is equal to $\frac{H}{M} \times \text{log. of } \frac{q}{p} \times \frac{SP}{SA}$. But theſe rules require correction, becauſe they ſuppoſe the denſity of the air to vary as the force of compreſſion, which is not true unleſs the temperature be the ſame; and at different elevations, the temperature is found to be very different at the ſame time. In aſcending the Condelleras of Peru, the temperature of the air was obſerved to vary, through all degrees between the hotteſt ſummer and coldeſt winter; and the ſpecific gravity of mercury, which is encreaſed by heat and diminiſhed by cold, will be

be different at every different elevation, and the ſame weight may conſequently have different lengths, and different weights the ſame length, in a barometer tube. According to Father Feuillees' obſervations upon the Pic of Teneriffe, the altitude of mercury in the barometer, at the bottom of the mountain, was 27 in. 10 lin.; and at the top, or 13158 Paris feet above the ſurface of the ſea, it was 17 in. 5 lin., or 2 in. 2 lin. lower than it ſhould be by the rule. The inconſiſtency between the theory and facts, ariſes from a difference of temperature, at the places of obſervation; for the weight of the mercury, in a barometer, varies as the altitude of the column ſuſpended multiplied into its ſpecific gravity, and the latter being variable, the height alone is not, though aſſumed to be, a true meaſure of the preſſure of the atmoſphere. And beſides, the abſolute force of elaſticity of air, or the force with which two particles, at equal diſtances, repel each other, is encreaſed by heat and diminiſhed by cold; and when therefore, in two ſtations, the denſity of the air is the ſame, and the temperature and conſequently elaſticity different, the columns of mercury ſupported at theſe ſtations will be different and cannot occupy the ſame parts of the tube, but muſt have different altitudes, and conſequently this denſity cannot meaſure the altitude truly.

195. PROP. *If any portion of a barometer tube, perfectly cylindrical,* FIG. LXI. *be occupied with air before inverſion; after inverſion the mercury will ſubſide below the ſtandard altitude, and the ſtandard altitude will be to the defect from it as the ſpace occupied by the air after inverſion, to the ſpace occupied by it before inverſion.*

Let *LM* be the ſtandard altitude of the mercury, *AE* and *AD* the ſpaces occupied by air before and after inverſion. The elaſticity of air (*E*) before its expanſion is equivalent to the weight of a column of mercury whoſe altitude is *LM*, and its elaſtic force when uniformly diffuſed through *AD* (*e*) is equivalent to the

weight

weight of a column whoſe height is LC, becauſe the mercury is depreſſed by this force from L to C; therefore $E : e :: LM : LC :: AC : AB$ (20). Q. E. D.

196. Cor. 1. Let the ſtandard altitude $LM = a$, the defect from it or $LC = d$, the height of the included air or $AB = b$, the difference between the length of the tube and the ſtandard altitude or $AL = l$; and $a : d :: \pm l + d : b$. In the third term of this analogy, the ſign of l is $+$ or $-$ according as the length of the tube, or AM, is greater or leſs than the ſtandard altitude LM.

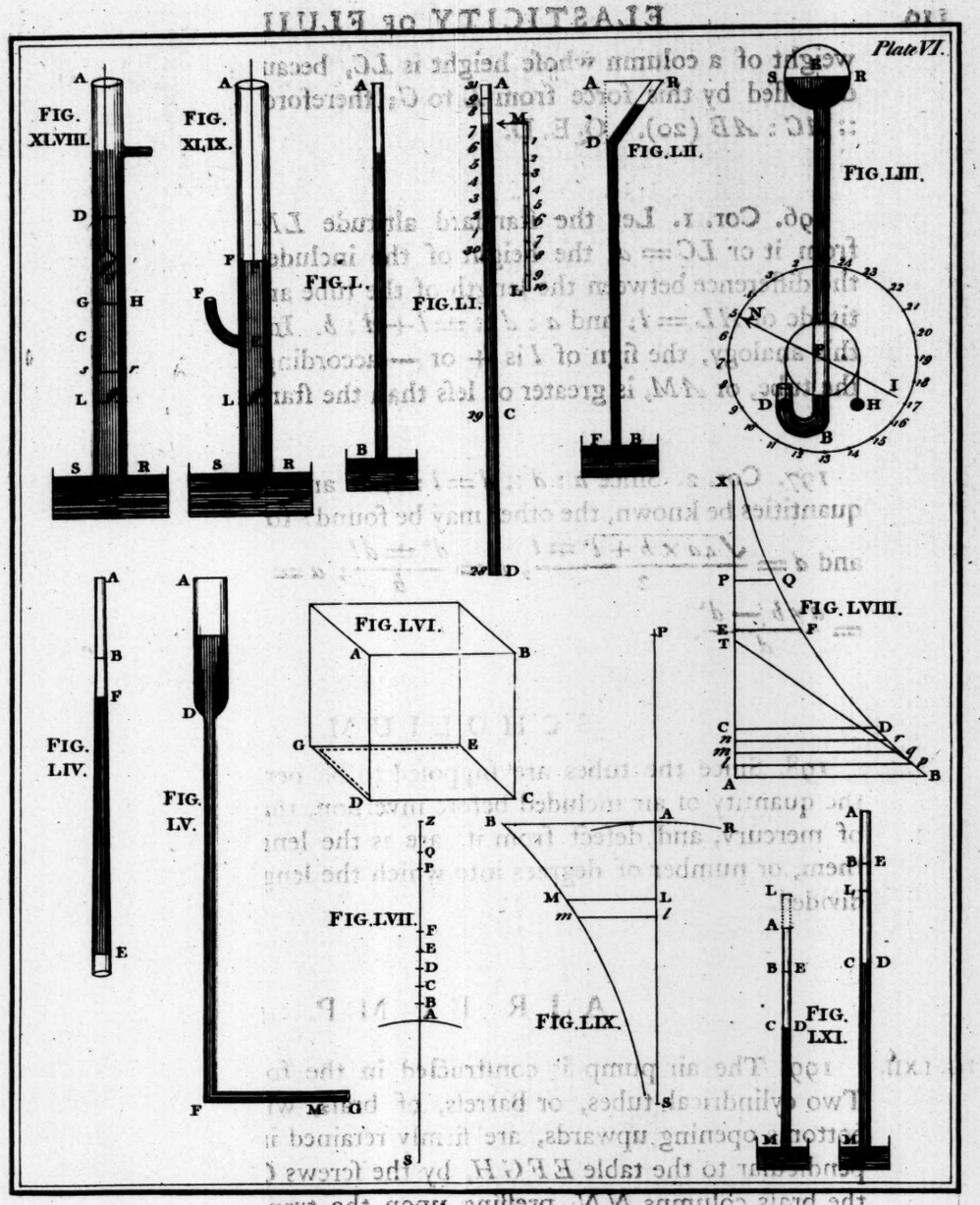
197. Cor. 2. Since $a : d :: d \pm l : b$, if any three of theſe four quantities be known, the other may be found: for $a \times b = d^2 \pm dl$, and $d = \frac{\sqrt{4a \times b + l^2} \mp l}{2}$; $b = \frac{d^2 \pm dl}{b}$; $a = \frac{d^2 \pm dl}{d}$; and $\pm l = \frac{a \times b - d^2}{d}$.

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198. Since the tubes are ſuppoſed to be perfectly cylindrical, the quantity of air included before inverſion, the ſtandard weight of mercury, and defect from it, are as the lengths occupied by them, or number of degrees into which the length of the tube is divided.

AIR PUMP.

FIG. LXII. 199. The air pump is conſtructed in the following manner: Two cylindrical tubes, or barrels, of braſs, with valves at their bottoms opening upwards, are firmly retained in a ſituation perpendicular to the table $EFGH$, by the ſcrews OO, at the top of the braſs columns NN, preſſing upon the tranſverſe beam TT; and


FIG. XLVIII.
FIG. XLIX.
FIG. L.
FIG. LI.
FIG. LII.
FIG. LIII.
FIG. LIV.
FIG. LV.
FIG. LVI.
FIG. LVII.
FIG. LVIII.
FIG. LIX.
FIG. LXI.


BRITISH MUSEUM
30
61
APR

and communicating with a cavity in the table at *D*, and with a receiver *LM*, fixed upon a braſs plate ſmeared with wax, or covered with wet ſheepſkin, by means of a canal or tube, one end of which opens into the cavity at *D*, and the other into the receiver at *K*. Each barrel has a piſton in it exactly adapted to its cavity, that air cannot paſs between it and the ſides of the barrel; and each piſton has a valve in it opening upwards, and is connected to a rack *CC*, and raiſed or depreſſed by a winch or handle *B*, fixed to the axis of a cog-wheel, whoſe teeth catch the teeth of the rack; and one piſton being raiſed and the other depreſſed by the ſame turn of *B*, the weight of the atmoſphere, preſſing upon the riſing piſton, is counterbalanced by the ſame weight, preſſing upon the other piſton deſcending. There is a ſcrew at *V*, which cloſes the orifice of a pipe terminating at *K*, to admit air, when required.

200. Cor. When either of the piſtons is drawn upwards, a vacuum is left behind it; and all preſſure being thus removed from the valve in the bottom of the barrel, it will be opened by the elaſticity of the air in the receiver, and the air will be uniformly diffuſed through the receiver, canal from *K* to *D* and barrel. Upon depreſſing the piſton, the valve in the bottom of the veſſel will be cloſed, and that in the piſton opened, through which all the air in the barrel will be diſcharged; and thus, by every turn of the winch, a quantity of air equal to the content of the barrel, and equally denſe with that in the receiver, is exhauſted.

201. PROP. *If* R *be equal to the content of the receiver and canal* D, B *the content of the barrel, the quantity of air before any turn of the pump, is to the quantity extracted by it, as* R + B : B.

DEM. Let *L* be the quantity of air in *R* before any ſtroke, *l* the quantity exhauſted by it; and ſince, the piſton being raiſed, the air

air is uniformly diffuſed through R and B, and that in B extracted by the ſtroke (laſt cor.); $L : l :: R + B : B$. Q. E. D.

202. Cor. 1. Let L, M, N, &c. be the quantities of air before any ſucceſſive ſtrokes; l, m, n, &c. the quantities exhauſted by each ſtroke; and $L : l :: R + B : B :: M : m :: N : n$, &c. and, by diviſion of proportion, $L : L - l\,(M) :: M : M - m\,(N) :: N : N - n\,(O)$, &c. or $L : M :: M : N :: N : O$, &c.: therefore L, M, N, &c. are in decreaſing geometric progreſſion, whoſe common ratio is that of $R + B : R$. If $R = 2B$, then $R + B : R :: 3 : 2$, and $M = \frac{2}{3}L$, $N = \frac{2}{3}M$, &c. and the quantities of air are equal to L, $\frac{2}{3}L$, $\frac{4}{9}L$, $\frac{8}{27}L$, &c.

203. Cor. 2. Since $L : M :: l : m$, $M : N :: m : n$, $N : O :: n : o$, &c.; l, m, n, &c. are in decreaſing geometric progreſſion, whoſe common ratio is that of $R + B : R$. If, as in the laſt cor, $R + B : R :: 3 : 2$, then $m = \frac{2}{3}l$, $n = \frac{2}{3}m$, &c. and the quantities of air exhauſted by the ſucceſſive ſtrokes are l, $\frac{2}{3}l$, $\frac{4}{9}l$, &c.

204. Cor. 3. If R be to B in any finite ratio, as 3 : 2, the receiver can never be perfectly exhauſted by any finite number of turns; for, let the number of turns be n, and Q the laſt remainder, and $Q = L \times \frac{2^n}{3^n}$ (203), ſuppoſing L to be the quantity of air in the receiver at firſt; and this quantity $L \times \frac{2^n}{2^n}$ is finite, ſince n is finite.

205. DEF. *The* gage, *which is uſed to meaſure the exhauſtion of the receiver, conſiſts of a tube divided by a ſcale annexed to it, of inches, and fractional parts of an inch, whoſe higher orifice communicates with the receiver, and lower is immerſed into a ciſtern of mercury below the table,* EFGH.

206. Cor.

206. Cor. 1. Before any exhauſtion has taken place, the mercury in the tube and ciſtern is upon the ſame level; and, after any number of turns of the pump, the air in the receiver and tube is equally rarefied, and the mercury will aſcend in the tube, till the weight of the column above the ſurface of the mercury in the ciſtern, and the elaſticity of the air in the receiver, taken together, be equivalent to the weight of the atmoſphere; and when the altitude of the column is equal to the ſtandard altitude, the vacuum in the receiver, and that above the mercury in the barometer, are the ſame.

207. Cor. 2. The elaſticity of the air in the receiver, being equivalent to, and meaſured by, the weight of a column of mercury, whoſe length is the difference between the ſtandard altitude and that of the column of mercury in the gage-tube, and this elaſticity being as the denſity, or, ſince R is given, as the quantity of air contained in R; the defects from the ſtandard altitude, after any number of ſucceſſive ſtrokes of the pump, will be as the quantities of air remaining, and conſequently (203) decreaſe in geometric progreſſion, whoſe ratio is that of $R + B : R$.

208. Cor. 3. Since the quantity of air in R, before any exhauſtion, is as the ſtandard altitude, and the elaſticity of the air remaining after 1, 2, 3, &c. ſtrokes, is as the defect of the column raiſed from the ſtandard altitude; the elaſticity, or quantity, of the air exhauſted by each ſtroke, is as the aſcent of the mercury produced by it, and conſequently theſe aſcents are in geometric progreſſion, whoſe ratio is that of $R + B : B$.

209. PROP. *If the capacity of the receiver, and canal* D, *be to that of the pump barrel as* r : b, *the air will be rarefied* f *times by a number of turns of the pump equal to* $\frac{\text{log. of } f}{\text{log. of } r + b - \text{log. } r}$.

Let a be the quantity of air in the receiver before any exhauſtion, n the number of turns, and a, $\frac{ar}{r+b}$, $\frac{ar^2}{\overline{r+b}^2}$, $\frac{ar^n}{\overline{r+b}^n}$, are the quantities of air after 0, 1, 2, n turns. Therefore the denſity of air, when rarefied f times, is to the denſity in its natural ſtate as $\frac{ar^n}{\overline{r+b}^n} : a :: \frac{r^n}{\overline{r+b}^n} : 1 :: 1 : f$, from the ſuppoſition; and $f = \frac{\overline{r+b}^n}{r^n}$, and log. of $f = n \times$ log. $r+b$ — log. of r, and $n = \frac{\text{log. of } f}{\text{log. } r+b - \text{log. of } r}$. Q. E. D.

210. Cor. 1. Let $r = b = 1$, and $n = \frac{\text{log. of } f}{\text{log. } 2}$, and, ſubſtituting for f, 1, 2, 3, 4, &c. the value of n, or number of turns correſponding to each, will be given. And from the value of $n = \frac{\text{log. of } f}{\text{log. } 2}$, the following table is conſtructed, which exhibits the number of turnsand correſponding rarity of air in the receiver.

Rarity.	No. of Turns.	Rarity.	No. of Turns.	Rarity.	No. of Turns.
1	0	60	5.907	900	9.814
2	1	64	6.	1000	9.966
3	1.585	70	6.129	1024	10.
4	2.	80	6.322	2000	10.966
5	2.222	90	6.492	2048	11.
6	2.585	100	6.444	3000	11.551
7	2.807	128	7.	4000	11.966
8	3.	200	7.644	4096	12.
9	3.170	256	8.	5000	12.288
10	3.322	300	8.229	6000	12.551
16	4.	400	8.644	7000	12.773
20	4.322	500	8.966	8000	12.966
30	4.907	512	9.	8192	13.
32	5.	600	9.229	9000	13.136
40	5.322	700	9.451	10000	13.288
50	5.644	800	9.644	16384	14.

211. Cor.

211. Cor. 2. If the ratio of r to b be unequal, the number of turns, by which the receiver will be exhausted f times, is found by the general value of n or $\frac{\text{log. of } f}{\text{log. } r+b - \text{log. } r}$; or by supposing $r = b = 1$, and consequently the number of turns to be equal to $\frac{\text{log. } f}{\text{log. } 2}$, and finding a multiplier for this number, when r and b are unequal, and the rarefaction continues to be the same as when they were equal. Let this number be m, and therefore $m \times \frac{\text{log. } f}{\text{log. } 2} = n = \frac{\text{log. } f}{\text{log. } r+b - \text{log. } r}$, and $m = \frac{\text{log. } 2}{\text{log. } r+b - \text{log. } r}$. By this equation the following table is constructed, shewing the value of m. Let $f = 100$; $r = 10$; $b = 1$; and $\frac{\text{log. } 100}{\text{log. } 2} = \frac{2.0000}{0.30103} = 6.644$ turns, and $m = \frac{\text{log. } 2.}{\text{log. } 11 - \text{log. } 10} = \frac{0.3010300}{0.0413927} = 7.273$, and $m \times \frac{\text{log. } 100}{\text{log. } 2} = n = 6.644 \times 7.273 = 48.322$ turns.

Capacity of the receiver.	Multiplier.	Capacity of the receiver.	Multiplier.	Capacity of the receiver.	Multiplier.
1	1.	20	14.207	300	208.291
2	1.710	30	21.139	400	277.605
3	2.409	40	28.071	500	346.920
4	3.106	50	35.003	600	416.235
5	3.802	60	41.394	700	485.549
6	4.497	70	48.866	800	554.864
7	5.191	80	55.798	900	624.179
8	5.885	90	62.729	1000	693.694
9	6.579	100	69.661		
10	7.273	200	138.976		

212. PROP. *If the diameters of different pumps, by which the air is reduced to a given degree of rarefaction* (f), *be the same, the time* (T) *is least, when the altitude of the pump is least.*

For T is equal to the time of one ſtroke of the pump, multiplied into the number of ſtrokes; but the time of one ſtroke varies as the length of the pump, or as B (ſuppoſing B = its capacity) and conſequently T varies as $\frac{\log. f \times B}{\log. R + B - \log. R}$, which, if $R = 1$, is equal to $\frac{\log. f \times B}{\log. B + 1}$. Let $Bm - 1$ = length of the pump, and T is as $\frac{\log. f \times Bm - 1}{\log. Bm}$, or, f being given, as $\frac{Bm - 1}{\log. Bm}$. But if the length, or B, be encreaſed, T will encreaſe, becauſe the numerator encreaſes faſter than the denominator, from the nature of log., and T therefore is leaſt when B, or the length, is leaſt. Q. E. D.

SCHOLIUM.

*213. Inſtead of one hole in the bottom of the barrel, there are ſeven of an hexagonal ſhape, and of an equal ſize, in the beſt air pumps, contrived by Mr. Smeaton; and the valve is thus ſupported by a kind of grating. To remedy the inconvenience, neceſſarily reſulting when the piſton is not exactly adapted to the cavity of the barrel, the top of the barrel is covered by a plate, having in the middle a collar of leather, through which the ſtem of the piſton, being cylindrical, paſſes. For the gage, already deſcribed, a ball of glaſs is ſubſtituted, reſembling a pear in ſhape, open at one end and hermetically ſealed at the other, and capable of containing half a pound of mercury. This ball being filled with mercury, and a bubble of air admitted into it before inverſion; if the open end be immerſed in a veſſel of mercury, placed under the receiver, the mercury in the bulb will be ſupported, and deſcend, as the receiver is exhauſted, by the expanſion of the bulb of air above it, whoſe denſity is always equal to that of the air in the receiver; and this expanſion is meaſured by a ſcale annexed, and divided into parts of $\frac{1}{10}$th of an inch, or $\frac{1}{1000}$th part of the capacity of the bulb.

214. PROP.

* Phil. Tranſ. 47. 415.

214. PROP. *If the quantities of air in a receiver be augmented by a condenſer or forcing ſyringe, they will encreaſe in arithmetic progreſſion.*

When the piſton (which in the condenſer is without any vacuity) is drawn upwards, a vacuum is left behind it, and when at its higheſt elevation, air is admitted through an aperture in the barrel, juſt below the piſton, and the barrel is filled with natural air: and, by depreſſing the piſton, this air is impelled through a valve at the bottom, opening downwards, into the receiver; and conſequently the quantity of air in the receiver is uniformly encreaſed by a quantity of air, in its natural ſtate, equal to the capacity of the pump. Q. E. D.

215. Cor. 1. If R and B be the capacities of the receiver and barrel reſpectively, the quantities of air in the receiver, or denſities, after $0, 1, 3, \ldots n$ terms, will be as $R, R+B, R+2B, R+3B, \ldots R+nB$.

216. Cor. 2. If the open end of a tube, whoſe other extremity is hermetically ſealed, be plunged in a veſſel of mercury placed within a receiver, and the air in the receiver be condenſed, the mercury will aſcend up the tube, by the encreaſe of preſſure upon the external ſurface, till the air above it be equally denſe with that in the receiver, for then only the mercury will be quieſcent, and conſequently this denſity encreaſes in arithmetic progreſſion; but the denſity being inverſely as the ſpace occupied by a given quantity, the parts of the tube, above the mercury, are inverſely as a ſeries of terms in arithmetic progreſſion, and after $0, 1, 2, 3, \ldots n$ ſtrokes of the piſton, the diſtances of the mercury, from the top of the tube, will be as $\frac{1}{R}, \frac{1}{R+B}, \frac{1}{R+2B}, \frac{1}{R+3B} \ldots \frac{1}{R+}$

$\frac{1}{R+nB}$. Since this is a property of numbers in musical progression, these distances, after 0, 1, 2, 3, ... n strokes, if measured, will form a series of numbers in musical progression *.

FIG. LXIII. 217. PROP. *Suppose the density of condensed air, contained in the vessel* ABDC, *&c. to be* D, *and its elastic force* E, *to find the velocity of efflux, through a small aperture at* O, *into the ambient medium, whose density and elasticity* d *and* e, *are less than* D *and* E.

The whole force generating velocity is evidently equal to $E-e = \frac{D \times e}{d} - e$ (because the density is as the elastic force) $= e \times \frac{D-d}{d}$; but this is as the quantity of motion generated in a given time, or substituting Q for the quantity of air issuing through O in this time, as $Q \times V$, or as $D \times V^2$; and consequently V varies as $\sqrt{e \times \frac{D-d}{Dd}}$, and may be discovered, when e, D and d, are known. Q. E. I.

218. Cor. 1. If the densities D and d be equal, then $\sqrt{e \times \frac{D-d}{Dd}} = 0$, and consequently $V = 0$.

219. Cor. 2. If $d = 0$, then $e = a$, and $\sqrt{e \times \frac{D-d}{Dd}} = \frac{a}{v} \times \frac{D}{D} = 1$; and the velocity does not vary.

220. Cor.

* The gage tube is, in this corollary, supposed to be horizontal; for, if it be inclined to the horizon, the elasticity of the air in the receiver is equivalent to that in the tube above the mercury, together with the weight of a column of mercury, whose perpendicular altitude is equal to that of the mercury in the tube; and consequently the air in the tube and receiver cannot be equally dense.

220. Cor. 3. If d be greater than D, V becomes negative, or the ambient air will flow into the veſſel $ABDC$.

221. Cor. 4. Let d = the denſity of air at the ſurface of the earth, and D be equal to $10 \times d$, then the whole expulſive force $E - e = e \times \frac{D-d}{d} = 9 \times e$, and is equivaleht to the weight of a column of water whoſe altitude is 32×9 (ſuppoſing e to be equivalent to 32 feet of water) $= 288 \times 85$ feet of air, whoſe denſity is D, and conſequently (113. & Mech. 537) $V = 8 \times \sqrt{24480}$ feet in 1″.

SCHOLIUM.

222. Fire engines, wind-guns, fountains, &c. derive their efficacy from the elaſticity of condenſed air. 1. The fire engine is compoſed of two barrels E and D, in each of which a piſton, without any vacuity in it, is worked by means of a double lever, one piſton deſcending and the other aſcending at the ſame time. Theſe barrels are fixed in a veſſel of water, with which they communicate by valves at M and K, opening into them, and they communicate with a ſtrong veſſel CC, by pipes terminated by valves at L and T, opening into it. If either of the piſtons, as that in D, be raiſed, the water ruſhes out of the receiver through the valve at K, and the piſton in E, being depreſſed, forces the water through T into CC; and by repeated ſtrokes of the piſtons, the water in CC condenſes the air above it, whoſe elaſticity is at length ſufficient, by preſſing upon the ſurface of the water, to force it, in a continued ſtream, through a pipe $PORQR$. 2. In a wind-gun, juſt behind the ball, which is placed at the bottom of the barrel, is a cavity, terminated at both ends, by valves opening within it; and in the ſtock, or butt end of the piece, is a condenſer or forcing ſyringe, by which air is forced into this cavity, and when it is ſufficiently condenſed by repeated ſtrokes of the piſton, the valve, next to the ball, is opened by drawing back a trigger

FIG. LXIV.

trigger connected with it. The included air, being much more elaſtic than air in its natural ſtate, ruſhes through the valve, thus opened, and expels the ball with a velocity, that may be determined by the preceding propoſition. 3. Artificial fountains may be conſtructed in this manner: *ABCD* is a ſtrong veſſel, nearly
FIG. LXV. full of water; at the top *M*, a tube *EO* is ſcrewed in, and made air-tight, and at *K* is a ſtop-cock; which being opened and a forcing ſyringe ſcrewed on, and air injected, the air above the ſurface of the water, *SR*, is condenſed till its elaſticity be ſufficient, by preſſing upon that ſurface, to force the water up the tube *OE*. The velocity with which the fluid ſpouts from *E*, may be found, if the elaſticity of the included air, and weight of the column
FIG. LXVI. raiſed, be given. 4. The depth of the ſea is meaſurable, where a line cannot be uſed, by this property of air in the following manner: *A* is a ball of fir, or other light wood; *B* is a hollow glaſs veſſel, whoſe capacity is preciſely known; its neck *C* terminates in a ſmall orifice, and is bent downwards to prevent the eſcape of the included air, when immerſed in water. At *E* is a ſpring hook, which, if at liberty, would ſtand in the poſition *e*; but is preſſed through a ſlit in the ſtem, and kept to the poſition *E*, by a weight *D*. The inſtrument being ſuffered to deſcend in water, and the preſſure of the water encreaſing, it forces its way into *B* and condenſes the air contained in it; but when it reaches the bottom, *D* firſt ſtrikes the ground, and the reſt of the apparatus proceeding with the motion acquired, is diſengaged from the hook, and the apparatus riſes to the ſurface. The depth of the deſcent is known by the dimenſions of the air in *B*; for the denſity of the air is as the force of compreſſion, and, when the quantity is given, inverſely as the ſpace occupied by it; and, this being diſcovered, the force of compreſſion, compared with that of the atmoſphere, or of 32 feet of water, and conſequently the depth may be found. 5. The phenomena of the diving bell are explicable from the elaſticity of air. It conſiſts of a large veſſel, reſembling a bell in ſhape, ſo loaded with lead, as to ſink when empty with the mouth downwards. As the machine is lowered into the water, it will aſcend up the bell,

bell, and contract the dimenſions of the included air; but air is ſent down in ſmaller bells, and received into the cavity of the larger, to expel the water which riſes as the preſſure or depth encreaſes, and to replenish the air within the bell become foul by breathing, which eſcapes by a cock in the upper part of the bell.

CHAP. XII.

OF WINDS*.

223. WIND is a ſtream or current of air, denominated eaſt, weſt, north, ſouth, or by ſome intermediate appellation, according to its direction upon the compaſs; and with reſpect to their direction all winds are included in the three following kinds: viz. *general*, blowing always nearly in the ſame directions; *periodical*, blowing nearly in certain directions, at certain periods of time; and *variable*, not obſerving the ſame direction, but blowing from every point of the compaſs, and not changing their directions at any ſtated times. Before the cauſes of theſe different winds are attempted to be explained, it is neceſſary to know where they prevail; and therefore it is expedient to conceive the whole ocean to be divided into three parts: 1. The Atlantic or Ethiopic ſea, lying between America upon the weſt, and Africa upon the eaſt. 2. The Indian ocean, lying between Africa upon the weſt, and the Indian iſlands and New Holland upon the eaſt. 3. The Pacific ocean, lying between America upon the eaſt, and the Philippine iſlands, China, Japan, and New Holland, upon the weſt of it.

224. In

* Miſcellanea Curioſa. Phil. Tranſ. No. 183. Robertſon's Elements of Navigation, Book VI. Sect. VI.

224. In the Atlantic and Pacific oceans, between the north and ſouth latitudes of twenty-eight degrees nearly, a wind between the north and eaſt, blows always upon the north ſide, and more northerly in places nearer to the northern limit; and one between the ſouth and eaſt, and more ſoutherly in places nearer to the ſouthern limit, blows always upon the ſouth ſide, of the equator; and under the equator the wind is generally due eaſt. Theſe winds, uſually called *tropical*, or *general* trade winds, are liable to a few exceptions, which are contained in the following obſervations, made by Dr. Halley. 1. In the Atlantic ocean, near the coaſt of Africa, the north-eaſt trade wind extends only to within 10 deg. of the equator, and the ſouth-eaſt trade wind croſſes the line and extends to the latitude of 4 deg. north; and in the intermediate ſpace, or between 10 and 4 deg. north lat. there are calms and irregular ſqualls of wind from every point of the compaſs. 2. Upon approaching the coaſt of America, the north-eaſt winds become more eaſterly, are ſometimes eaſt and eaſt by ſouth, but generally to the northward of the eaſt; and their ſtrength generally decreaſes to the weſtward. 3. Upon the American ſide of the Atlantic, the north-eaſt trade winds extend to 30, 31, or 32 deg. of latitude, and upon the African ſide only to 27 or 28 deg.; and near the cape of Good Hope, the limit of the trade winds is three or four degrees nearer to the equator, than upon the coaſt of Brazil. 4. From the latitude of 4 deg. north, to the limits of the trade wind upon the ſouth ſide of the equator, the winds are perpetually between the ſouth and eaſt, and moſt commonly between the ſouth-eaſt and eaſt; but upon the ſide next to Africa, they are more ſoutherly, and next to Brazil more eaſterly, almoſt due eaſt, their ſmall deflection from it being to the ſouth. 5. The ſeaſon of the year has ſome ſmall effect upon the trade winds; for when the ſun's declination is conſiderably north, the ſouth-eaſt winds, between the coaſt of Guinea and Brazil, vary a point or two to the ſouthward, and the north-eaſt winds become more eaſterly; and when the ſun's declination is conſiderably ſouth, the ſouth-eaſterly winds become more eaſterly, and the north-eaſterly winds,

winds, upon the north ſide of the equator, become more northerly. 6. Upon the weſtern ſide of the coaſt of Africa, the wind ſets in upon the land from various points of the compaſs, north-weſt, weſt, ſouth by weſt, ſouth-weſt, and almoſt ſouth towards the cape of Good Hope; and along the coaſt of Guinea, for above 500 leagues, from Sierra Liona to the iſle of St. Thomas, and within 80 or 100 leagues of the coaſt; the wind is ſouth-ſouth-eaſt, then ſouth-ſouth-weſt, and near the coaſt it is ſouth-weſt; but theſe winds are ſometimes interrupted by calms and violent guſts in every direction. 7. Between 4 and 10 deg. of north latitude, and between the meridians of cape De Verd and the northernmoſt of theſe iſlands, or between 20 and 30 deg. of weſt longitude, there is a tract of ſea, called the *Rains*, which is ſubject to perpetual calms and tornadoes. 8. Between the parallels of 28 and 40 deg. in ſouth latitude, and between 30 deg. weſt, and 100 deg. eaſt, longitude, the wind is uſually between north-weſt and ſouth-weſt, though it does ſometimes vary from theſe directions; and between the ſame parallels, in north latitude, a weſterly wind prevails, but with leſs certainty than in ſouth latitude.

225. *Periodical* winds are the land, and the ſea, breezes, and monſoons or ſhifting trade winds; the former change from one direction to its oppoſite, every twelve hours, blowing towards the the land by day, and from it by night, and obtain near the ſhores of all countries, in different degrees, according to the nature of the ſoil, cultivation, &c. but are moſt conſpicuous in countries ſituated between the tropics. The latter or monſoons, are chiefly prevalent in ſome parts of the Indian ocean, changing from one direction to its oppoſite every ſix months, and the ſubſtance of the following hiſtory of them is taken from the obſervations of Dr. Halley. 1. Between the latitudes of 10 and 30 deg. ſouth, and between the coaſts of Madagaſcar and New Holland, the general trade wind, between the ſouth and eaſt, obtains in the ſame manner, as in the Atlantic and Pacific oceans. 2. This ſouth-eaſt wind extends within about two deg. of the equator, from June to November; and a

contrary wind from the north-weſt ſets in, and continues from the beginning of December to May, between the ſouth latitudes of 3 and 10 deg. in the meridian of the north end of Madagaſcar, and between 2 and 12 deg. about the meridian of Sumatra or Java; and this monſoon alſo prevails between Madagaſcar and the Moluccа iſles. 3. To the northward of 3 deg. in ſouth latitude, over the whole Arabian or Indian ſea and gulph of Bengal, from Sumatra to the eaſt of Africa, a monſoon, between the north and eaſt points, blows from October to April; and during the other half year, or from April to October, it blows from the ſouth-weſt. Theſe winds are not ſo conſtant, either in direction or ſtrength, in the gulph of Bengal, as in the Indian ſea; and the ſouth-weſt winds, in theſe ſeas, are generally moſt ſoutherly upon the ſide of Africa, and moſt weſterly upon the ſide of India. 4. In that tract of ſea, between Madagaſcar and the coaſt of Africa, and from thence northwards to the equator, the ſame ſouth wind prevails from April to October; but the direction of the wind, during the other ſix months, is not certainly known. 5. To the eaſtward of Sumatra and Java, in north latitude, and along the coaſt of Camboia and China, the monſoons blow nearly from the north and ſouth, or the north-eaſt winds are here more northerly, and the ſouth-eaſt winds more ſoutherly; the northern monſoon beginning in October or November, and the ſouthern in May, and extending to the eaſtward of the Pilippine iſles, and as far to the north as Japan. 6. In ſouth latitude, between the ſame meridians, being that tract which is ſituated between Sumatra and Java to the weſt, and New Guinea to the eaſt, the ſame northerly monſoons obtain, as in the laſt obſervation, with this difference, that the inclination of the northerly monſoon is towards the north-weſt, and of the ſoutherly towards the ſouth-eaſt; but their directions in both are variable through five or ſix points of the compaſs. The changes of theſe monſoons are, about a month or ſix weeks, later than in the Chineſe ſeas. 7. The breaking up of the monſoons, or their change to an oppoſite direction, is gradual, and attended with calms in ſome, and variable winds in others; and the end of the weſterly monſoon, upon the coaſt of Coromandel, and the two laſt months

months of the ſoutherly monſoon, in the ſeas of China, are ſubject to tempeſts, ſimilar, by their violence, to the Weſt India hurricanes.

226. PROP. *Suppoſing the temperature to be given, a partial rarefaction, or condenſation, of the atmoſphere, will produce a wind, blowing towards the parts rarefied, and from the parts condenſed.*

If at the ſame altitude, the denſity of the air were the ſame, no portion of it could move, becauſe the preſſures upon oppoſite ſides of any portion would be equal and deſtroy each other; but when any portion of air is either denſer, or rarer, than the ambient air, the equilibrium is deſtroyed, and motion muſt enſue, diverging from it, when denſer, and when rarer converging towards it. Q. E. D.

227. Cor. 1. The denſity of the air being as the compreſſing force, when the temperature is given, and the compreſſion which the earth ſuffers from it, according to the common courſe of things, being nearly the ſame, little motion of it, or little wind, can reſult from hence; and winds muſt therefore chiefly ariſe from partial rarefactions, owing to the different operations of heat upon land and water, and upon different kinds of land. When any portion of air is hotter than the ambient air, its elaſticity is increaſed, and being no longer in equilibrio with the ſurrounding medium, it will endeavour to expand itſelf and aſcend, the reſiſtance above being leſs than that below; and being thus rarefied, the ſurrounding air will ruſh in, to reſtore the equilibrium, as the rarefied air becomes cooler; and this influx will be encreaſed as the preſſure upon the ambient air is encreaſed, by the expanſion of the aſcending air in the higher regions of the atmoſphere. In this manner, air always moves from cold to warmer places below, and often in

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an oppoſite direction above, and winds are generated, as far as the cauſes of them are diſcovered, by the action of heat only *.

228. Cor. 2. The air contiguous to the equator is, in general, hotter than near any circle parallel to it, and would always be ſo, were the whole earth covered with water; and conſequently, air will move, from the north and ſouth, towards it, and there will be a north wind upon the north ſide, and a ſouth wind upon the ſouth ſide, of the equator. But the circle of greateſt heat, and therefore of greateſt rarefaction, may be altered by the magnitude and ſituation of continents, and the nature of their ſoil; as in the Atlantic ocean, near to the coaſt of Africa, this circle is a few degrees north of the equator, and wind therefore, coming from the ſouth, paſſes the line to the fourth degree of north latitude; which it never does in open extenſive ſeas, far removed from the influence of land.

229. Cor. 3. Land and ſea breezes ariſe from this partial rarefaction of the atmoſphere; for the air lying over land being hotter, during the day, and colder, during the night, than that over the adjacent ſea, a current of air will ſet in towards the land by day, and from it by night; and theſe breezes prevail upon the maritime coaſts of all countries, but moſt remarkably in iſlands ſituated between the tropics. Upon the coaſts of great continents, whoſe lands are very ſuſceptible of heat, the air lying over them may be ſo heated, and retain the heat ſo long, that, in the abſence of the ſun, it may always be hotter than that over the ſea; and in this caſe the ſea breezes will be perpetual. This is the cauſe of the conſtant ſouth-weſt winds upon the weſtern coaſts of Africa, which often obtain alſo upon the weſtern coaſts of America.

SCHO-

* If the door of a warm room be ſhut, a current of air is ſenſibly perceived to iſſue into the room, through an aperture at the bottom, and out of it through an aperture at the top, of the door, by placing a candle near them.

SCHOLIUM.

230. Air * being a heavy fluid, will be influenced by the action of the ſun and moon upon it, and there muſt be tides of air, as well as of water; but they are too ſmall to affect the mercury in a barometer, and winds therefore produced by this cauſe, are ſcarcely ſenſible. Mr. Henry Eeles accounts for the irregular motion of the atmoſphere, from the aſcent and deſcent of vapours and exhalations, attended with electric effluvia.

231. PROP. *The motion † of the earth, round its axis, will produce an eaſterly wind, in air moving towards the equator, and a weſterly wind, in air moving from it.*

DEM. The atmoſphere being a gravitating fluid, and partaking of the earth's diurnal motion from weſt to eaſt, performs one revolution in twenty-four hours nearly; and conſequently the velocity of air at the equator is to the velocity of air in any parallel of latitude, as the peripheries of theſe circles to each other, or as the radius to the coſine of latitude. If therefore air move towards the equator, it muſt appear to come from the eaſt, becauſe the places, viſited by it, are moving eaſtward with a greater velocity; and if it move from the equator, it muſt appear to come from the weſt, becauſe its motion eaſtward is greater than that of the places, over which it paſſes. Q. E. D.

232. Cor. 1. This motion of air to the eaſt, combined with that from the north and ſouth (228) will produce a north-eaſt wind upon the north ſide, and a ſouth-eaſt wind upon the ſouth ſide of the circle of greateſt rarefaction, and generally, of the equator. And from theſe two cauſes combined, are derived the general trade-winds, conſtantly blowing in moſt parts of the Pacific and Atlantic oceans.

233. Cor.

* Phil. Tranſ. No. 437. Muſſchen. Vol. II. pag. 1102.
† Phil. Tranſ. Vol. XLIX. Art. XXV. pag. 124.

233. Cor. 2. If the ſurface of the earth were covered with water, the rarefaction of air, in the ſame parallel, being equally expoſed to the ſun's rays and equally impregnated with water, would be the ſame; and the cauſe being the ſame, the effect would be ſo too, or the winds would always blow with the ſame ſtrength, and in the ſame directions. Variable and periodical winds are therefore chiefly to be attributed to the different degrees of heat, communicated by land and ſea, under the ſame parallel, to the air lying over them. And in extenſive ſeas therefore, as in the Pacific and Atlantic oceans, the general trade winds are ſubject to few variations, and to fewer in the firſt than in the ſecond, which is broken by numerous iſlands, and bounded by large continents, one of which is remarkably ſuſceptible of heat.

234. Cor. 3. In the Atlantic, a ſouth-eaſt wind blows conſtantly between the equator and four deg. of north latitude; becauſe this wind comes from the ſouth, and, when it paſſes over the equator, has not the velocity of places under the equator, but that of the parallel, from whence it came, augmented indeed during its paſſage; and it muſt appear to be eaſt, whilſt this velocity is leſs than that of the places viſited by it. The northerly trade wind extends only to ten degrees in north latitude, becauſe the circle of greateſt rarefaction lies upon the north ſide of the line; and between the latitudes of four and ten degrees, calms will be produced by an equilibrium of winds coming from oppoſite points, and irregular ſqualls by their ſtruggles againſt each other. To the eaſtward of this, a conſtant ſouth-weſt wind ſets in upon the coaſt of Africa, and in the intermediate ſpace, the air muſt be extremely rarefied by the blowing of oppoſite winds from it, and is therefore ſubject almoſt to conſtant rains.

235, Cor. 4. The general trade winds ought to extend, as they do, to higher latitudes upon the American, than upon the African, ſide of the Atlantic; becauſe the land in America extends to higher

higher latitudes, both north and ſouth, than in Africa; and unequal rarefactions of air, lying over land and water, and conſequently the trade winds, ariſing from them, will be extended to higher latitudes. They are alſo enclined to the eaſt upon the ſide of America; for, coming from leſs parallels, the relative velocity will be greater, and this effect will alſo be augmented, by the ſea breezes, which, in this caſe, come from the eaſt.

236. Cor. 5. The ſeaſon of the year ought to have ſome effect upon the direction of the trade winds; for when the ſun's declination is north, the air to the north of him, will be rarefied, and move towards the equator with leſs velocity, than when his declination is ſouth, and conſequently the wind in north latitude will be more eaſterly, and that to the ſouth of the line, more ſoutherly. And for the ſame reaſon, when the ſun's declination is ſouth, the winds in ſouth latitude are more eaſterly, and thoſe in north latitude, more northerly.

SCHOLIUM I.

237. Since a large portion of air, ſituated between twenty-eight degrees of north and ſouth latitude, is conſtantly moving towards the circle of greateſt rarefaction, and there becomes hotteſt and moſt rarefied, it will then aſcend; and, as the barometer is not ſenſibly affected by this accumulation of air, it muſt return to the north and ſouth in upper currents. After this air has been ſufficiently cooled and condenſed, by moving at ſuch an elevation from the earth, it will become ſpecifically heavier than the air below, and from the nature of fluids, deſcend, and form the north-weſt and ſouth-weſt winds, deſcribed to prevail (224.Obſ.8.) between the limits of the trade winds, and about forty degrees of latitude. And continuing to move near the earth, till it be ſufficiently heated and rarefied, it muſt aſcend again in an oppoſite direction; and, being again cooled, deſcend again and become the general trade

wind. The winds beyond the fortieth degree of latitude, are variable and not reducible to any known rules; becauſe very different degrees of heat are communicated to the ſea and land, in nearly ſimilar ſituations of the ſun, on account of the weather; but wind in theſe climates ariſes from the ſame cauſes, as in the torrid zone, a partial rarefaction of the atmoſphere.

SCHOLIUM II.

238. Since the radius of the earth is to the coſine of the latitude of either tropic, as 1000 : 917 nearly; air coming from the tropic, with the velocity eaſtward which it had there, to the equator, will appear to move to the weſt at the rate of 2083 miles in 24 hours, or of one mile and $\frac{1}{3}$th of a mile in 1', which is not conſiſtent with experiment, being greater than the velocity of wind in a violent ſtorm. But air is not transferred from the tropic to the equator, in an inſtant, and the relative velocity of air under them, or under any two unequal parallels of latitude, is gradually diminiſhed, its motion being affected by the diurnal motion of the earth. Thus the velocity of air, moving from the tropic to the equator, will be gradually augmented, by the greater velocity of the earth in the ſame direction; and when it arrives at the equator, be equal to that in ſome intermediate parallel, the nearer to the equator as its motion towards it is ſlower.

SCHOLIUM III.

239. The cauſe of the general trade winds, according to Dr. Halley, is different from the preceding, and, for the ſake of thoſe who may not think with me, here inſerted. "The ſun appears to move from eaſt to weſt, and in conſequence of this, there muſt be ſome point upon the globe hotter than any other, which is the point of greateſt rarefaction. The greateſt heat at any place is about three o'clock in the afternoon, and the point of greateſt rarefaction is therefore behind the ſun; but the air, eaſt of this point,

point, will be grown cool again, and therefore follow it to reſtore the equilibrium. In this manner, a conſtant current of air, from the eaſt, will take place all over the globe, which compounded with a north wind on the north ſide, and a ſouth wind on the ſouth ſide, of the equator, will produce the general trade winds." That this account is unſatisfactory, is preſumed, for the following reaſons. 1. It is true, that air will always move towards the point of greateſt rarefaction, or 45 deg. eaſt of the ſun, and that, from the eaſt of this point, there will be an eaſterly wind: but, when there is the leaſt inequality of rarefaction, it will, for preciſely the ſame reaſon, viz. the nature of air, move from the parts that are more, to thoſe that are leſs, denſe. And as the air that is 30, 40, 50, &c. degrees weſt of the ſun, is denſer than that to which the ſun is vertical, and ſtill more ſo than that of 45 deg. to the eaſt of him, it muſt, and cannot, whilſt poſſeſſed of this property of air, but move from thoſe parts where it is denſer, and conſequently become a weſterly wind. And thus for the morning, and for ſome time after the ſun had paſſed the meridian, there would be a conſtant north-weſterly, and ſouth-weſterly wind upon the north and ſouth ſides of the equator reſpectively; which is contradicted by experience. 2. A current of air, actually moving in a direction oppoſite to the earth's diurnal motion, would certainly, in a great number of years, have ſenſibly retarded its motion; which has not been yet obſerved. But ſuppoſing the earth and air to move in the ſame direction, the magnitude of impact, and conſequent retardation, is much leſs: and this is, in ſome degree, compenſated by the current of air returning from the equator (laſt ſcholium), and conſequently moving eaſtward with a greater velocity than the places over which it paſſes.

240. PROP. *Air, incumbent upon land, imbibes a given quantity of heat ſooner, and loſes it ſooner, than that upon water, eſpecially between the tropics, and during the ſummer, in countries reaching to higher latitudes.*

DEM. In this country, and indeed in greater latitudes, land, about noon in ſummer, is hotter than water, and air incumbent upon it muſt therefore be ſo too; and a thermometer expoſed for a given time, to air lying over land, is higher than upon the adjacent ſea or lake. In the Indian iſlands, a freſh breeze ſets in towards land, about nine o'clock in the morning, encreaſes to two or three, then decreaſes, and quite vaniſhes about nine o'clock at night; then a breeze conſtantly blows from land, which encreaſes till two, and dies away about nine o'clock in the morning, and theſe changes prevail throughout the year. Q. E. D.

SCHOLIUM.

241. This propoſition is not generally true: for winds, coming over large tracts of ice and ſnow, are felt to be much colder than ſea winds blowing over the contiguous ſeas. And in this kingdom, vapours ariſing from water, during froſty and cold weather, are warmer than the incumbent air, becauſe they are condenſed and become viſible as ſoon almoſt as they riſe into it; and theſe vapours, riſing out of the ſea, in great abundance, muſt communicate heat to the atmoſphere lying over it, and render it warmer than that over land.

242. Cor. 1. The monſoons, in the Indian ocean and bay of Bengal, ariſe from the difference of heat, communicated to, and loſt by, air lying over land and water, in the ſame time; for the Indian ocean is ſurrounded, to the eaſt and north, by part of Africa, Arabia, Perſia, and India, whoſe ſhores lie within the limits of the trade winds; and the air ſituated over theſe immenſe tracts of land, becoming hotter than that over the adjoining ſea, after the ſun has croſſed the equator, (about the 22d of March) a wind ſets in about April, three weeks afterwards, upon land, and, coming from the equator, muſt have a weſterly direction. This ſouth-weſt wind continues from April to October, when the ſun's declination becomes ſouth; and then the air, lying over the land towards

towards the north, being ſooner cooled than that lying over the ſea, a breeze ſets in from the land, and, coming towards the equator, has an eaſterly direction; and thus a ſouth-eaſt wind will blow from October to April.

243. Cor. 2. The monſoon, blowing between 3 and 10 deg. of ſouth latitude, in the Indian ocean, from the ſouth eaſt during the months of June, July, Auguſt, September, October, November, and from December to May, from the north-weſt, may be explained in the ſame manner, as the preceding. During the ſummer months, or from the end of May to the end of November, the general trade winds (always blowing in the ſouthern part of the Indian ocean, from 28 to 10 deg. of ſouth latitude) now reach within 2 or 3 deg. of the line; and the monſoon, ariſing from the extreme rarefaction of the air lying over the lands in north latitude (laſt cor.) is extended to between 2 and 3 deg. of ſouth latitude, and would have extended ſtill farther to the ſouth, had the land been nearer to the line. The direction of this monſoon is ſouth-eaſt, becauſe its velocity eaſtward, when it paſſes the equator, is leſs than that of places lying in this circle, and it muſt appear to come from the eaſt, till its velocity be greater than that under the parallels viſited by it, which happens between 2 and 3 deg. of ſouth latitude. During the winter months, or from the beginning of December to May, the northern monſoon extends to 10 deg. of ſouth latitude; its direction being north-eaſt to between 2 and 3 deg. of latitude, where the velocity of the parallel and its velocity eaſtward are equal, and north-weſt between this and 10 deg. of latitude. This monſoon ſets in later than that on the north ſide of the line, becauſe ſome time muſt elapſe before the cauſe, producing the general trade wind from the ſouth-eaſt, can be ſet aſide by that producing the current of air from the north.

244. Cor. 3. Between Madagaſcar and the coaſt of Africa, a ſouth-weſt wind blows from April to October; becauſe at that time,

time, the ſun's declination is north, and that part of Africa oppoſite to Madagaſcar, being ſooner cooled than the ſea, a wind will blow from it, or be between the ſouth and weſt.

245. Cor. 4. To the eaſtward of Sumatra, in north latitude, the north-eaſt monſoon is more northerly, and the ſouth-eaſt monſoon more ſoutherly, owing to the immenſe continents ſituated to the weſt and north of this tract of ſea.

SCHOLIUM I.

246. The monſoons, or ſhifting trade winds, are differently explained by Dr. Halley, in the following manner. The air, which is carried to the equator, being there much rarefied, muſt aſcend and fall again in an oppoſite direction, to preſerve the equilibrium; and if this returned air be rarer than that moving towards the equator, it muſt be carried along with it in the ſame direction; and if it be denſer, it will prevail over it, and produce a current in a contrary direction. Now to the north of the Indian ocean, there are lands every where within 30 deg. of latitude, which, like Mediterranean parts of Africa, are ſubject to exceſſive heats, during the ſummer, but are temperate, in winter, from a ridge of mountains, at ſome diſtance within the land, which at that ſeaſon are often covered with ſnow, and over which the air that paſſes is much chilled. And hence, from October to April, the lower current will be denſer than the upper, or be north-eaſt; and from April to October, when the ſun's declination is north, the lower current becomes rarefied, and the upper current prevails, or the wind is ſouth-weſt. But the preceding account ſeems to be more ſatisfactory than this of Dr. Halley, which is liable to many objections. If all the air be returned within the limits of the monſoon, there will be a perpetual accumulation of air in this latitude, as freſh air is always blowing towards the equator, and the mercury in the barometer would be higher every year, which is

is not true; and if a part of it only were returned, there would ſtill be an accumulation, and the remainder, returned beyond the limits of the monſoon, would clearly affect the direction and velocity of the general trade winds from the ſouth-eaſt, blowing beyond the limit of this monſoon.

SCHOLIUM II.

247. If any portion of ſpace were entirely exhauſted of air, the velocity of the contiguous air, ruſhing into it, would be nearly 1319 feet in 1″ (149); and the degree of rarefaction being given, the velocity of air might be aſcertained by art. 217. But this propoſition ſuppoſes the temperature to be given, and cannot be applied to diſcover the velocity of wind, which ariſes only from a difference of temperature, and is much leſs than when the temperature is the ſame. For the air which is rarefied, is hotter and more elaſtic, than the ambient air, and by this encreaſe of elaſticity, diminiſhes the velocity of influx. Dr. Derham, from repeated obſervations upon the motion of light, downy feathers, moving freely in the wind, concluded that air, in a great ſtorm, moved at the rate of 45 miles in an hour; and when the ſtorm was moſt violent, it moved at the rate of 50 or 60 miles in an hour. Mr. Brice diſcovered, from ſome obſervations upon the motion of clouds, or their ſhadows moving upon the ſurface of the earth, that the velocity of wind in a ſtorm, was 62.9 miles in an hour, 21 miles in a freſh gale, and 9.9 miles in a ſmall breeze.

CHAP.

CHAP. XIII.

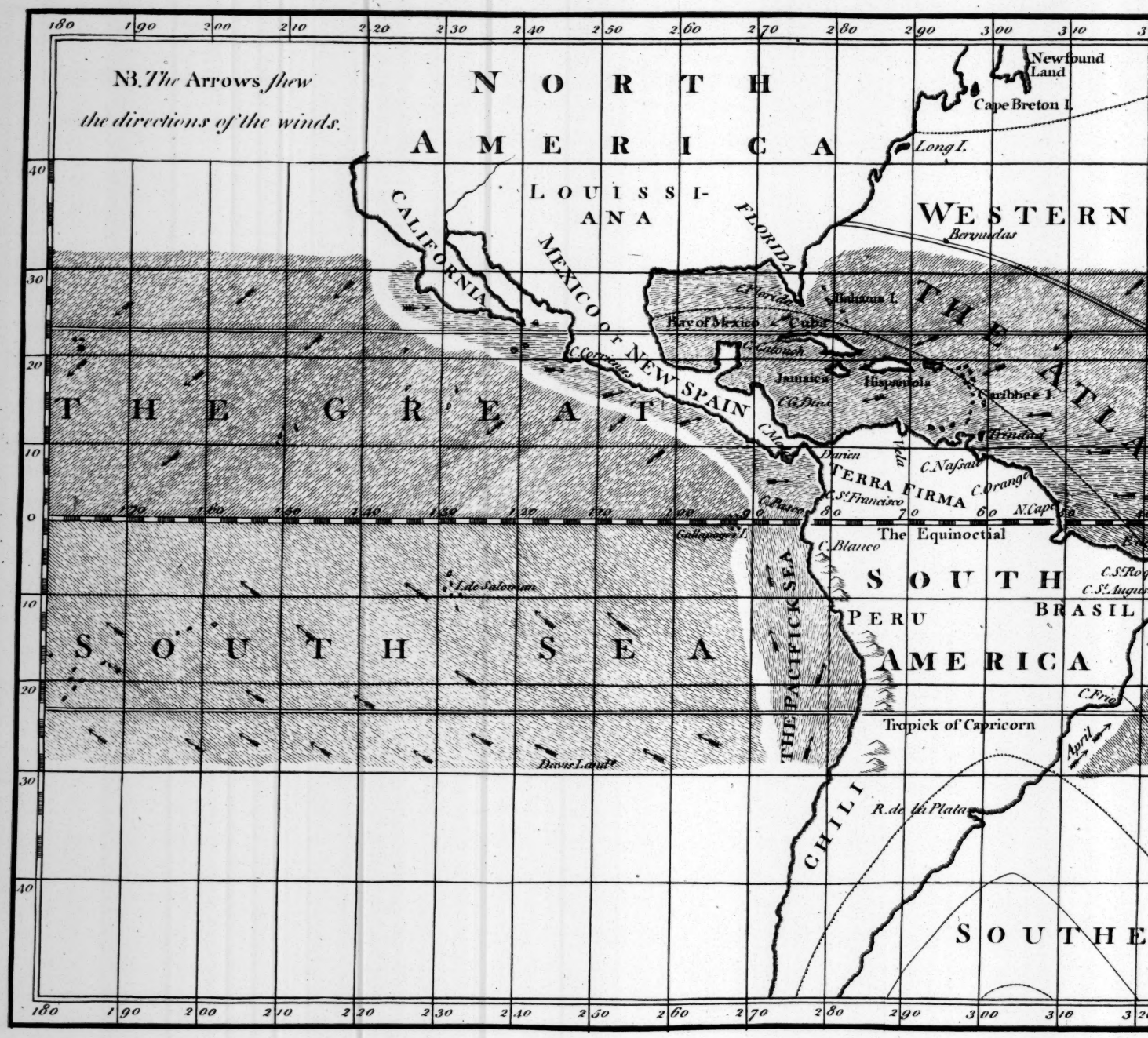
OF SOUND.

248. PROP. *SOUND is propagated from the ſounding body by the motion of air.*

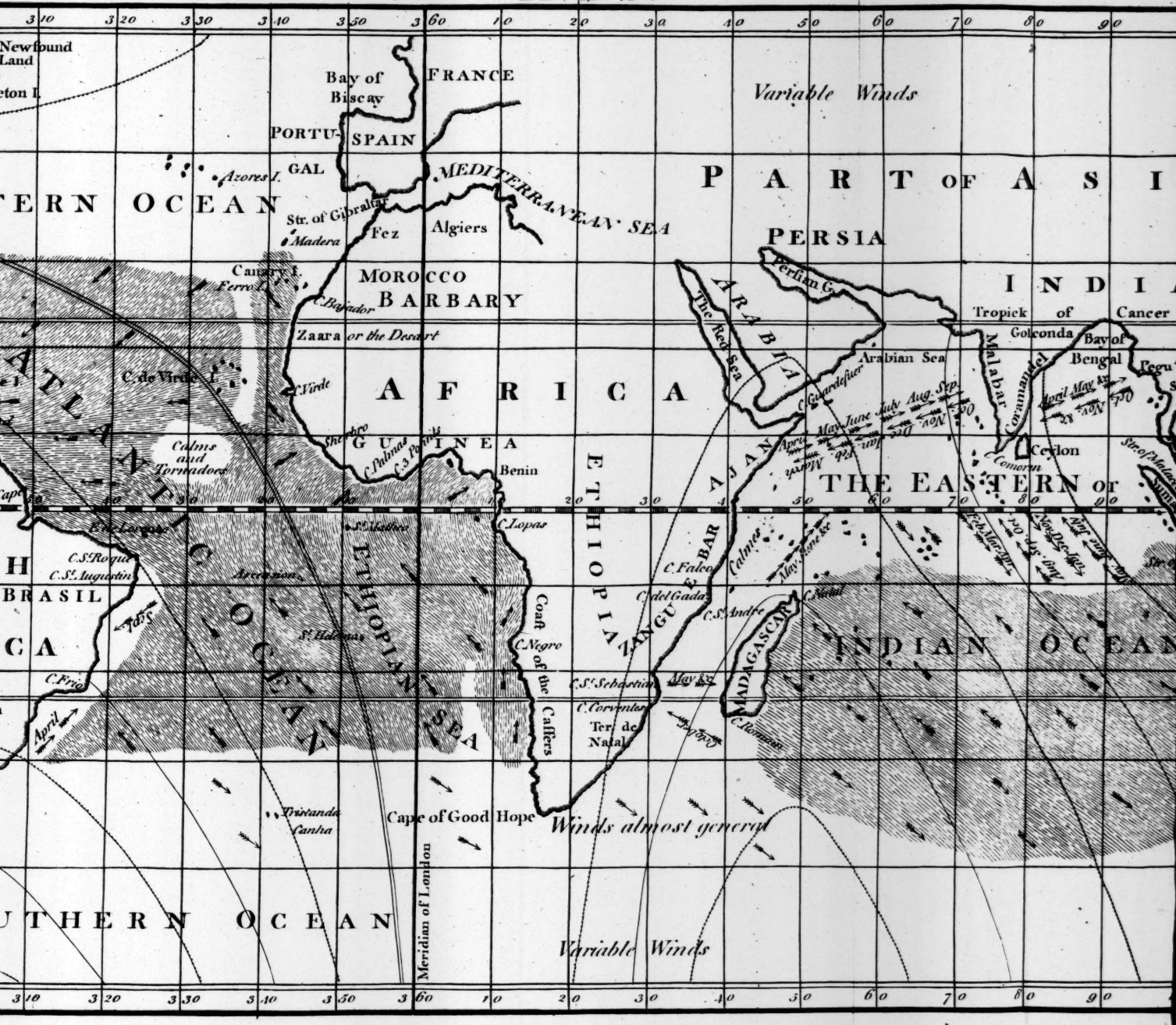
DEM. 1. The perception of ſound, without the actual impulſe of matter upon the organ of hearing, is impoſſible, and it muſt therefore be conveyed by ſome intermediate fluid. 2. The ſound of a bell, included under a receiver, is weaker when the quantity of air is diminiſhed, and ſtronger when augmented by a condenſer. 3. A ſtrong receiver filled with natural air, in which a bell was ſuſpended, was ſcrewed down to a braſs plate covered with wet leather, ſo that no air could eſcape, and included under a larger receiver; and when the air contained between the two receivers, was exhauſted, the ſound of the bell could not be heard, and was not therefore tranſmitted through the vacuum; but it was heard again after the air was readmitted. 4. That motion is communicated, by the ſounding body, to the contiguous air, is evident from the viſible motion of ſmall particles of duſt floating in it; and in very loud ſounds, as of artillery or large bells, the ſurface of any contiguous water is ſenſibly agitated, and glaſs windows are ſometimes broken, by the impulſe of air upon them. Q. E. D.

249. PROP. *All ſonorous bodies are elaſtic.*

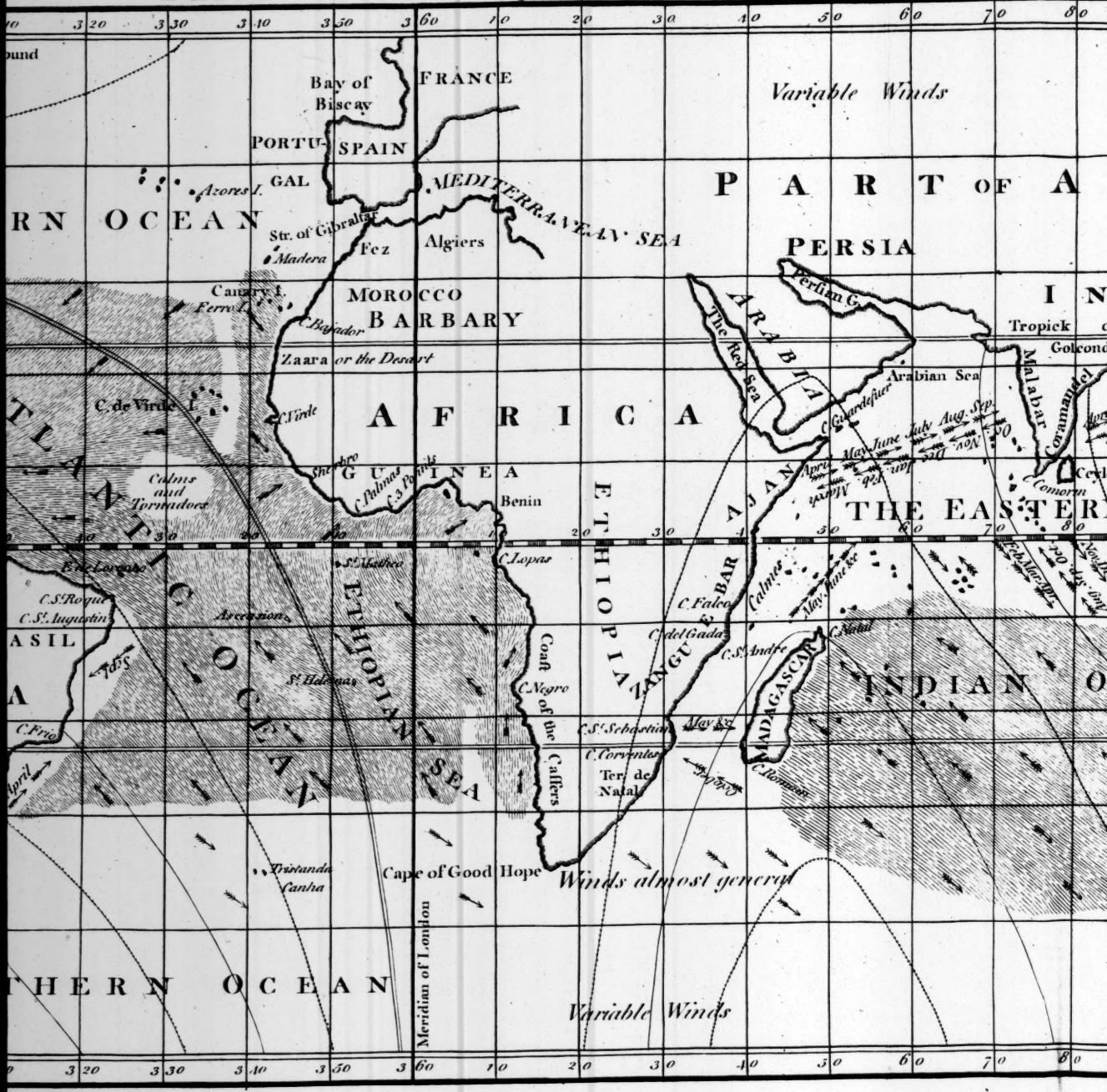
1. Let

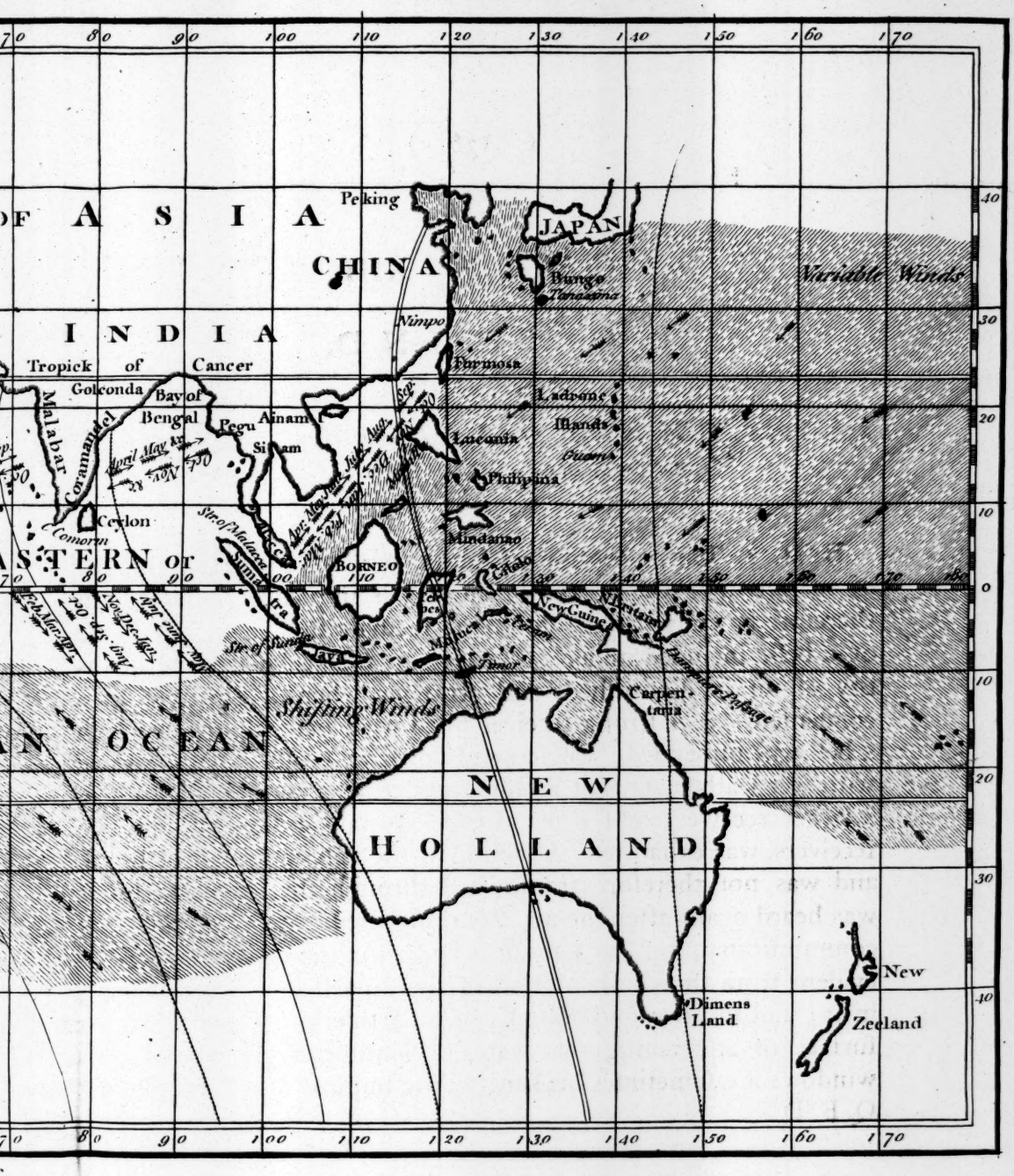

N.B. The Arrows shew the directions of the winds.
NORTH AMERICA
LOUISSI-ANA
CALIFORNIA
MEXICO or NEW SPAIN
FLORIDA
WESTERN
THE GREAT
SOUTH SEA
THE PACIFICK SEA
TERRA FIRMA
PERU
BRASIL
AMERICA
CHILI
SOUTHE
Newfound Land
Cape Breton I.
Long I.
Bermudas
Bahama I.
Bay of Mexico
Cuba
Jamaica
Hispaniola
Caribbee I.
Trinidad
Darien
C. Nasau
C. Orange
C. St. Francisco
C. Blanco
N. Cape
The Equinoctial
Tropick of Capricorn
Galapagos I.
I. de Salomon
Davis Land
R. de la Plata
April
180 190 200 210 220 230 240 250 260 270 280 290 300 310
40 30 20 10 0 10 20 30 40

MAP of the WINDS.



MAP of the WINDS.




Peking
JAPAN
CHINA
Bungo
Tanaxima
Variable Winds
Ningpo
Formosa
Tropick of Cancer
Golconda
Bay of Bengal
Pegu
Ainam
Siam
Malabar
Coromandel
Ladrone Islands
Philippina
Ceylon
Comorm
Str. of Malacca
Sumatra
BORNEO
Mindanao
Celebes
New Guinea
New Britain
Str. of Sunda
Java
Molucca
Timor
Shifting Winds
Carpentaria
Passage
NEW HOLLAND
Dimens Land
New Zeeland
0
10
20
30
40
70
80
90
100
110
120
130
140
150
160
170

1. Let glaſs, bell-metal, &c. be covered with a little duſt when they are ſtricken, and their parts will evidently appear to move, from the tremulous motion of the particles of duſt. 2. This tremulous motion is alſo collected from that of water contiguous to the ſides of a glaſs veſſel containing it, which is made to emit a ſound by friction. 3. If the parts of a body were immoveable by impact, they could not communicate motion to the contiguous air; but ſound is conveyed to the ear by the motion of air communicated to it by the body, whoſe parts muſt therefore move, and continue to move forward and backward for ſome time, becauſe the ſound is heard for ſome time. Q. E. D.

SCHOLIUM.

250. Sonorous bodies, are conceived to be made up of elaſtic laminas or fibres, which move by the impact of another body upon them, and communicate the motion received to the next contiguous, and theſe to the next, &c. and their tremors continue till deſtroyed by the reſiſtance of the ſurrounding fluid or by friction.

251. LEMMA. *Every point of an elaſtic chord or fibre* FAE, *equally ſtretched and perfectly elaſtic, will, in performing very ſmall vibrations, return to the axis, or ſtate of quieſcence,* FE, *at the ſame inſtant of time.*

Take equal and very ſmall parts of the fibre, *EM*, *MN*, *NO*, &c. and becauſe the tenſion is the ſame, their lengths, at any one inſtant of time, muſt be equal; but if *EM* coincided with *Em*, ſooner or later than *MN*, *NO*, &c. with *mn*, *no*, &c. their lengths, and conſequently their tenſions, would be unequal, contrary to the hypotheſis. All the points *M*, *N*, &c. arrive therefore at *FE*, at the ſame time. Q. E. D. FIG. LXVII.

SCHOLIUM.

252. If the length of FE be very ſmall, an encreaſe of tenſion ariſing from pulling, ſtriking, &c. is communicated almoſt inſtantaneouſly to it; but if it be great, the middle point A may return towards the axis before any motion in the direction BA be communicated to the diſtant parts of the fibre; and conſequently different parts of it will move in oppoſite directions, and form a ſerpentine line. Though the tenſion be the ſame through the fibre FAE, yet the accelerating forces of different points, in directions perpendicular to the axis, are plainly unequal, being greateſt at A and decreaſing from that point to F and E, where the accelerating force is nothing, becauſe thoſe points are fixed.

253. LEMMA. *If a perfectly elaſtic chord or fibre* FAE, *be ſtretched by a force equal to* P, *and inflected through an infinitely ſmall ſpace* BA, *by a force equal to* F, *acting at the middle point,* F *will vary as* $\frac{P \times AB}{FE}$.

Let the force P be repreſented in quantity and direction by AE, and reſolved into two forces BE and BA, and that part of it which acts in oppoſition, and is equal, to F is BA; and conſequently $F : P :: AB : AE :: AB : BE$ nearly, and $F = \frac{P \times AB}{BE}$, and varies as $\frac{P \times AB}{FE}$. Q. E. D.

254. Cor. 1. The tenſion of any part of the fibre is evidently the ſame, at the ſame diſtance, AB, from the axis, FE, whether it be only inflected to A, or be returning from ſome remoter inflection, GB; and conſequently, when P and the length of the fibre

fibre are given, F will vary as the diſtance AB, in all ſmall inflections, or A will always arrive at the axis in the ſame time. And as other points arrive at FE, at the ſame time with A (251), all vibrations of the ſame fibre are iſochronous, and it will aſſume ſuch a figure that the accelerating force of each point may vary as its diſtance.

255. Cor. 2. Since the accelerating force is as the ordinate, and alſo as the ſpace deſcribed in an infinitely ſmall time, the ordinates, in different ſituations of the fibre, will be to one another in a given ratio.

256. Cor. 3. The curvature of the fibre FE, in different poſitions, will vary as the correſponding ordinates; for let DB and CI be very near each other, and TB and TP be tangents, and $CG : CI :: DP : DB :: CK : CH$, and divid. $DP : DB :: KG : IH :: \angle KPG : \angle HBI ::$ curvature at P : curvature at B. This concluſion is derived differently by the following proceſs: DE, an infinitely ſmall part of the curve, is acted upon by equal tenſions at D and E in the directions of the tangents, and the elaſticity, being equivalent to them, is therefore as the diagonal of the rhombus GH paſſing through the center of curvature O. The abſolute force to move DE in the direction GH (A) : tenſion :: $DE : EO$; but the accelerating force, F, varies as $\frac{A}{DE}$, and therefore as $\frac{\text{tenſion} \times DE}{DE \times EO}$, or the tenſion being given, as $\frac{1}{EO}$, or as the curvature. It follows from hence, that the ordinate DB, being as F, varies as $\frac{1}{EO}$.

FIG. LXVIII.

FIG. LXIX.

257. LEMMA. *To find the radius of curvature of the curve* FDE, *whoſe length is* L *and axis* b.

Let the radius of curvature at the middle point $M = R$, and if $\dot{z}$ be given, the radius at any other point $D \left(= \frac{\dot{x}\dot{y}}{\ddot{x}}\right) : R :: b : y$, and $y\dot{y}\dot{z} = Rb\ddot{x}$ and $Rb\dot{x} = \frac{\dot{x}y^2}{2}$; but at M, $\dot{x} = \dot{z}$, and $y = b$, and $Rb\dot{x} - Rb\dot{z} = \frac{y^2 - b^2}{2} \times \dot{z}$; and $Rb\dot{x} = Rb\dot{z} + \frac{y^2 - b^2}{2} \times \dot{z} = \frac{2Rb + y^2 - b^2}{2} \times \sqrt{\dot{x}^2 + \dot{y}^2}$, and $\dot{x} = \frac{2Rb + y^2 - b^2}{\sqrt{4Rb^3 - 4Rby^2 - y^4}} \times \frac{\dot{y}}{-b^4 + 2b^2y^2} =$ (since R is much greater than y or b) $\frac{2Rb\dot{y}}{\sqrt{4Rb^3 - 4Rby^2}} = \frac{\dot{y} \times \sqrt{Rb}}{\sqrt{b^2 - y^2}}$, and $x = \sqrt{Rb} \times$ circular arc, whose sine is $\frac{y}{b}$ and radius unity; and when $y = b$, then $x = FL = \frac{1}{4}L = \sqrt{R \times b} \times \frac{3.1416}{2} = \sqrt{R \times b} \times \frac{p}{2}$, (substituting p for 3.1416) and $R = \frac{\frac{1}{16}L^2 \times 4}{p^2 \times b} = \frac{L^2}{p^2 b}$, and the radius of curvature in any other point $D = \frac{L^2}{p^2 y}$. Q. E. I.

258. PROP. *The length* L, *and weight* W, *of the fibre* FDE, *and tending force* P, *being given, to find the periodic time* T.

Let $A =$ the absolute force of tension, $a =$ the force with which DE moves, $w =$ the weight of DE. $A : a :: DO : DE :: R : \dot{z}$; but the accelerating force varies as the absolute force directly and quantity of matter to be moved inversely, and accel. force of $\dot{z}$: accel. force of gravity $:: \frac{a}{w} : \frac{A}{P} :: \frac{\dot{z}}{w} : \frac{R}{P}$; and, supposing the accelerating force of gravity $= 1$, the accelerating force of $\dot{z}$ $(F) = \frac{P \times \dot{z}}{R \times w}$. But the ratio of $P : w$ is equal to the ratios of $P : W$ and

$W : w$,

Plate VII.

FIG. LX.

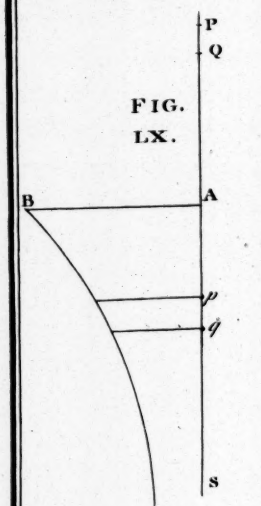


FIG. LXII.

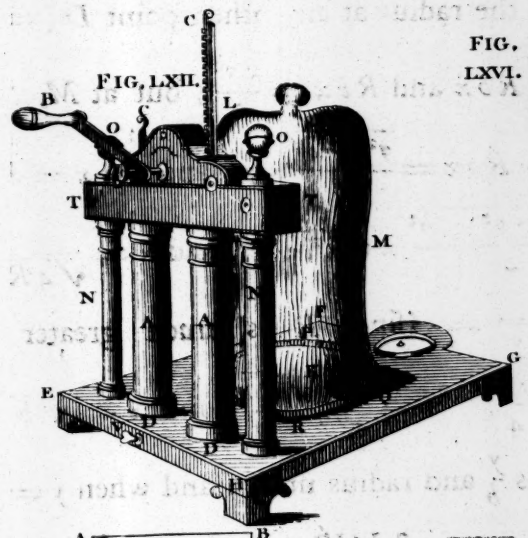


FIG. LXVI.



FIG. LXV.

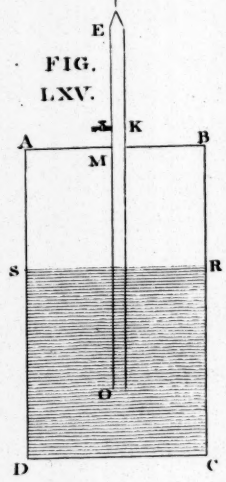


FIG. LXIII.

FIG. LXIV.

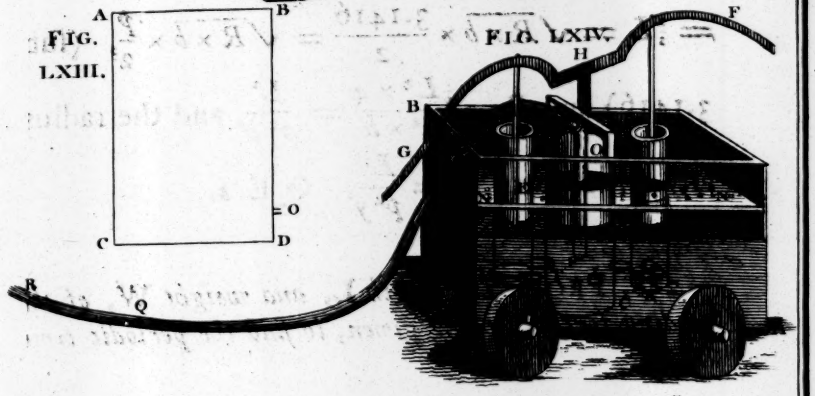


FIG. LXVII.

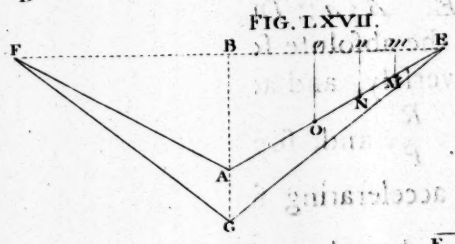
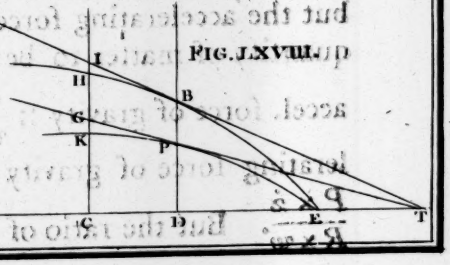


FIG. LXVIII.





$W : w$, or of $P : W$ and $L : \dot{x}$, and $\frac{P}{w} = \frac{P \times L}{W \times \dot{x}}$, and consequently F varies as $\frac{P \times L}{W \times R}$, or, substituting for R its value, as $\frac{P \times p^2 \times b}{L \times W}$. Let there be a pendulum, whose length is D and time of vibration t; and since the time varies as $\sqrt{\frac{\text{space}}{\text{force}}}$, $T : t :: b^{\frac{1}{2}} \times \sqrt{\frac{L \times W}{P \times p^2 \times b}} : D^{\frac{1}{2}}$ and $T = t \times \frac{\sqrt{L \times W}}{\sqrt{P \times p^2 \times D}}$, and the number of vibrations of the fibre performed in the time t is equal to $\frac{p \times P^{\frac{1}{2}} \times D^{\frac{1}{2}}}{L^{\frac{1}{2}} \times W^{\frac{1}{2}}}$. Q. E. I.

259. Cor. 1. Since p and D are given, the periodic times of different fibres will vary as $\frac{L^{\frac{1}{2}} \times W^{\frac{1}{2}}}{P^{\frac{1}{2}}}$.

260. Cor. 2. When the diameter of a fibre is given, L varies as W, and the times will therefore be as $\frac{L}{P^{\frac{1}{2}}}$.

SCHOLIUM.

261. Since the fibres, which compose sonorous bodies, during infinitely small vibrations, move according to the same law with a cycloidal pendulum, the particles of air immediately contiguous to them must move in the same manner, as their motion is derived from, and governed by, that of the fibre in contact with them. And that the motion of the particles more remote is similar to that of the string, seems to be apparent from their elasticity; and is still more clear from this experiment. If of two fibres, whose diameters, lengths, and tensions are equal, one of them be stricken

so

ſo as to produce a forcible tone, the other will receive, from the intermediate air, a tone and motion exactly ſimilar to the firſt.

262. DEF. *Pulſes* are thoſe parts of air which vibrate forward and backward, and in their progreſs ſtrike againſt obſtacles; and the latitude of a pulſe is the rectilineal ſpace, through which motion is communicated to the air, during one vibration backward and forward of the ſounding body, or particles of air.

263. PROP. *If particles of air, during any part of their vibration, be agitated according to the law of a cycloidal pendulum, they will continue to move according to that law to the end of their vibration.*

FIG. LXX. AB, BC, CD, &c. are ſpaces through which air moves during the times of three ſucceſſive vibrations; E, F, G are three phyſical points; Ee, Ff, Gg are three equal ſpaces through which they vibrate; ε, ϕ, γ, are any intermediate ſituations of theſe points.

Let $PS = Ee$; and let the periphery of a circle, whoſe diameter is PS and its parts, repreſent the time of a whole vibration and its parts, ſo that in the times PH, $PHSh$, the particles may have deſcribed PL, or $PS + Sl$.

If PH be the time elapſed from the beginning of E's motion, PI and PK will be the times elapſed from the beginning of the motions of F and G (becauſe $EF : BC$ :: time of deſcribing EF : time of deſcribing BC :: HI : whole periphery); and in the progreſs of the particles, $E\varepsilon$, $F\phi$, $G\gamma$ are reſpectively equal to PL, PM, PN; and $\varepsilon\phi = EF + F\phi - E\varepsilon = EF - LM =$ the expanſion of the air in EF when transferred to $\varepsilon\phi$; and the expanſion of air in $\varepsilon\phi$: mean expanſion :: $EF - LM : EF$; but $LM : IH :: IM : OP$, $IH : EF$ :: whole periphery : $BC :: OP : V$ (the radius of a circle whoſe circumference is BC) $LM : EF :: IM : V$; and conſequently expan. in $\varepsilon\phi$: mean expan. :: $V - MI : V$.

But the elaſticity of air varies inverſely as the ſpace occupied by it, and

and the elaſt. in $\varepsilon\phi$: mean elaſt. :: $\frac{1}{V-IM} : \frac{1}{V}$. By a ſimilar proceſs the elaſt. in $\phi\gamma$: mean elaſt. $\frac{1}{V-KN} : \frac{1}{V}$; and the difference between theſe forces : mean elaſt. :: $\frac{IM-KN}{V^2-V\times KN+IM+IM\times KN}$: $\frac{1}{V}$ :: $IM-KN : V$, the vibrations being very ſmall, and IM, KN, infinitely leſs than V. But the difference of elaſticity in $\varepsilon\phi$ and $\phi\gamma$ is the force with which the point ϕ is agitated, which varies as $IM-KN$ (V being given) or as $KL-IM$, becauſe KH is biſected in I, or, ſince $KL-IM : IH :: OM : OI$ or OP and IH and OP are given quantities, as OM, or biſecting Ff in Ω, as $\Omega\phi$.

In the ſame manner it may be ſhewn, that if $PHSb$ repreſent the time elapſed from the beginning of E's motion, $PHSi$, $PHSk$ will repreſent the times elapſed from the beginning of the motion of F and G reſpectively; and that the expanſion of EF in $\varepsilon\phi$: mean expanſion :: $EF+F\phi-E\varepsilon$ or $EF+lm : EF :: V+bl : V$, and the elaſtic force in $\varepsilon\phi$: mean elaſtic force :: $\frac{1}{V+bl} : \frac{1}{V}$, and the difference of the elaſtic forces in $\varepsilon\phi$ and $\phi\gamma$: mean elaſtic force :: $kn-im : V$, and this difference of elaſtic forces varies directly as $\Omega\phi$. But this is the force with which the phyſical point ε is agitated in its regreſs; and this force conſequently varies both in its progreſs and regreſs as the diſtance from the middle point of its vibration. Q. E. D.

SCHOLIUM.

264. If three particles had moved for times denoted by PK, PI, PH, the difference of the elaſtic forces of the extreme particles, upon the intermediate one, would have been as $KL-IM$, or as MO; or if the time be diminiſhed till I coincide with P, that is, in the initial motion of the ſecond particle, and before the third has

has moved at all, the point M will fall upon P, and $MO = PO$: conſequently the elaſtic force of the ſecond particle, at the beginning of its motion and before the third particle has received any motion, varies as the diſtance from the middle point of its vibration, ſuppoſing the firſt particle to move according to that law. If, for inſtance, the point E be conſidered as a particle of air, vibrating according to this law, and moving, for an infinitely ſmall time, before F begins to move; the force agitating F varies as the diſtance from the middle point, and F begins to move according to the law of a cycloidal pendulum, and it will impel G juſt in the ſame manner as it was impelled by E, and the comparative elaſtic force of G will vary as the diſtance. Such are the initial motions of the particles, and, from this propoſition, theſe motions are continued, and pulſes of air are propagated, from the ſounding body, according to the law of a cycloidal pendulum.

265. Cor. 1. The number of pulſes propagated is equal to the number of vibrations of the ſounding body; for the line of air $\varepsilon\gamma$, after returning to its firſt ſituation, will be quieſcent, its velocity being as IM, and there conſequently evaneſcent.

266. Cor. 2. In the extreme points of the ſpace, through which the particles vibrate, or at P and S, their expanſion is equal to that of air in its natural ſtate; for the expanſion in $\varepsilon\gamma$, is to the natural expanſion, as $V \mp IM$ to V, and IM is there equal to nothing. In the middle of the progreſs, the condenſation is greateſt, becauſe IM is there greateſt, and conſequently $V - IM$, or the expanſion is leaſt. During the remainder of the progreſs, and to the middle point of the regreſs, the medium grows rarer, and there the rarefaction is greateſt, becauſe im and $V + im$, which expreſſes the expanſion, is there greateſt.

267. Cor.

267. Cor. 3. It is evident, from the nature of an elaſtic fluid, that theſe condenſations and rarefactions will be diffuſed in every direction. For the condenſed air, having more elaſticity, will expand itſelf into the contiguous parts all around, and theſe into the next, &c.; and the rarefied air, having leſs elaſticity, the contiguous air will ruſh in from every direction; and in this manner ſpherical ſhells of condenſed and rarefied air, at equal intervals, will encompaſs the ſonorous body.

SCHOLIUM.

268. The propagation of ſound is often illuſtrated by the obvious inſtance of a heavy body, thrown into a pond of ſtagnant water. The ſurface of the water forms itſelf into circular waves, which are propagated not only directly forwards, but if any obſtacle, with a ſmall aperture in it, obſtruct their progreſs, thoſe, which paſs through the aperture, will iſſue from it as from a center, and be diffuſed over the whole ſurface of the water lying behind the partition; not by a direct motion from the place in which they were firſt excited, being prevented by the partition, but by an oblique and lateral progreſs, their courſe being bent as they paſs through the aperture. In a ſimilar manner, the tremulous motions of a ſonorous body communicate undulations to the air. In the water, the waves are elevated above, and the intervals between them depreſſed below, the contiguous unruffled ſurface, and it is obvious, that the waves, and cavities between them, will both be diffuſed to the lateral parts of the medium by gravity. And in air the ſame effect is produced by elaſticity. The alternate condenſations and rarefactions of air, and meaning of a pulſe, will be eaſily conceived by obſerving the ſucceſſive effects produced by an elaſtic fibre, in different parts of its vibration. Let AE be a ſlender column of air, perpendicular to the fibre at the middle point, and touching it at A; and whilſt the fibre performs one vibration, backward and forward, let motion be communicated through AE, and conſequently, taking $AB = BC = CD = DE$, it will be communicated through

PLATE VIII. FIG. LXXI.

each of these spaces in $\frac{1}{4}$th of the time of a vibration. 1. During the first half of the fibre's progress, the particles of air between *A* and *B* will move successively with an accelerated velocity, and be condensed, and the condensation will be greatest in the middle of the progress. 2. During the remainder of the fibre's progress, the air between *A* and *B* will be still progressive, but retarded and rarefied, and, at the limit of the progress, its density will be equal to that of air in its natural state. Let it be observed, that, in the middle of the fibre's progress, the condensation nearest to it is first greatest, and successively transmitted to the following parts of the medium; and at the limit of the progress, the density of air, next to it, is first equal to that in the natural state, and successively transmitted to the following particles. In this half of the fibre's progress, the air between *B* and *C* will be accelerated and condensed, by the greater elasticity of the medium between it and the fibre, exactly in the same manner as that between *A* and *B*, in the first part of the progress. 3. In the first half of the fibre's regress, the the air between it and *B* will be regressive and rarefied, and its rarefaction will be greatest at the middle of its regress: the air between *B* and *C* will still be progressive, but retarded and become of the same density with air in its natural state; and that between *C* and *D* will be condensed in the same manner as that between *B* and *C* was before. 4. In the last half of the regress, the air between it and *B* will return to its first situation, and there remain quiescent; that between *B* and *C* is regressive, and at the greatest rarefaction; that between *C* and *D* is progressive, and rarefied to its natural state; and the greatest condensation is between *D* and *E*; and as motion is thus communicated from *A* to *E*, during one vibration of the fibre, or of a particle of air, backward and forward, the length *AE* is a pulse.

269. PROP. *If* A *be the altitude of a homogeneal atmosphere, of the same density with air at the place where sound is propagated, a cycloidal pendulum, whose length is* A, *will perform an oscillation in the same time that*

that a pulſe of air deſcribes the circumference of a circle, whoſe radius is A.

If the phyſical line, EG, were urged, in the extreme points of its vibration, by an elaſtic force equal to its weight, the time would be equal to that of a body vibrating forward and backward in a cycloid, whoſe length is PS, by its weight. Let this time $= t$; and the time of a ſimilar vibration of a cycloidal pendulum, whoſe length is A, be equal to T; and $t : T :: \sqrt{PO} : \sqrt{A}$ (Mechanics); elaſticity at P or S: whole elaſticity :: $HK : V$; whole elaſticity or weight incumbent : weight of $EG :: A : EG$; and conſequently the elaſticity at P or S : weight of $EG :: HK \times A : V \times EG :: PO \times A : V^2$. But the times, in which bodies deſcribe equal ſpaces, are inverſely as the ſquare roots of the forces (Newt. Princip. Vol. II. Prop. 24. Cor. 3.); and therefore calling τ the time of a vibration by the action of elaſticity, $\tau : t :: \sqrt{\text{weight of } EG} : \sqrt{\text{elaſticity at } P \text{ or } S} :: \sqrt{V^2} : \sqrt{PO \times A}$; and $\tau : T :: V : A :: BC :$ periphery of a circle whoſe radius is A (P). But τ (= time in which the pulſe deſcribes BC) : time of deſcribing $P :: BC : P :: \tau : T$, and conſequently $T =$ the time of deſcribing P. Q. E. D. FIG. LXX.

270. Cor. 1. The velocity with which a pulſe of air moves : velocity acquired in falling, by the action of gravity, through $\frac{1}{2}A :: \frac{P}{T} : \frac{A}{\text{time of falling through } \frac{1}{2}A}$, becauſe the velocity varies as the ſpace uniformly deſcribed directly and time inverſely; but T : time of falling through $\frac{1}{2}A :: P : A$, and conſequently the velocity of a pulſe of air is equal to that acquired in falling through $\frac{1}{2}A$. All pulſes therefore move with the ſame velocity, whatever be the length of PS, or the time of its deſcription, that is, whether the tone be loud or low, grave or acute.

271. Cor. 2. Since A varies as the elaſticity (E) directly and denſity (D) inverſely, and the velocity of the pulſe varies as $\sqrt{A}$, it will alſo vary as $\sqrt{\frac{E}{D}}$. Hence ſounds move a little faſter in ſummer than in winter; for the elaſticity is not encreaſed in the ſame ratio with the denſity in winter, being diminiſhed by the cold. If A be known, the velocity of ſound may be diſcovered; for the periphery of a circle is to its radius as 710 : 113; and conſequently $P = A \times \frac{710}{113}$ = the ſpace through which ſound moves, whilſt a pendulum, whoſe length is A, performs a double vibration, or in $\sqrt{\frac{A \times 2''}{39\frac{1}{2}}}$, and conſequently $A \times \frac{710}{113} \times \frac{1}{\sqrt{\frac{4A}{39\frac{1}{2}}}}$ = the rate at which ſound moves, which, after having allowed for the magnitude of the particles of air, and the quantity of vapours contained in it, amounts to about 1142 feet in 1″.

272. Cor. 3. Since the velocity of air is uniform, the latitude of a pulſe is equal to the ſpace, deſcribed in a given time, divided by the number of vibrations of the ſonorous body in that time. The body which emits, what Monſ. Sauveur denominates his *fixed ſound*, performed 100 vibrations in 1″, and, in the ſame time, the pulſe deſcribed 1070 Pariſian feet, and conſequently the latitude of the pulſe is equal to $\frac{1070}{100} = 10.7$ feet.

273. Cor. 4. The latitude of a pulſe varies inverſely as the number of vibrations of the ſounding body in a given time, or directly as the time of one vibration, or as the length of the ſounding fibre (260), or as the greateſt ſpace through which the middle point of it vibrates (253). According to Monſ. Sauveur, the

the middle point of the chord, producing his *fixed ſound*, whoſe diameter is $\frac{1}{72}$th of a line, deſcribes in its ſmalleſt ſenſible vibrations $\frac{1}{18}$th of a line, and in its greateſt, $72 \times \frac{1}{18}$ of a line, or $\frac{1}{4}$ of an inch. The latitude of the pulſe of this fixed ſound was 10.7 feet; and conſequently the greateſt ſpace, through which the middle point of this chord vibrates, is to the latitude of a pulſe as $\frac{1}{4}$ of an inch to 10.7 feet.

274. PROP. *To find the force of ſound at different diſtances from the ſounding body.*

Suppoſe the air to be divided into ſpherical ſhells, and conſidered as elaſtic bodies, A, B, C, D, &c. whoſe radii, beginning at the body, are 1, 2, 3, 4, &c. and their reſpective velocities a, b, c, d, &c. Since $a : b :: A + B : 2A$, $b : c :: B + C : 2B$, &c., $b = \frac{2Aa}{A+B}$, $c = \frac{2Bb}{B+C}$, $d = \frac{2Cc}{C+D}$, &c. (Mech.). But ſpheres, whoſe radii are 1, 2, 3, 4, &c. are 1, 8, 27, 64, &c.; and their differences, or A, B, C, D, &c. are 1, 7, 19, 37, &c., and conſequently their velocities, or a, b, c, d, &c. are as $1, \frac{2}{8}, \frac{7}{52}, \frac{266}{2912}$, &c. And in a given medium, theſe velocities encreaſe as the force of ſound at the diſtances 1, 2, 3, &c. Q. E. I.

SCHOLIUM.

275. The principal cauſe of the decay of ſound ſeems to be the want of perfect elaſticity in the air, as from hence the whole motion of the preceding ſhell is not communicated to the next ſubſequent, nor of this to the next, &c.; and conſequently the velocity of ſound decreaſes in receding from the ſonorous body, and PS and IM are alſo diminiſhed on this account. But in all vibrations, the latitude of the pulſe, and conſequently V, remain the ſame;

ſame; therefore $\frac{1}{V-IM}$, which expreſſes the condenſation, or magnitude of impulſe upon the ear, or loudneſs of ſound, decreaſes in receding from the ſounding body. In conformity with this it is found, that ſounds are heard moſt diſtinctly when north, north-eaſt, and eaſterly winds prevail (Phil. Tranſ. ann. 1705); at which time the air is dry (De Luc, Vol. II. p. 203.), and therefore more elaſtic; and the ſpaces deſcribed by the vibrating particles decreaſe leſs, though the barometer be then higheſt (ibid. Vol. I. p. 203.) and the maſs of air to be moved the greateſt.

276. PROP. *Pulſes of ſound, incident upon any reflecting ſurface, diverge from the point of incidence in all poſſible directions.*

Whatever be the angle, in which the pulſe is reflected, motion will be communicated, from the point of incidence, to the lateral parts of the medium, in the ſame manner and for the ſame reaſon, as from a point of the ſounding body. Q. E. D.

277. Cor. In the impact of air upon an obſtacle, the angle of incidence cannot properly be ſaid to be equal to the angle of reflection, ſince every impulſe becomes a center from which a new ſeries of pulſes is propagated in every direction.

PLATE VIII. FIG. LXXII.

278. PROP. *If the pulſes of any ſound, propagated from a center* A, *ſtrike againſt a number of obſtacles* C, D, E, F, G, *&c. and the ſum of the lines drawn from* A *to each obſtacle, and from each obſtacle to a ſecond point* B *be equal, an echo will be heard, provided the interval* AB *be leſs than* AC + CB *by* 127 *feet nearly.*

Each of the obſtacles *C*, *D*, *E*, *F*, &c. will be a new center of pulſes, and one ſeries of each will paſs through *B*, and ſince

AC +

$\overline{AC+CB}$, $\overline{AD+DB}$, $\overline{AE+EB}$, &c. are equal to each other, the pulſes propagated from A to C, D, E, &c. and from thence to B, will arrive there at the ſame time, and concur in producing a perception of ſound. Now (Muſſchen. Vol. II. p. 210.) the ear of an exerciſed muſician can only diſtinguiſh ſuch ſounds as ſucceed each other about 9 times in 1″, and conſequently two ſounds, at a leſs interval than $\frac{1}{9}$ of 1″, muſt coaleſce and form one ſingle ſound; but ſound deſcribes 1142 feet in 1″, and $\frac{1142}{9}$ or 127 feet nearly in $\frac{1}{9}$ of 1″. If therefore AB be leſs than $\overline{AC+CB}$ by 127 feet, two diſtinct ſounds will be heard at B. Q. E. D.

279. Cor. Since $AC+CB=AD+DB=AE+EB$, &c. the points of the obſtacles, producing an echo, lie in the ſurface of an oblong ſpheroid, whoſe foci are A and B. As there may be ſeveral ſpheroids having the ſame foci, there may be ſeveral echos from the ſame original ſound; and the ſecond or third echo may be louder than the firſt, provided the greater number of reflected pulſes be more than ſufficient to compenſate for the decay of ſound which ariſes from its moving through a greater ſpace. It is unneceſſary that the points C, D, E, &c. ſhould be placed accurately in an elliptic curve, for if the ſums of the lines $AC+CB$, $AD+DB$, &c. do not differ from each other by more than 127 feet, the pulſes will conſpire and be undiſtinguiſhable.

SCHOLIUM.

280. Though the loudeſt echo be produced when the ſounding body is in one focus of an ellipſe, and the hearer in the other; yet echoes will be heard in other ſituations, when a ſufficient number of reflected pulſes arrive at the ear to excite a diſtinct perception. A ſpeaker may often hear the echo of his own voice, when the reflecting obſtacles are properly ſituated. The common rate of ſpeaking is three ſyllables and an half, or ſeven half ſyllables in

1″; and as ſound moves at the rate of 1142 feet in 1″, if the echo arrive at the ear of the ſpeaker, after 1, 2, 3, 4, &c. half ſyllables are pronounced, it is evident that the ſpace deſcribed by it will be $\frac{1142}{7}$, $\frac{1142\times2}{7}$, $\frac{1142\times3}{7}$, &c. feet, and conſequently the diſtance of the reflecting object will be half of this fraction.

281. PROP. *In an elaſtic tube, one of whoſe orifices is very ſmall, and the other very large, the ſound of a human voice applied to the ſmaller orifice will be augmented.*

The lateral expanſion of the air is diminiſhed by the ſides of the tube, and conſequently the direct expanſion and velocity of the included air muſt be encreaſed. Every point of the tube, againſt which the air is impelled, has a tremulous motion, and becomes a new center of propagation, and conſequently the number of pulſes, ſtriking againſt the ear at the ſame time, muſt alſo be encreaſed. As the diameter of the tube perpetually encreaſes, and the parts of it vibrate in directions perpendicular to the ſurface, theſe vibrations will partly conſpire in impelling the particles forward, and thus encreaſe their velocity; and from this and the preceding cauſes, the intenſity of the ſound ought to be, and is in fact, augmented. Q. E. D.

SCHOLIUM.

282. Theſe ſeem to be the principal cauſes of the augmentation of ſound, in the ſtentorophonic tube, or ſpeaking trumpet; and that figure is to be preferred, in which their combined influence is the greateſt. That figure is aſſerted to be beſt which is generated by the revolution of a logarithmic curve round its axis; becauſe, if the included air be conceived to be divided into ſhells, perpendicular to the axis, and each to receive motion in the ſame manner with the elaſtic

Plate VIII.

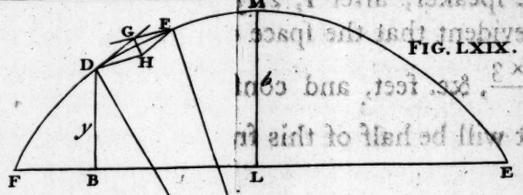


FIG. LXIX.

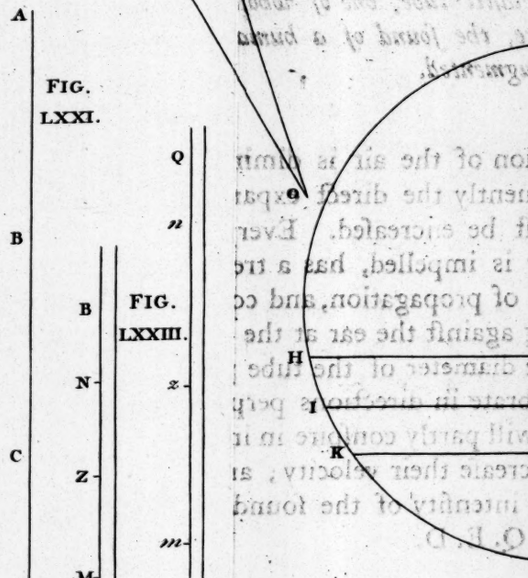


FIG. LXXI.

FIG. LXXIII.

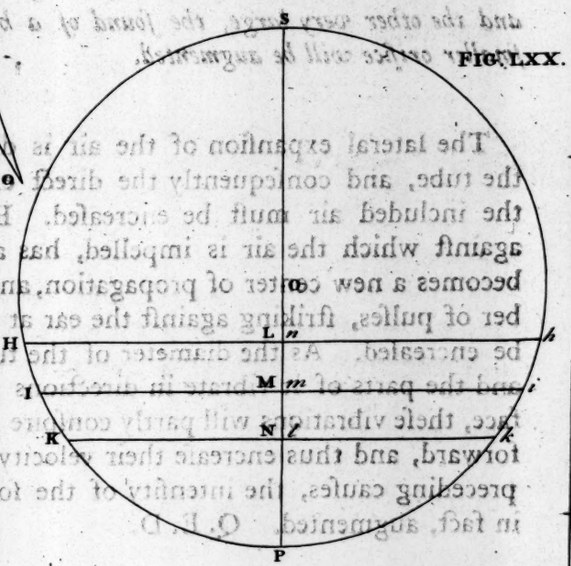
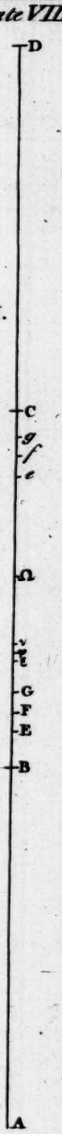


FIG. LXX.

FIG. LXXII.




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APR

elaſtic bodies, the quantity of motion communicated to the laſt ſhell, and conſequently the intenſity of the ſound, is the greateſt poſſible. This concluſion is certainly unſatisfactory; for the loudneſs of a ſound is meaſured by the number of particles ſtriking againſt the ear in a given time, and force of each, but it certainly does not follow, that, becauſe the moment of the laſt ſhell is greateſt, the moment of that ſmall cylinder of air which impinges againſt the drum of the ear is alſo the greateſt poſſible. This is leſs than it would be were the tube cylindrical. It is alſo aſſerted, that the intenſity of the naked voice is to that, with which it iſſues from the trumpet, as the orifice of the tube to the ſurface of a ſphere, whoſe radius is the radius of the tube; becauſe theſe intenſities are to each other as the maſſes of air to be moved inverſely, and ſound, by receding from the ſpeaker, is dilated in every direction through that entire ſurface, and iſſuing out of the tube, it is confined within the area of the orifice. But it is clear, that this confinement of the voice cannot be the ſole cauſe of the augmentation of ſound, for if it were, a cylindrical tube would be better than one encreaſing in diameter, and the effect would alſo be greater as the diameter was diminiſhed, which are repugnant to experience *.

* Comp. Syſt. Part II. page 55. Graves, L. I. C. XXIV. Newt. Princip. L. II. Sect. VIII. Cotes, pag. 174. Muſſchenbroek 2189, &c. Helſham, Lect. V. Young's Enquiry into the principal Phenomena of Sounds and Muſical Strings. Cyclopædia, Art. Sound.

CHAP. XIV.

THERMOMETERS.

283. DEF. 1. *TEMPERATURE* denotes the heat or coldneſs in bodies, whether within the limits of perception or not. 2. *Thermometer* is an inſtrument calculated to meaſure the different temperatures in bodies contiguous to it, being uſually a cylindrical glaſs tube, terminating in a bulb, and containing air, water, ſpirit of wine, mercury, &c. which fluids are found to occupy different portions of the tube, in different temperatures, and theſe portions being meaſured exhibit the different expanſions of the included fluid. 3. *Correſponding* points, in different thermometers, are thoſe which indicate the ſame temperature. 4. *Fundamental interval*, in different thermometers, is the ſpace intervening between two correſponding points, generally thoſe of boiling water and melting ice. 5. A *degree* is one of the diviſions into which the tube of a thermometer is divided, which are marked with numbers in arithmetic progreſſion, beginning from ſome known point; and the temperature of the included fluid is expreſſed by that number, above or below this point, which is in the plane of its upper ſurface. 6. *Similar* thermometers are thoſe, in which the ſame number is affixed to the ſame temperature.

SCHOLIUM.

284. The ſame body aſſumes different appearances, when ſubject to different degrees of heat, or of different temperatures. It may be converted into a fluid or vapour, or decompounded by the fuſion or volatilization of ſome chemical principle, or its dimenſions

ſions may be merely encreaſed or diminiſhed, without any viſible decompoſition or diſperſion of its component parts. The degree of temperature muſt be eſtimated by ſome of theſe changes, and particularly by the variation of the magnitude of the ſame maſs of matter, which only of theſe effects is determinate and meaſurable. The matter applied to this purpoſe is generally ſome fluid, as air, ſpirit of wine, linſeed oil, mercury, &c. and that is to be preferred which is affected moſt eaſily, uniformly and ſenſibly by heat and cold, capable of a conſiderable rarefaction without boiling or being diſſipated, and of condenſation without being congealed: and when equal increments of dimenſion ariſe from equal increments of heat, or bear any proportion to them, the changes of the latter are eaſily aſcertained from thoſe of the former. Linſeed oil, and every other viſcid fluid, are found to adhere to the ſides of the glaſs, and prevent the freedom and quickneſs of motion neceſſary to indicate ſudden changes of temperature. Rectified ſpirits, are well calculated to exhibit the different degrees of heat of temperate climates, but they are incapable of bearing much heat without boiling, as they cannot be heated beyond 180 deg. in Fahrenheit's thermometer, and are apt to be congealed when the weather is very cold; ſo that the latitude of heat which ſpirit of wine is capable of expreſſing, is too circumſcribed to be of general uſe. Air and water are ſtill worſe adapted to the purpoſe, being mixed with each other in various and ever changing proportions, and their dilations conſequently very irregular. Mercury ſeems not to be liable to theſe objections, being pretty homogeneous and uniformly dilated by heat, very ſenſible of the ſmalleſt changes of temperature, ſooner heated or cooled than oil or ſpirit of wine, not congealed without extreme cold, nor expanded to boiling without intenſe heat, and when properly purified, it does not adhere to the inſide of the glaſs, but aſcends and deſcends freely.

285. PROP. *The increments of expanſion in a mercurial, ſpirit of wine, or linſeed oil, thermometer, are as the increments of heat nearly.*

DEM. Let 1, 2, 3, &c. equal portions of boiling water, when the barometer is at any given height, be mixed with the ſame quantity of equally cold water, and the fluid of a mercurial, linſeed oil, or ſpirit of wine, thermometer, graduated ſo as to ſhew equal expanſions of it, and immerſed ſucceſſively in theſe mixtures, will be expanded through ſpaces which are nearly as the quantities of hot water, or as 1, 2, 3, &c. Or, let a thermometer be graduated as before, and it appears, by an immerſion of it in two equal meaſures of water, of unequal temperatures, ſeparately and when mixed together, that the common temperature is nearly an arithmetic mean between the original temperatures before the mixture. Q. E. D.

SCHOLIUM.

286. The contraction of ſpirit of wine, a little before its congelation, is found to be irregular and not to vary as the decrements of heat; and poſſibly the variation in the dimenſions of all fluids, a little prior either to their being congealed or boiling, is ſubject to a ſimilar irregularity with that of the ſpirit, and as the diſtance between theſe two points is wider in mercury than other fluids, it is on that account to be preferred. In the foregoing propoſition, it is preſumed that equal bodies of the ſame kind and of equal temperature contain the ſame quantity of heat; and if two equal bodies of the ſame kind be in contact for a ſufficient time, the heat will be uniformly diffuſed through them both, or the temperature will be the ſame in every part, which is abundantly proved by experiments; and this is alſo true, within certain limits, when the bodies are diſſimilar.

287. Cor. 1. If two equal bodies of the ſame kind, but of different temperatures, be brought into contact, the hotter body will communicate one half of its ſurplus of heat to the other; becauſe they acquire a common temperature, and this can only be effected by

by an equal diviſion of the ſurplus of heat between the two bodies. And if two unequal bodies of the ſame kind, but of unequal temperatures, be brought into contact, the ſurplus of heat in one body, above that in the other, will be divided proportionally to their maſſes, becauſe they acquire the ſame temperature. And if two diſſimilar bodies of unequal temperature be in contact, for a ſufficient time, the ſurplus of heat will be divided between them in the ratio of their maſſes, provided the colder body be not heated to its limit, or boiling heat if it be a fluid; and hence a thermometer, whoſe bulb is ſmall, may be conſidered as poſſeſſing the ſame temperature with the ſurrounding air, or any large body in contact with it.

288. Cor. 2. If two equal bodies, but of unequal temperatures, be in contact, and the common temperature be either higher or lower than the arithmetic mean, the ſurplus of heat muſt be unequally divided, and the diſpoſition to be heated greater in one body than in the other.

289. PROP. *Suppoſe any one determinate point of heat, as that of boiling water, to be given, to conſtruct thermometers which will correſpond with each other, or ſhew the ſame temperature.*

Suppoſe the ſpaces, occupied by the ſame fluid in two thermometers, when at the temperature of boiling water, to be A and B; and dividing A and B into m equal parts, mark upon each tube 1, 2, 3, . . . n of theſe parts reſpectively, and in any given temperature, the fluid will ſtand at the ſame number, in both thermometers. For the expanſion and contraction, ariſing from the ſame heat, is as the quantity of fluid; or, the denſity being given, as the magnitude or ſpace occupied by it, or as A and B; if therefore A's expanſion

or

or contraction, from the boiling point, be equal to $\frac{A}{m} \times n$, $A : B ::$ $\frac{A}{m} \times n$: to B's expanſion or contraction, which is conſequently equal to $\frac{B}{m} \times n$. If the thermometers contain different fluids, whoſe variations of magnitude from the ſame temperature, by an equal increment or decrement of heat, are as p to q, the capacities of the tubes, between the numbers 1, 2, 3, &c. reſpectively muſt be to each other as p to q, and the ſurface of the fluid, in each tube, will, in a given temperature, ſtand at the ſame number.

SCHOLIUM.

290. Suppoſing the tubes of thermometers could be graduated with accuracy into fractional parts of the whole bulk of the included fluid, different thermometers might be eaſily compared together. But it is more convenient, ſafe, and eaſy, to determine, at leaſt, two fixed points of heat, at a conſiderable diſtance from each other, and to divide the intermediate ſpace into any number of equal parts. The two points generally uſed are thoſe of water *boiling* and juſt *freezing*; the laſt of which is a conſtant and ſettled degree of heat, and the former, being dependent upon the preſſure of the air, and therefore a little variable, is adjuſted to a middle ſtate of the atmoſphere, in places nearly level with the ſea, and when, in this country, the mercury in the barometer ſtands at about 30 inches.

FIG. LXXIII.

291. PROP. *The fundamental intervals,* M N *and* m n, *of two thermometers,* A B *and* P Q, *being known, to find a point* (z) *in* P Q, *which will correſpond with any point* Z *in the other thermometer.*

In

In the ſame kind of fluid, the expanſion and contraction, ariſing from a given change of temperature, are as the quantities of fluid of a given temperature, and, in different fluids, as their quantities and capacities of changing their dimenſions conjointly; and, conſequently, all correſponding intervals are proportional to each other, or ſuppoſing Z and z to correſpond, $MN : mn :: MZ : mz$. Otherwiſe, let P be the difference of temperature at M and N, or m and n, and Q the difference of temperature at M and Z, or m and z; and (285) $P : Q :: MN : MZ :: mn : mz$. Q. E. I.

292. Cor. Since the number of degrees in MN is to that in MZ as MN to MZ, and the ſame is true of mn and mz; the number of degrees in MN: number in mn:: number in MZ: number in mz. In all ſimilar thermometers therefore, whether the fluids uſed be the ſame or different, correſponding points will always be oppoſite to the ſame number, or they will always exhibit the ſame temperature, whatever be the magnitude or ſpecies of the included fluid. And in diſſimilar thermometers, whatever numbers be affixed to M and m, and whatever be the diviſions in M and m, the numbers oppoſite to correſponding points, in any thermometers may be found by the above analogy. Thus in Newton's and Fahrenheit's thermometers, the numbers oppoſite to the freezing points are 0 and 32, and the number of degrees in the fundamental intervals are 180 and 34 reſpectively. A number in Newton's, correſponding to any number, 96, in Fahrenheit's thermometer, is found by this analogy; 180 : 64 :: 34 : 12 nearly, and 96 and 12 are nearly correſponding points. In Mr. Amonton's thermometer, the freezing and boiling points are at 51 deg. 6 lin. and 73 deg. reſpectively, and the fundamental interval conſequently is 21.5; any point in this correſponding with one, as for inſtance 112, in Fahrenheit's, is found by this analogy, 180 : 215 :: 112 : 61 nearly. By this proceſs the correſponding points of other thermometers are diſcovered, and exhibited in fig. 74.

SCHO-

SCHOLIUM.

293. Fluid thermometers differ from each other either in the mode of graduation, or in being compoſed of different fluids, according to the judgment of the inventor, and hence the ſame temperatures are marked with different numbers, and different fluids are uſed as air, ſpirit of wine, linſeed oil, mercury; the laſt of which is beſt, as its dimenſions are affected moſt uniformly by different degrees of heat; and it is capable of enduring greater degrees of heat and cold, without boiling or congelation, than any of the reſt. They conſiſt uſually of a cylindrical glaſs tube annexed to a glaſs ball, or wider cylinder, which and part of the tube, to a convenient height, are filled with ſome of the above fluids; and they are conſtructed nearly in the ſame maner, as may be eaſily underſtood from the ſhort deſcription ſubjoined, and fig. 74. Let it be obſerved, however, that ſuppoſing the magnitude of the fluid, contained in the ball and tube to vary directly as the heat, its different magnitudes are not accurate meaſures of the temperature, becauſe the glaſs is dilated and contracted by heat and cold, together with the fluid, and conſequently the apparent variation in the dimenſions of the fluid, is the difference between the real cotemporary expanſions, or ſum of the cotemporary contractions, of the fluid and glaſs. But the latter, in thermometers properly conſtructed, are inconſiderable, and unleſs very great preciſion be required, may be neglected. The change of dimenſion in the glaſs is prior to that in the fluid, and hence the fluid is found to ſink before its riſe, upon an increaſe of temperature; and if the ball be ſpherical and large, ſome time will elapſe before the fluid acquire the ſame temperature with the glaſs, or poſſibly this may never happen, if one change of temperature be ſuddenly ſucceeded by another. And, for this reaſon, the ball ſhould be oblong, or if ſpherical, not very large. The tube ſhould be capillary or ſmall, that ſmall variations of heat may be ſenſible; and graduated that the contents between the points of diviſion may be equal; which is effected, when the tube

is

is not perfectly cylindrical, by ſuffering a given quantity of mercury to paſs ſucceſſively from one end of the tube to the other; or, after the ball is filled, by pouring in preciſely the ſame weight of mercury, and marking the ſpace occupied by it; or by any other methods which the ingenuity of the conſtructor may deviſe.

1. The common *air thermometer* conſiſts of a glaſs tube *ACD*, connected at one end to a glaſs ball *B*, which being moderately heated ſo as to expel part of the air through the orifice at *D*, and that being immerſed in ſome coloured fluid, the fluid will be impelled, by the external preſſure of the atmoſphere, into the tube, as the air in *B* becomes cooler, and in a mean temperature, aſcend to ſome point *C*; and the deſcent of the fluid below *C*, or aſcent about it, indicates the exceſs or defect of the heat of the weather from the mean temperature, which is meaſured by a ſcale of the ſame length on each ſide of *C*, divided into 100 equal parts. This thermometer is very imperfect, as a change of temperature is uſually attended with a change in the weight of the atmoſphere, and conſequently of preſſure upon the ſurface of the fluid at *D*. FIG. LXXV.

2. *Amonton's thermometer* conſiſts of a glaſs tube four feet in length, open at one end and recurved at the other, which terminates in a ball, into which air is compreſſed by pouring mercury into the tube. The elaſticity of this included air is equivalent to the weight of the atmoſphere, or of the mercury in the barometer at that time, together with the column of mercury poured in above the ſurface of the mercury in the recurved part, and is therefore known. The quantity of mercury in the tube is ſo adjuſted that its height, when the ball is plunged in boiling water, together with the height of the barometer, may be 73 inches; and the height of the barometer being ſuppoſed to be 28 inches, that of the mercury in the tube is 45 inches, and here the number 73 is marked, and the tube is divided into inches, 72, 71, &c. deſcending from the point of boiling, which are ſubdivided into lines. This thermometer is inconvenient from its great length; and imperfect, becauſe the boiling point or 73 is variable, being dependant upon the height of the barometer; and beſides, air is not always equally dilated by FIG. LXXVI.

the ſame heat, its dilatation being affected by the quantity of water contained in it.

3. *Polenis's thermometer* is conſtructed in the ſame manner with that of Amonton's, but with leſs mercury, 47 inches correſponding, according to Dr. Martin, with 51 in that of Amonton's, and 53 with $59\frac{1}{2}$.

The conſtruction and compariſon of the thermometers of Florence, Fahrenheit, Newton, &c. compoſed of fluids leſs uncertain than air, viz. ſpirit of wine, linſeed oil, mercury, are obvious from art.292. and fig.74. in which their correſponding points are placed in the ſame horizontal line; and the following tables of experiments, made by thoſe of Fahrenheit and Newton, diſplay the uſes to which they may be applied.

Experiments by Fahrenheit's thermometer.

height		
600	Mercury	boils, according to Muſſchenbroek, the barometer being 29 Rhinland inches in height.
546	Oil of Vitriol	
242	Spirit of Nitre	
240	Lixivium Tartar	
213	Cow's Milk	
212	Water	boils.
206	Freſh Human Urine	
190	Brandy	
176	Alcohol, according to Muſſchenbroek	
174	Alcohol	
156	Serum of Blood and white of Egg hardens.	
146	Heat that kills Animals in a few minutes.	
108	Heat of a Hen hatching Eggs from this to 92.	
from 107 to 103	Heat of the ſkin in Ducks, Geeſe, Hens, Pigeons, Partridges and Swallows.	
106	Heat of the ſkin in an Ague-fever.	
from 103 to 100	Heat of the ſkin in Dogs, Cats, Sheep, Oxen, Swine, and other quadrupeds.	
from 99 to 92	Heat of the Human ſkin in health.	

97 Heat

height

97 Heat of a Hive of Bees.
96 In Water of this heat a Perch died in three minutes.
80 Heat of the Air in the ſhade when the weather is very hot.
74 Butter begins to melt.
64 Heat of the Air in the ſhade in warm Weather.
48 Temperate Air in England and Holland.
43 Oil of Olive begins to ſtiffen, and become opake.
32 Water juſt freezing, or Ice and Snow juſt thawing.
30 Milk freezes.
28 Urine and common Vinegar freezes.
25 Blood out of the Body freezes.
20 Good Burgundy, ſtrong Claret and Madeira freezes.
7 One part of Spirit of Wine, mixed with three parts of Water, freezes.
0 A mixture of Snow and Salt, which freezes Oil of Tartar, per deliquium, but not Brandy.
—39 Mercury freezes.

Experiments by Sir I. Newton's thermometer.

height

192 Heat of burning Coals in a ſmall Fire made of bituminous Pit-coal, and not blown with the bellows.
161 Heat with which bodies burning in the twilight, ſhine manifeſtly, but not at all or very obſcurely by day.
136 Heat with which burning bodies ſhine in a dark night, but not at all in the twilight.
114 Heat with which burning bodies, by cooling, ceaſe to be viſible in a dark night, and by heating begin to ſhine, in the ſame degree of darkneſs, ſcarce ſenſible.
96 The leaſt heat by which Lead melts.
81 Heat with which Biſmuth melts, as alſo a mixture of four parts of Lead and one of Tin.
68 The leaſt heat requiſite to melt one part of Biſmuth and eight parts of Tin.

height

57 Heat with which a mixture of two parts of Tin and one part of Biſmuth melts.

48 The leaſt heat with which a mixture of equal parts of Tin and Biſmuth melts.

$40\frac{4}{11}$ The leaſt heat with which a mixture of one part of Lead, four of Tin, and five of Biſmuth, melts and continues in fuſion.

34 Heat with which Water boils vehemently.

$28\frac{6}{11}$ Heat between Water boiling and Wax melting.

24 Heat of Water in which floating Wax melts.

$20\frac{2}{11}$ Heat of Water in which floating melted Wax begins by cooling to loſe its fluidity and tranſparency.

17 Heat of Water ſupportable to the hand, at reſt.

$14\frac{3}{11}$ Heat of Water ſupportable to the hand, in motion, and of Blood juſt out of the body.

12 Heat of the thermometer in contact with the human Body and of a Bird hatching her eggs.

6, 5, 4 } Heats of the Air in ſummer.

4, 3, 2 } Heats of the Air in ſpring and autumn.

2, 1, 0 } Heats of the Air in winter.

0 Water begins to freeze.

The different degrees of heat in this table were computed by the thermometer to the point where tin begins to melt, and the reſt by obſervations upon hot iron. By a thermometer made of linſeed oil, it was found, that dividing the whole quantity of oil, when placed in melting ſnow, into 10000 equal parts, it occupied 10256, 10705, 10725, 11516 of theſe parts, when of the ſame tem-

temperature reſpectively with the human body, water juſt beginning to boil, water boiling vehemently, and melting tin; and ſuppoſing theſe dilatations to be as the heat (285) and the heat of the human body to be numbered 12, the number correſponding to the other degrees of heat are 33, 34, 72. Theſe heats being found, a piece of iron, of a ſufficient thickneſs was heated till it became red hot, and placed where there was a conſtant current of air, ſo that the decrements of heat in the iron would be as its whole heat above that of the contiguous air, and conſequently the heats loſt and retained in geometric progreſſion; different metals and other fuſible bodies were placed upon it, and the times, in which they loſt their fluidity and began to coagulate, and laſtly when the heat of the iron became equal to the external heat of a human body, were accurately obſerved; and theſe times being taken in arithmetic progreſſion, the heats, varying in geometric progreſſion, were determined as in the table. This theory, however beautiful it may be, does not quite agree with fact. See Dr. Martin's Eſſays, p. 51.

294. PROP. *The fundamental interval of a thermometer being known, when the altitude of the barometer is 27 Paris inches, to find that interval in any other altitude of the barometer.*

Let y be a number expreſſing the height of the barometer, in 16ths of a Paris line, and the fundamental interval of the thermometer uſed, when the barometer is 27 Paris inches, be divided into 80 equal parts or degrees, and it is collected, from a number of experiments upon the heat of boiling water, at different altitudes above the level of the ſea, by M. de Luc, that $\frac{99}{200000} \times \log. y - 10387$ is always equal to the height of the thermometer, when plunged in boiling water, above melting ice, expreſſed in hundredth parts of a degree. Q. E. I.

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*295. M. de Luc conſiders the index and ſeven following figures as integer numbers, and according to the common uſe of logarithms therefore, the expreſſion above, or $\frac{99}{200000} \times \log. y - 10387$ is equal to $\frac{99 \times 100}{2} \times \log. y - 10387$, or $4950 \times \log. y - 10387$, which is uſed in the following corollaries.

296. Cor. 1. Let y = a number expreſſing the height of the barometer in 16ths of a Paris line;
v = the ſame in Paris lines;
x = the ſame in 10ths of a Paris inch;
z = the ſame in 10ths of an Engliſh inch.

And $y = 16v$; $v = \frac{x \times 12}{10}$; and becauſe $x \times P = z \times E$ (ſubſtituting P and E for a Paris and Engliſh inch reſpectively), $x : z :: E : P :: 2 : 2.1315$, and $x = \frac{z \times 2}{2.1315}$; and conſequently log. of y $=$ log. $16 +$ log. $v =$ log. $16 +$ log. $12 -$ log. $10 +$ log. $x =$ log. $16 +$ log. $12 +$ log. $2 -$ log. $10 -$ log. $2.1315 +$ log. z, and $4950 \times$ log. $y - 10387 = 4950 \times$ log. $z +$ log. $16 +$ log. $12 +$ log. $2 -$ log. $10 +$ log. $2.1315 - 10387 = 4950 \times$ log. $z - 4171.55 =$ the height of de Luc's thermometer, in 100th parts of a degree of his ſcale, when z is expreſſed in 10ths of an Engliſh inch. Divide both ſides of the equation by 100, and for z ſubſtitute 30×10, and $\frac{4950}{100} \times \log. 300 - 41.7155 = 80.902 =$ the height of de Luc's thermometer, expreſſed in degrees of his ſcale, when the height of the barometer is 30 Engliſh inches.

297. Cor.

* See De Luc Reſerches ſur les Mod. de l'Atmoſphere, Tom. I. p. 382, &c. and Tom. II. p. 398, &c. Phil. Tranſ. Vol. LXIV. Art. XX. and XXX.

297. Cor. 2. In Fahrenheit's thermometer conſtructed by Bird, the boiling point is fixed when the height of the barometer is 30 inches, and the interval, between that and the freezing point, being 180 deg., the correſponding intervals, in Bird's Fahrenheit and de Luc's thermometers, are reſpectively 180 and 80.902. Let $u =$ the interval in Fahrenheit, correſponding with the point 80 in de Luc, and (291) $80.902 : 80 :: 180 : u$, and $u = \frac{80 \times 180}{80.902} = 177.99$, and $177.99 + 32$ (32 being Fahrenheit's freezing point) $= 209.99$ = a number in Fahrenheit correſponding with 80, the boiling point of de Luc.

298. Cor. 3. Let F and L be numbers expreſſing any correſponding intervals in Bird's Fahrenheit and de Luc's thermometers, in degrees, and $F : L :: 180 : 80.902 :: 1800 : 809$, and $L = \frac{F \times 809}{1800} = \frac{F \times 899}{2000}$ nearly. But $L = \frac{4950}{100} \times \log. z - 41.7155$, and conſequently $F = \frac{4950 \times 20}{899} \times \log. z - 92.80 =$ the height of Bird's Fahrenheit in degrees, when the height of the barometer is expreſſed in 10ths of an Engliſh inch. From this equation the following table is computed. The firſt column is the height of the barometer in Engliſh inches; the ſecond ſhews the equation to be added to, or ſubtracted from, 212, the boiling point of Fahrenheit, and the ſum or difference, according as the ſign is + or —, is the true boiling point. This point may be found with ſufficient accuracy, by the difference in the third column, and the rule of proportion, for any intermediate height of the barometer.

Height of barom.	Equation.	Difference.
31.0	+ 1.57	0.78
30.5	0.79	0.79
30.0	0.00	0.80
29.5	— 0.80	0.82

29.0

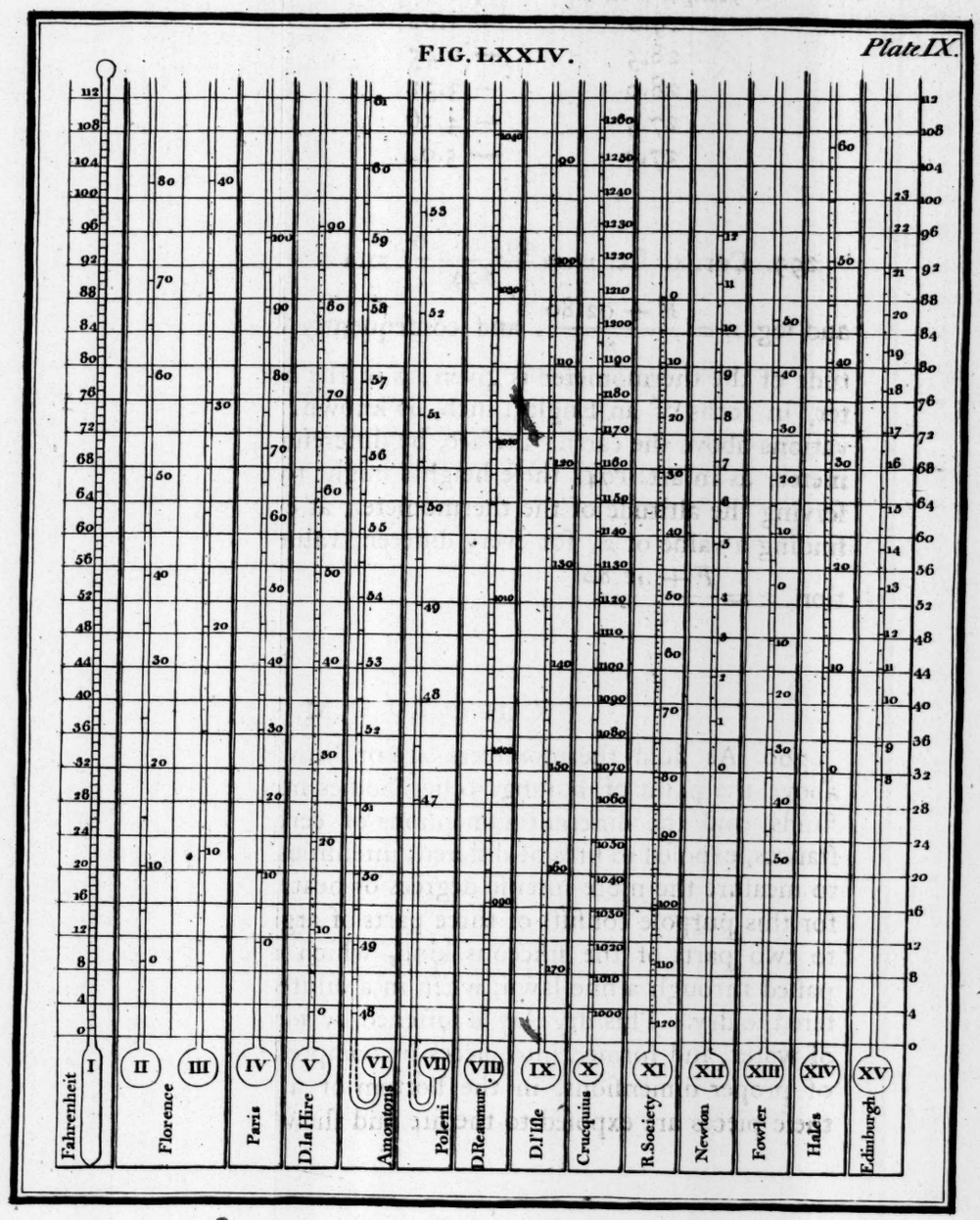
Height of barom.	Equation.	Difference.
29.0	— 1.62	0.82
28.5	— 2.45	0.83
28.0	— 3.31	0.86
27.5	— 4.16	0.85
27.0	— 5.04	0.88

299. Cor. 4. Let $n = \frac{4950 \times 20}{899}$, and $n \times \log. z - 92.80 = F$, and $\log. z = \frac{F + 92.80}{n}$, and conſequently when F, or the altitude of the thermometer is given, z or the height of the barometer, in 10ths of an Engliſh inch, is known. In aſcertaining elevations above the earth's ſurface, by different heights of the barometer, as in art. 194, theſe heights ought to be corrected by obſerving the altitude of the thermometer, at every obſervation, and finding a value of z, for every different value of F, by this equation, $z = \frac{F + 92.80}{n}$.

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300. As fluid thermometers are of little uſe in temperatures above the point of boiling, other bodies muſt be ſubſtituted for fluids, and the different dimenſions of certain argillaceous ſubſtances, expoſed to fires of different intenſities, ſeem well calculated to meaſure the more intenſe degrees of heat. The ſubſtance uſed for this purpoſe conſiſts of three parts of argillaceous earth, added to two parts of the ſiliceous kind, which are well waſhed, and paſſed through a fine lawn, when in a diluted ſtate, and then ſuffered to dry. This dry clay is ſoftened by adding $\frac{1}{5}$ths of its weight of water, and formed into pieces by compreſſing it through holes, of proper dimenſions, in the bottom of a veſſel, and when dry, theſe pieces are expoſed to the fire and ſhew its intenſity by a contraction

FIG. LXXIV. *Plate IX.*




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traction of their dimenſions; which are aſcertained by a gauge made of two ſtraight pieces of braſs, 24 inches long, divided into inches and tenths, fixed $\frac{1}{10}$ of an inch aſunder at one end, and $\frac{3}{10}$ at the other, upon a braſs plate, ſo that one of the thermometric pieces, when properly pared, may juſt fill the wider end. If this piece be diminiſhed by the air $\frac{1}{5}$ of its bulk, it will paſs through half the length of the gauge; if diminiſhed $\frac{2}{5}$ it will paſs on to the narrower end; and in any intermediate contraction, the degree at which the piece ſtops againſt the converging ſides, will meaſure its contraction, each diviſion of the ſcale anſwering to $\frac{1}{200}$ part of the breadth of the piece of clay.

Experiments by this thermometer.

Deg.

156 Common Chineſe Porcelain becomes ſoft and applies itſelf cloſe to any rough ſurface it reſts upon.
150 A Heſſian crucible melted into a flag-like ſubſtance.
130 Caſt iron melts.
125 The greateſt heat of a common ſmith's forge.
124 Vitrification of the materials of plate glaſs.
114 Do of flint glaſs.
121 Bow-China
112 Derby China } vitrified.
105 Chelſea China
102 Stone ware fixed.
95 Welding heat of iron.
86 Queen's ware fixed.
32 Gold
28 Silver
27 Sweediſh copper } melted.
21 Braſs
0 Red heat viſible in the day *.

* Phil. Tranſ. Vol. LXXII. Part II. p. 305.

CHAP. XV.

OF THE MOTION OF BODIES IN FLUIDS.

301. DEF. A POINT in any ſurface, moving in a reſiſting quieſcent medium, is ſaid to be *reſiſted directly*, when the line deſcribed by it is perpendicular to the ſurface at that point; or when this line and perpendicular are both parallel to the direction of the fluids motion, when both the plane and fluid move; or when the perpendicular coincides with the direction of the ſtream, ſuppoſing the fluid only to move. And it is *reſiſted obliquely* when theſe conditions do not obtain.

302. Cor. 1. The direction of motion of any plane ſurface, moving in a reſiſting medium, is not affected when the reſiſtance upon every point is direct. And the effect is evidently the ſame whether the plane move in any direction, or the fluid impinge upon it, with the ſame velocity, in an oppoſite direction.

303. Cor. 2. The direct reſiſtance upon a body is meaſured by its retardation, or decrement of motion produced in a given time; and if the time be not given, it varies as the decrement of motion, directly and time inverſely. If therefore Q, R repreſent the quantity of matter in a body and reſiſtance upon it, and V, T, S, be the velocity, time and ſpace reſpectively; R varies as $\frac{Q \times -\dot{V}}{\dot{T}}$, or as $\frac{Q \times -V\dot{V}}{\dot{S}}$; $\dot{S}$ varies as $\frac{Q \times -V\dot{V}}{R}$, $-V\dot{V}$ varies as $\frac{R \times \dot{S}}{Q}$.

304. Cor.

304. Cor. 3. If a given body aſcend or deſcend rectilineally by a reſiſting fluid, by the action of a centripetal force (F); $\dot{V}$ varies as $\overline{F-R} \times \dot{T}$; and $-\dot{V}$ varies as $\overline{F+R} \times \dot{T}$; $V\dot{V}$ varies as $\overline{F-R} \times \dot{S}$, and $-V\dot{V}$ as $\overline{F+R} \times \dot{S}$. When the reſiſtance and centripetal force are equal, or $F=R$, the increment of velocity is equal to nothing, and the body will continue to deſcend uniformly.

305. PROP. *The direct reſiſtance of fluids* (R) *ariſing from inertia, to plane ſurfaces moving in them, varies as the area of the ſurface* (A), *the denſity of the fluid* (D), *and the ſquare of the velocity* (V).

The reſiſtance evidently varies in the ſame ratio with the area of the ſurface, the number of equal particles ſtriking it in a given time, and the force or velocity of each; and, as the number of particles varies as the denſity and velocity conjointly, the reſiſtance will vary in a compound ratio of the area, denſity, and ſquare of the velocity, or R varies as $A \times D \times V^2$. Q. E. D.

306. Cor. When the ſurface is given, R varies as $V^2 \times D$, and, the denſity alſo being given, as V^2. But the reſiſtance of a body, moving in air or water, is found by experiment to vary as V^2, and as $V^2 \times D$, comparing the reſiſtance upon it when moving in air and water (38, 39); and conſequently the reſiſtance of theſe fluids ariſes from their inertia, and the tenacity of the particles of water is comparitively evaneſcent.

307. PROP. *The direct and oblique reſiſtances, of a given fluid upon a given plane ſurface* A B, *are to each other as the ſquare of the radius to the ſquare of the ſine of the inclination of the ſtream.* FIG. LXXVII.

The whole force of reſiſtance varies as the number of equal particles impinging upon the plane, in a given time, and the force or velocity of each. Let PA repreſent, in quantity and direction, the force or velocity of one particle, and be reſolved into two forces, PR perpendicular and AR parallel, to the plane; AR is loſt, making the particle only move parallel to the ſurface, and PR is the only part of the force impelling the plane; and the direct force of one particle is therefore to its oblique force, as $PA : PR$. But the number of particles impelling the plane, when the impulſe is direct, is to the number, when the impulſe is oblique, as $EA : EB$ (ſuppoſing EB to be perpendicular to the ſtream) or as $PA : PR$; and conſequently the reſiſtance, when the fluid impels the plane perpendicularly, is to the reſiſtance when it impels it obliquely, as $PA^2 : PR^2 ::$ rad.^{2} : $\overline{\text{ſine of inclination}}^2$. Q. E. D.

308. Cor. 1. Since the reſiſtance, when the impulſe is direct, varies as $A \times D \times V^2$, the reſiſtance, when the impulſe is oblique, varies as $A \times D \times V^2 \times \frac{\overline{\text{ſin. of } \angle \text{ incl.}}^2}{\text{rad.}^2}$, or as $\overline{\text{ſin. of incl.}}^2 \times V^2$, ſuppoſing the area, denſity and radius to be given.

309. Cor. 2. Suppoſing the fluid to impel the plane obliquely, and the reſiſtance perpendicular to it to be repreſented by PR, and reſolved into two forces, RD perpendicular to, and PD coincident with the ſtream; PD is the magnitude of reſiſtance eſtimated in the direction of the fluids motions, and is to the perpendicular reſiſtance as $PD : PR :: PR : PA ::$ ſine of $\angle$ of incl. : radius. And conſequently the reſiſtance, eſtimated in the direction of the fluids motion, varies as $A \times D \times V^2 \times \frac{\overline{\text{ſin. } \angle \text{ incl.}}^3}{\text{rad.}^3}$, or the reſt being given as $\frac{\overline{\text{ſin. } \angle \text{ incl.}}^3}{\text{rad.}^3} \times V^2$.

310. Cor.

310. Cor. 3. The effect of the fluid in the direction PR is to its effect in the direction RD, as $PR : RD ::$ radius : cosine of $\angle$ of incl.; and consequently the force in the direction RD varies as $\frac{\text{sin. of } \angle \text{ of incl.}^2 \times \text{cosine of } \angle \text{ of incl.}}{\text{rad.}^3}$, supposing A, D and V to be given.

311. Cor. 4. If another plane Ea be joined to EA, equal to it and equally inclined to the direction of motion EX, and the resistances PA and pa be resolved in the same manner, those in the directions RD, and rd are equal and opposite, and consequently destroy each other; those in the directions PR and pr are equal, and vary as $A \times D \times V^2 \times \frac{\text{sin. of } \angle \text{ of incl.}^2}{\text{rad.}^2}$; and those in the direction of motion are also equal, and vary as $A \times D \times V^2 \times \frac{\text{sin. of } \angle \text{ of incl.}^3}{\text{rad.}^3}$. And if AEa represent a curved surface, formed by the revolution of a curve line AEa round an axis EX, and this surface, or the fluid, move in a direction parallel to EX, and Aa be a small annular part of the surface; the resistance of the fluid upon this annulus varies as its area $\times D \times V^2 \times \frac{PR^3}{PA^3}$, or as the area $\times D \times V^2 \times \frac{AN^3}{AE^3}$, supposing AN to be perpendicular to the axis, and AE a tangent at A in the same plane with EX.

312. Prop. *The resistance of a fluid impelling a plane surface* (A) *directly, with a velocity equal to* V, *is equivalent to the weight of a column of the same fluid perpendicularly incumbent upon an orifice equal to* A, *in the bottom of a vessel, whose altitude* (h) *is such that the velocity of efflux is equal to* V.

The

The reſiſtance upon A is meaſured by the number of equal particles impelling it directly in a given time, and velocity of each, or, when eſtimated for an inſtant, by one lamina of the fluid whoſe area is A multiplied into V. But if this lamina were to impinge directly againſt the loweſt lamina of the column with a velocity equal to V, it would clearly prevent the efflux of the fluid, or be equivalent to the weight of the column perpendicularly incumbent, which generates the velocity of efflux. Q. E. D.*

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313. The reſiſtance of a fluid is compoſed of the moment of every particle impinging againſt the reſiſted ſurface, and impact cannot be ſaid properly to be equal to a weight. But conſidering impact and weight as forces, whoſe magnitudes are eſtimated by their cotemporary effects, they may be ſaid to be equal; and the direct reſiſtance of a given fluid upon a plane ſurface is equal to the weight of a column of that fluid whoſe magnitude is $A \times h$, and oblique reſiſtance, in the direction of motion, is equal to the weight of a column of fluid whoſe magnitude is $A \times h \times \frac{(\text{ſin. of } \angle \text{ of incl.})^3}{\text{rad.}^3}$.

FIG. LXXVIII.

314. PROP. *If a ſphere and a cylinder, of the ſame diameter, move in a reſiſting medium with the ſame velocity* V, *in a direction parallel to the axis of the cylinder, the reſiſtance, ariſing from inertia, upon the baſe of the cylinder is equal to twice that upon the ſurface of the ſphere.*

Suppoſe theſe bodies to be ſo poliſhed that their poſterior ſurfaces ſuffer no reſiſtance; let ABD be a ſemicircle upon the ſurface

* It is ſuppoſed, in this propoſition, that the impreſſions tranſmitted to the loweſt lamina by the weight of thoſe above it, are communicated in an inſtant, and its velocity conſequently is equal to that acquired in falling through the whole altitude. The propoſition is not true except upon this ſuppoſition.

face of the ſphere and in the ſame plane with the line CT deſcribed by the center C; $CE = r$; $FH = y$; $CG = x$; $p =$ the periphery of a circle whoſe diameter is unity. Draw the ordinates FH, EG, infinitely near to each other, and $2py$ is the periphery of a circle whoſe radius is y; and $2py\dot{x}$ is the fluxion of the ſurface of the ſphere, the reſiſtance of which varies as $2py\dot{x} \times \frac{\overline{CG}^3}{r^3} \times v^2$, or, becauſe $2py\dot{x} = 2pr\dot{x}$, as $2pr\dot{x} \times v^2 \times \frac{x^3}{r^3}$, whoſe fluent $\frac{2prx^4 \times v^2}{4r^3}$, or, when $x = r$, $\frac{pr^2 \times v^2}{2}$ is the reſiſtance upon the hemiſphere. But the reſiſtance upon the baſe of the cylinder varies as its area $\times v^2$, or as $pr^2 \times v$; and conſequently the reſiſtance of the ſphere : reſiſtance of the cylinder :: $\frac{pr^2 \times v^2}{2} : pr v^2$ :: 1 : 2. *Q. E. D.*

315. Cor. 1. Since the reſiſtance upon the ſphere varies as $\frac{pr^2 \times v^2}{2}$, the reſiſtances upon different ſpheres, moving in the ſame fluid, will vary as the ſquares of their diameters and ſquares of the velocities, and moving in different fluids, they will vary in a compound ratio of the ſquares of their diameters, ſquares of their velocities, and denſities.

316. Cor. 2. The reſiſtance of any fluid upon the baſe of a cylinder, moving in a direction parallel to its axis, is equal to the weight of a column of that fluid, whoſe magnitude is $pr^2 h$ (ſuppoſing h to be the height of a column of the fluid whoſe weight generates a velocity equal to v, as in art. 312). The reſiſtance alſo upon the circular annulus $2py\dot{x}$ is equal to the weight of a column of the fluid, whoſe magnitude is $2py\dot{x} \times \frac{x^3}{r^3} \times h = \frac{2pr\dot{x} \times x^3 \times h}{r^3}$, whoſe

whose fluent, when $x = r$, $= \frac{p r^2 \times b}{2}$, whose weight is the resistance upon the hemisphere.

317. PROP. *The force of resistance to a spherical body, moving in an uniform resisting medium with a velocity equal to* v, *is to the constant force* F, *which would generate the velocity* v *in the sphere, in the time of describing* $\frac{2}{3}$ *parts of its diameter uniformly with this velocity, as the density of the fluid to the density of the sphere.*

Let D = the diameter of the sphere; W = its weight; b = its density; h = the space through which a body must fall from rest, in vacuo, to acquire the velocity v; f = the density of the fluid.

The time of describing $\frac{2}{3}d$ uniformly with the velocity v, is equal to the time of describing half that space, or $\frac{1}{3}d$, from rest, by the constant action of F; and (Mechan. art. 532.) $F : W :: \frac{v^2}{\frac{4d}{3}} : \frac{v^2}{h}$, and $F = \frac{3Wh}{4d}$. But $W = \frac{p \times d^3 \times b}{6}$, and consequently $F = \frac{p \times d^2 \times h \times b}{8}$. But the resistance of the sphere is equal to the weight of a portion of the fluid, whose magnitude is $\frac{p r^2 h}{2}$ or $\frac{p d^2 h}{8}$, and is equal to $\frac{p d^2 h \times f}{8}$; and consequently F : resistance of the sphere $:: \frac{p d^2 hb}{8} : \frac{p d^2 hf}{8} :: b : f$, or as the density of the sphere : density of the fluid. Q. E. D.

318. Cor.

318. Cor. 1. The reſiſtance therefore is always equal to $\frac{3Wbf}{4db}$. Or, let $v = 2c$, and b will be equal to $\frac{c^2}{s}$ (s being $16\frac{1}{12}$ feet), and $F = \frac{3W \times c^2}{4ds}$; therefore $\frac{3W \times c^2}{4ds}$: reſiſtance of the ſphere :: $g : f :: W$: weight of an equal magnitude of the fluid; and if $w =$ the relative weight of the ſphere, the reſiſtance will be equal to $\overline{W - w} \times \frac{3c^2}{4ds}$.

319. Cor. 2. The greateſt velocity, acquired by a body deſcending in a reſiſting medium, is when the reſiſtance becomes equal to the force of deſcent, or its relative weight, or when $\overline{W - w} \times \frac{3c^2}{4ds} = w$. And this velocity conſequently, or $2c$ at that time $= 2\sqrt{\frac{w \times 4ds}{3 \times \overline{W - w}}}$.

320. Cor. 3. If S be the ſpace through which the ſphere deſcends by its relative weight, in a medium without reſiſtance, to acquire the greateſt velocity $2c$ it can acquire in the ſame medium, when reſiſted; then the ſquare of the velocity acquired in falling through s by the force $W : 4c^2$ (or ſquare of the velocity acquired in falli g through S by the relative weight w) :: $W \times s : w \times S$ :: $4s^2 : \frac{W}{W - w} \times \frac{16ds}{3}$, therefore $S = \frac{W \times \frac{4}{3}d}{W - w}$, and $S : \frac{4}{3}d :: W : W - w$ :: denſity of the ſphere : denſity of the fluid.

321. Cor. 4. If T be the time in which this greateſt velocity ($2c$) is generated, without reſiſtance, $s : S :: 1^2 \times W : T^2 \times w$ and $T = \sqrt{\frac{S \times W}{ws}}$.

322. Cor. 5. If a ſpherical body were to move, in the ſame fluid, with different velocities, till the velocities were deſtroyed, and the reſiſtances were to continue the ſame as at firſt, the ſpaces deſcribed would be the ſame, being as the ſquares of the velocities directly, and the reſiſtances or ſquares of the velocities inverſely.

FIG. LXXVIII.

323. PROP. *To find the reſiſtance of a line, plane figure, or ſolid formed by a revolution about an axis, when they move in a direction parallel to their axis.*

1. The reſiſtance upon the ſmall arc $\dot{z}$ is equal to $h \times \dot{z} \times \frac{FL^3}{FG^3}$ (313) $= h \times \frac{\dot{z} \times \dot{y}^3}{\dot{z}^3} = \frac{h \times \dot{y}^3}{\dot{y}^2 + \dot{x}^2} = \frac{h \times \dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}}$; and the reſiſtance upon $FH = \dot{y} \times h$; and the reſiſtance upon the curve line ABD: reſiſtance upon AD as the fluent of $\frac{h\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}}$: fluent of $h\dot{y}$.

2. If the curve line ABD revolve round an axis BC; the reſiſtance upon the annular ſurface generated by the arc $\dot{z}$ is equal to $2p\,y\dot{z} \times \frac{y^3}{FG^3} \times h$ (313, 314) $= 2p\,y\dot{z} \times \frac{\dot{y}^3}{\dot{z}^3} \times h = \frac{2p\,y\dot{y} \times h}{1 + \frac{\dot{x}^2}{\dot{y}^2}}$. And the reſiſtance upon the annulus generated by $H1$ is equal to $2p\,y\dot{y} \times h$. Theſe reſiſtances are therefore to each other as $\frac{y\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}} : y\dot{y}$; and the reſiſtances upon the ſolid and its baſe are as the fluents of theſe quantities, which may be found by ſubſtituting the value of $\dot{x}^2$ in terms of y derived from the properties of the curve. Q. E. I.

324. Cor.

324. Cor. 1. Let ABC be an iſoſceles triangle, and move in the direction of the perpendicular, at the biſection of the baſe, DC; and let $Dc = a$, $AC = c$, $AD = b$. From ſimilar triangles $x = \frac{y \times a}{b}$ and $\dot{x}^2 = \frac{a^2 \dot{y}^2}{b^2}$, and the reſiſtances upon AD and AC are as the fluents of $\dot{y}$ and $\frac{\dot{y}}{1 + \frac{a^2}{b^2}}$, or as $a^2 + b^2 : b^2$, or as $AC^2 : AD^2$. FIG. LXXIX.

325. Cor. 2. If ABD be a circular arc and the radius $= r$; then $\dot{x} = \frac{y\dot{y}}{r - x} = \frac{y\dot{y}}{\sqrt{r^2 - y^2}}$, and $\frac{\dot{x}^2}{\dot{y}^2} = \frac{y^2}{r^2 - y^2}$; therefore $\frac{\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}}$ $= \frac{\dot{y}}{1 + \frac{y^2}{r^2 - y^2}} = \frac{\sqrt{r^2 - y^2} \times \dot{y}}{r^2}$, and its fluent $= y - \frac{y^3}{3r^2}$. The reſiſtances therefore upon the diameter and the ſemicircular arc ABD are to each other as $y : y - \frac{y^3}{3r^2} :: 3r^2 : 3r^2 - y^2$ and when $y = r$ as $3 : 2$.

326. Cor. 3. Let ABD be an hemiſphere, then $\frac{y\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}} =$ $\frac{\sqrt{r^2 - y^2}}{r^2} \times y\dot{y}$ whoſe fluent $= \frac{y^2}{2} - \frac{y^4}{4r^2}$, and the reſiſtance againſt the baſe, is to the reſiſtance upon the convex ſurface as $y^2 : y^2 - \frac{y^4}{2r^2}$ as $2r^2 : 2r^2 - y^2$, or when $y = r$, as $2 : 1$, the ſame as in art. 314.

327. Cor. 4. Let the cycloidal arc BDC revolve round the baſe AC, and from the nature of the curve $y = z - \frac{z^2}{4a}$ and $\dot{y} =$ $\dot{z} -$ FIG. LXXX.

$\dot{x} - \frac{z\dot{z}}{2a}$, and $\frac{y\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}} = \frac{\dot{y}^2 \times y\dot{y}}{\dot{x}^2} = \frac{y\dot{y} \times \overline{2a - z}^2}{4a^2} = \frac{\overline{a - y} \times y\dot{y}}{a}$ whose fluent $= \frac{y^2}{2} - \frac{y^3}{3a}$. The resistance therefore upon the base and convex surface are to each other as $y^2 : y^2 - \frac{2y^3}{3a} :: 1 : \frac{1 - 2y}{3a}$, or when $y = a$ as $3 : 1$.

FIG. LXXIX. 328. Cor. 5. If ABC be a cone, the resistances upon the base and surface are to each other as the fluents of $y\dot{y}$ and $\frac{y\dot{y}}{1 + \frac{\dot{x}^2}{\dot{y}^2}}$, or because $x = \frac{ay}{b}$ and $\dot{x}^2 = \frac{a^2\dot{y}^2}{b^2}$, as $y^2 : \frac{y^2 b^2}{b^2 + a^2} :: AC^2 : AD^2$.

FIG. LXXXI. 329. PROP. *Suppose a frustum of a cone,* ADCF, *of a given base and altitude, to move in an uniform resisting medium, in a direction parallel to the axis* BE, *with its smaller end first, to find the altitude of the cone, of which it is a frustum, when the resistance is the least possible.*

Let AGC be the surface of the cone; AC its base; $BG = x$; $AB = b$; $BE = a$. And the resistance upon AGC : resistance upon $AC :: AB^2 : AG^2$; and the resistance upon AGC therefore $=$ resistance upon $AC \times \frac{AB^2}{AG^2}$. For the same reason the resistance upon $DGC =$ resistance upon $\frac{DF \times DE^2}{DG^2}$. Let the resistance upon $AC = AG^2$, and consequently the resistance upon $DF = DG^2$ (because resistance upon AC : resistance upon $DF :: AC^2 : DF^2 :: AG^2 : DG^2$) : and the resistances upon AGC and DGC are respectively

tively equal to AB^2 and DE^2. The difference of these resistances together with the resistance upon DF are equal to the resistance upon the frustum $= AB^2 - DE^2 + DG^2 = AB^2 + EG^2 = AB^2 + \overline{BG - BE}^2 = AB^2 + BG^2 - 2BG \times BE + BE^2 = AG^2 + BE^2 - 2BG \times BE$. Therefore the resistance of the frustum : resistance of the base $AC :: AG^2 + BE^2 - 2BG \times BE : AG^2$; and substituting unity for the resistance upon the base, the resistance of the frustum $= 1 + \frac{BE^2 - 2BG \times BE}{AG^2} = 1 + \frac{a^2 - 2ax}{b^2 + x^2}$, whose fluxion or $- \frac{2ab^2\dot{x} - 2ax^2\dot{x} - 2a^2x\dot{x} + 4ax^2\dot{x}}{\overline{b^2 + x^2}^2} = 0 = 2ax^2 - 2a^2x - 2ab^2$ and $x = \sqrt{\frac{a^2}{4} + b^2} + \frac{a}{2}$. Q. E. I.

330. PROP. *Suppose a plane surface begin to revolve about an axis, by the impulse of a fluid moving parallel to the axis, to find the angle in which the plane is enclined to the direction of the fluid's motion, when the impulse is the greatest possible* FIG. LXXVII.

Let PA be the direction and velocity of the fluid and equal to r; AE the plane surface fixed to an arm issuing from the axis; and let PA be resolved into two forces, PR perpendicular, and AR parallel to the plane, and let PR be revolved into two forces PD coincident with the direction and RD perpendicular to it. RD is the force of the fluid to make the plane revolve round the axis and varies as $\frac{PR^2 \times AR}{PA^3}$ (310), or as $\frac{\overline{r^2 - x^2} \times x}{r^3}$, which being a maximum, its fluxion, or $\frac{\dot{x}}{r} - \frac{3x^2\dot{x}}{r^3} = 0$; consequently $r^2 = 3x^2$, and $r : x :: \sqrt{3} : 1 ::$ radius : cosine of inclination; and the angle of inclination $= 54°. 44'$. Q. E. I.

331. Cor.

331. Cor. If the angle of inclination of the plane to the axis be 54°. 44′. the force of the wind is greateſt when its direction is parallel to the axis.

332. PROP. *Suppoſe a plane ſurface be inclined to the axis in an angle of* 54°. 44′. (a) *the velocity of the wind to be* f *feet in* 1″, *and the time of one revolution to be* t″; *to find that point of the plane or diſtance* (x) *from the axis, upon which the wind does not act.*

FIG. LXXVII. Let PA be the direction and velocity of the wind, and reſolved as before, and ſo ſmall that RD may coincide with the circular arc deſcribed by A; A the point whoſe velocity round the axis is equal to RD, and its diſtance from the axis $= x$. And $f : PA :: 1'' : \frac{PA''}{f}$ the time of deſcribing PA or RD) $= \frac{RD \times r^2}{f \times \text{ſin. of } a \times \text{coſin. of } a}$. But the time of deſcribing RD by the plane $= \frac{t \times RD}{2px}$ (ſuppoſing h to be the periphery of a circle whoſe diameter is 1); and conſequently $\frac{RD \times t}{2px} = \frac{RD \times r^2}{f \times \text{ſin. of } a \times \text{coſin. of } a}$, and $x = \frac{f \times \text{ſin. of } a \times \text{coſ. of } a \times t''}{2p \times r^2}$. Q. E. I.

333. Cor. If the plane be longer than x, the velocities of all points more remote from the axis than the extremity of x, will be greater than that of the wind, and conſequently they will protrude the air and be retarded.

FIG. LXXXII. 334. PROP. *Suppoſe a ſtream of water to flow from the inclined plane* A C, *whoſe elevation above the horizon is* E°, *and the perpendicular* AH = PC = p *feet*; *to find the radius of the water wheel* DBQ, *when the effect of the ſtream upon it is a maximum.*

Let

Let the wheel touch AC at B and the horizon at D, and draw DE, CE, BE to the center. Let $EB = x$, $AC = a$; then the $\angle ACH = BED$ and $\angle BEC = \frac{E^\circ}{2}$; and rad. (1) : x :: tang. of $\frac{E}{2}$: $CB = x \times$ tang. of $\frac{E}{2} = x \times T$; and $AB = a - Tx$. From similar triangles $PC : FB :: AC : AB$, or $a : a - Tx :: p : FB = p - \frac{pTx}{a}$; but the effect of the ſtream, for an inſtant, is as the velocity of the ſtream × radius of the wheel, or as $\sqrt{p - \frac{pTx}{a}} \times x$, which, or $x^2 \times \overline{a - Tx}$, being a maximum, its fluxion, or $2axx - 3Tx^2\dot{x} = 0$, and $x = \frac{2a}{3T} = \frac{2p}{3T \times \text{fine } E}$. Q. E. I.

335. Cor. If $E = 30^\circ$, $2p = 8$ feet, $x = 39.8$ feet; and if the water deſcend perpendicularly, then $T =$ tang. of $45^\circ = 1$; $a = PC = p$, and $x = \frac{2}{3}a = 5\frac{1}{3}$ feet.

336. PROP. *Let the velocity of a ſpherical body, moving in a right line,* CP, *in an uniform reſiſting medium, be given at any point* C, *and equal to that acquired in falling from reſt by the force of gravity, without reſiſtance, through* c *feet, to find the velocity,* v, *at any other point* V. FIG. LXXXIII.

Let $CV = x$; $v =$ the velocity acquired in falling from reſt, in vacuo, by gravity through h feet; and the denſity of the body to that of the medium as $b : 1$; $d =$ the diameter of the ſphere, and $W =$ its weight.

The

The reſiſtance at $C = \frac{3Wc}{4db}$, and at V the reſiſtance $= \frac{3Wb}{4db}$ (318); but $v\dot{v} = 2Wsb^{\cdot}$ * $= -\frac{6Wsb\dot{x}}{4db}$, and $\frac{b^{\cdot}}{b} = -\frac{3\dot{x}}{4db}$, and the hyperbolic logarithm of $b = -\frac{3x}{4db}$ + correction $= -\frac{3x}{4db}$ + log. of c (for when $x = o$, $b = c$ and log. of b = log. of c), and log. of $\frac{c}{b} = \frac{3x}{4db}$. Let N equal a number (2.71828) whoſe hyp. log. is 1, and $\frac{c}{b} = N^{\frac{3x}{4bd}}$ and $b = \frac{c}{N^{\frac{3x}{4bd}}}$; but $v = \sqrt{4sb}$, and, ſubſtituting the value of b, $= \frac{\sqrt{4sc}}{N^{\frac{3x}{8bd}}}$. Q. E. I.

337. Cor. 1. The velocity at $C = \sqrt{4sc}$, and the velocity at $V = \frac{\sqrt{4sc}}{N^{\frac{3x}{8bd}}}$, and the velocity at C : vel. deſtroyed by the reſiſtance in moving through $CV :: \sqrt{4sc} : \sqrt{4sc} - \frac{\sqrt{4sc}}{N^{\frac{3x}{8bd}}} :: 1 : \frac{N^{\frac{3x}{8bd}} - 1}{N^{\frac{3x}{8bd}}}$.

The velocity loſt therefore = the velocity at $C \times \frac{N^{\frac{3x}{8bd}} - 1}{N^{\frac{3x}{8bd}}}$.

338. Cor.

* $\dot{v}$ repreſents the increaſe of velocity, or ſpace uniformly deſcribed by that increment in a given time, which varies as the force or weight, and time (i) in which it is generated conjointly, i being taken very ſmall that the action of the force may be conſtant. If therefore s be the ſpace deſcribed from reſt in 1″ by the conſtant action of a given force (1), $2s$ is the velocity generated in 1″, and $\dot{v} : 2s :: W \times i : 1 \times 1''$ and $\dot{v} = 2Ws \times i = \frac{2Ws \times b^{\cdot}}{v}$ and $v\dot{v} = 2Wsb^{\cdot}$. In the ſame manner, by comparing the velocities generated by the reſiſtance $\left(\frac{3Wb}{4db}\right)$ and this given force (1), $v\dot{v} = -\frac{6Wsb\dot{x}}{4db}$.

338. Cor. 2. If the velocities at any point C and V be given, their distance or x may be found. Let the velocity at C be to that at V as any number, $n : 1$; and consequently $n : 1 :: \sqrt{4sc} : \frac{\sqrt{4sc}}{N^{\frac{3x}{8bd}}}$ $:: N^{\frac{3x}{8bd}} : 1$, and $\frac{3x}{8bd} \times \log. \text{ of } N = \log. \text{ of } n$, and $x = \frac{\log. \text{ of } n \times 8bd}{3 \times \log. \text{ of } N}$.

339. Cor. 3. Let T be the time of describing CV or x, and since the velocity at $V = \frac{\sqrt{4sc}}{N^{\frac{3x}{8bd}}}$, and, in uniform motions, the time is equal to the space divided by the velocity, $T = \dot{x} \times \frac{N^{\frac{3x}{8bd}}}{\sqrt{4sc}}$, and $T =$ the fluent of this quantity. Let $N^{\frac{3x}{8bd}} = z$ and $\frac{3x}{8bd} =$ hyp. log. of z, and $\dot{x} = \frac{8bd}{3} \times \frac{\dot{z}}{z}$, and $\dot{x} \times N^{\frac{3x}{8bd}} = \frac{8bd}{3} \times \dot{z}$, and the fluent of $\frac{\dot{x} \times N^{\frac{3x}{8bd}}}{\sqrt{4sc}} =$ the fluent of $\frac{8bd}{3 \times \sqrt{4sc}} \times \dot{z} = \frac{8bd}{3 \times \sqrt{4sc}} \times z = \frac{8bd}{3\sqrt{4sc}} \times N^{\frac{3x}{8bd}} +$ cor. $= \frac{8bd}{3\sqrt{4sc}} \times N^{\frac{3x}{8bd}} - \frac{8bd}{3 \times \sqrt{4sc}} = \frac{8bd}{3\sqrt{4sc}} \times N^{\frac{3x}{8bd}} - 1$.

340. PROP. *Suppose a spherical body to descend by the action of gravity, from* C *in the right line* CP, *in an uniform resisting medium, to find the velocity* (v) *at any point* V. FIG. LXXXIII.

Let $CV = x$; diameter of the sphere $= d$; its specific gravity $= b$; that of the fluid equal 1; $b =$ the height through which a body falls from rest, unresisted, to acquire a velocity equal to v;

W = the weight of the ſphere, and w = the weight of an equal magnitude of the fluid $= \frac{W}{b}$.

Then $W - \frac{W}{b}$ = the weight of the body in the fluid, and this diminiſhed by the reſiſtance, or $W - \frac{W}{b} - \frac{3Wb}{4db}$ = the force of acceleration $= \overline{1 - \frac{1}{b} - \frac{3b}{4db}} \times W$. But $v\dot{v} = 2Wsb^{\cdot} = \overline{1 - \frac{1}{b} - \frac{3b}{4db}} \times 2sW\dot{x}$, and $\dot{x} = \frac{b^{\cdot}}{1 - \frac{1}{b} - \frac{3b}{4db}}$ and $\frac{\dot{x}}{4db} = \frac{b^{\cdot}}{4d \times \overline{b-1} - 3b}$.

Let $4d \times \overline{b-1} - 3b = z$ and $3b^{\cdot} = -\dot{z}$ and $\frac{b^{\cdot}}{4d \times \overline{b-1} - 3b} = -\frac{\dot{z}}{3z}$, and the fluent of $\frac{b^{\cdot}}{4d \times \overline{b-1} - 3b} = -\frac{1}{3} \times$ hyp. log. of $z = \frac{x}{4db} +$ cor. But when $x = 0$, b vaniſhes and $-\frac{1}{3} \times$ hyp. log. of $z = -\frac{1}{3} \times$ hyp. log. of $4d \times \overline{b-1}$, and conſequently $\frac{x}{4db} = -\frac{1}{3} \times$ hyp. log. of $\frac{4d \times \overline{b-1} - 3b}{4d \times \overline{b-1}}$. Let N = a number (2.71828) whoſe hyp. log. is unity, and $\overline{4d \times \overline{b-1} - 3b} = N^{-\frac{3x}{4db}} = 1 - \frac{3b}{4d \times \overline{b-1}}$; and $b = \frac{4d}{3} \times \overline{b-1} \times 1 - N^{-\frac{3x}{4db}}$. But the velocity at V or $v = \sqrt{4sh}$ = (ſubſtituting the value of h) $\sqrt{\frac{16ds}{3} \times \overline{b-1} \times 1 - N^{-\frac{3x}{4db}}}$.

341. Cor.

341. Cor. 1. Since the velocity $= \sqrt{\frac{16ds}{3} \times b - 1} \times 1 - N^{-\frac{3x}{4db}}$, if x be increaſed, $N^{-\frac{3x}{4db}}$ will be diminiſhed and vaniſh when x is infinitely great. The limit therefore of the velocity, which it can never exceed, nor arrive at till x become infinite, is equal to $\sqrt{16ds \times b - 1}$. Let the ſpecific gravity of the ſpherical body be twice that of the fluid, or $b = 2$, and $x = 16d$, and $\frac{3x}{4db} = 6$, and $N^{-6} = \frac{1}{403}$ and $1 - N^{-6} = \frac{402}{403}$, and $\sqrt{1 - N^{-6}} = \frac{805}{806}$, and the velocity of the ſphere, after deſcribing 16 diameters, is equal to $\sqrt{\frac{16ds}{3} \times b - 1} \times \frac{805}{806}$, or nearly equal to the limit. The velocity therefore, between this point and an infinite diſtance, may be deemed uniform.

342. Cor. 2. If the diameter of the ſpherical body be very ſmall, and its ſpecific gravity greater than that of the fluid, its velocity will be nearly uniform from the beginning of the motion, and equal to $\sqrt{\frac{16ds}{3} \times b - 1}$; becauſe it moves nearly at that rate, after deſcribing 16 diameters, and ſince $\frac{16s}{3}$ and $b - 1$ are given, it varies as $\sqrt{d}$. If d be infinitely ſmall, the velocity is alſo infinitely ſmall, and the body will not deſcribe a finite ſpace in any finite time.

343. Cor. 3. A ſpherical globule, whoſe ſpecific gravity is leſs than that of the fluid, will aſcend, and the velocity of aſcent is determined by a ſimilar progreſs with that in this propoſition

$w - W$, or $\frac{W}{b} - W$ diminiſhed by the reſiſtance, or $\frac{W}{b} - W - \frac{3Wh}{4bd} = \frac{1}{b} - 1 - 3b \times \frac{W}{4bd}$ is the force of acceleration; and by repeating the proceſs, the velocity after deſcribing any ſpace, x, is equal to $\sqrt{\frac{16ds}{3} \times \overline{1-b} \times 1 - N^{-\frac{3x}{4bd}}}$. If the diameter be infinitely ſmall or $N^{-\frac{3x}{4bd}} = 0$, the velocity, at any point $V = \sqrt{\frac{16ds}{3}}$.

344. Cor. 4. If a ſpherical body be projected upwards from P with a velocity, v, equal to that acquired in falling from reſt through p feet, the retarding force $= \frac{\overline{4d - 4db - 3b}}{4db} \times W = \frac{\overline{4d \times 1 - b - 3b}}{4db} \times W$, and $\frac{\dot{x}}{4db}$ (putting $PQ = x$, and retaining the ſame ſymbols) $= \frac{-\dot{b}}{\overline{4d \times 1 - b - 3b}}$ and making $4d \times \overline{1-b} - 3b = z$, $-3\dot{b} = -\dot{z}$ and $\frac{-\dot{b}}{\overline{4d \times 1 - b - 3b}} = \frac{\dot{z}}{3z}$, and $\frac{x}{4db} = \frac{1}{3} \times$ hyp. log. of $\overline{4d \times 1-b} - 3b +$ cor. $= \frac{1}{3} \times$ log. of $\overline{4d \times 1-b} - 3b +$ hyp. log. $\overline{4d \times 1-b} - 3p$.

345. Cor. 5. Since in uniform motions (and v is uniform for a very ſmall time) the time is equal to the ſpace divided the velocity; $\dot{T} = \sqrt{\frac{3}{16ds \times \overline{b-1}}} \times \frac{\dot{x}}{\sqrt{1 - N^{-\frac{3x}{4bd}}}}$, and $T = \sqrt{\frac{3}{14ds \times \overline{b-1}}} \times$ fluent of $\frac{\dot{x}}{\sqrt{1 - N^{-\frac{3x}{4bd}}}}$. Let $\sqrt{1 - N^{-\frac{3x}{4bd}}} = z$, and conſe-

quently

quently $1-z^2 = N^{-\frac{3x}{4bd}}$, and log. of $1-z^2 = -\frac{3x}{4bd}$ and $-\frac{2z\dot{z}}{1-z^2} = -\frac{3\dot{x}}{4bd}$, and $\dot{x} = \frac{8bd}{3} \times \frac{z\dot{z}}{1-z^2}$ and $\frac{\dot{x}}{\sqrt{1-N^{-\frac{3x}{4bd}}}} = \frac{8bd}{3} \times \frac{\dot{z}}{1-z^2}$, and the fluent of $\frac{\dot{x}}{\sqrt{1-N^{-\frac{3x}{4bd}}}}$ = the fluent of $\frac{8bd}{3} \times \frac{\dot{z}}{1-z^2} = \frac{8bd}{3} \times \frac{1}{1} \times$ hyp. log. of $\frac{1+z}{1-z} = \frac{8bd}{3\times 2} \times$ hyp. log. of $\frac{1+\sqrt{1-N^{-\frac{3x}{4bd}}}}{1-\sqrt{1-N^{-\frac{3x}{4bd}}}}$; and T therefore is equal to $\sqrt{\frac{3}{16ds \times b-1}} \times \frac{4bd}{3} \times$ hyp. log. of $\frac{1+\sqrt{1-N^{-\frac{3x}{4bd}}}}{1-\sqrt{1-N^{-\frac{3x}{4bd}}}} = \sqrt{\frac{b^2d}{3s \times b-1}} \times$ hyp. log. of $\frac{1+\sqrt{1-N^{-\frac{3x}{4bd}}}}{1-\sqrt{1-N^{-\frac{3x}{4bd}}}}$.

346. PROP. *Let an aeroſtatic globe aſcend from reſt, from the earth's ſurface at* A, *in the right line* AP, *to find its velocity at any point* O. FIG. LXXXIV.

Let the weight of the globe $= w$; the weight of an equal bulk of air at $A = W$; diameter of the globe $= 2r$; its magnitude $= 1$; denſity of the air at $A = 1$; $AO = x$ miles.

It is collected from experiments that the denſities of air, at A and 7 miles above it, are as 4 : 1 and therefore the denſity at the height of 7 miles $= \frac{1}{4}$. But altitudes being taken in arithmetic progreſſion, the reſpective denſities are in geometric, and conſequently $7 : x ::$ log. $\frac{1}{4}$: log. denſity at $O =$ log. $\frac{1}{4} \times \frac{x}{7} =$

$L \times$

$L \times \frac{x}{7}$, (putting L for the logarithm of $\frac{1}{4}$) and the denſity at any point $O = L^{-\frac{x}{7}}$, and the weight of a bulk of air, equal to the globe, at any point $O = W \times L^{-\frac{x}{7}}$. The force of aſcent $= W \times L^{-\frac{x}{7}} - w$, and the force of acceleration is equal to this diminiſhed by the reſiſtance, or equal to $W \times L^{-\frac{x}{7}} - w - \frac{3W}{8r} \times \frac{v^2}{4s} \times L^{-\frac{x}{7}}$ (318); but (ſince $\dot{v} : 2s :: \text{force} \times i : w \times 1''$ and $\dot{v} = \frac{2s \times \text{force} \times i}{w}$)

$2v\dot{v} = 4s\dot{x} \times \frac{W}{w} \times L^{-\frac{x}{7}} - \frac{w}{w} - \frac{3W}{8r \times w} \times \frac{v^2}{4s} \times L^{-\frac{x}{7}} =$ (making $\frac{W}{w} = a$) $4s\dot{x} \times aL^{-\frac{x}{7}} - 4s\dot{x} - \frac{3a}{8r} \times v^2 \times L^{-\frac{x}{7}}\dot{x}$, and making $\frac{3a}{8r} = \frac{1}{b}$) $2v\dot{v} + \frac{v^2}{b} \times \frac{\dot{x}}{L^{\frac{x}{7}}} = \frac{4s\dot{x}}{L^{\frac{x}{7}}} \times \overline{a-1}$*. The fluents being found by ſeparating the unknown quantities v and x, the velocity will conſequently then be found. Q. E. I. †

SCHOLIUM.

347. The next natural principle, which forms one diviſion in the arrangement of this branch of philoſophy, is the attraction of coheſion (1); but the phenomena apparently reſulting from it, though very numerous, being imperfectly underſtood and not reduced to preciſe menſuration, a few of the moſt popular of them only are therefore here added as corollaries from Prop. Art. 52.

Pre-

* To diſcover the fluent of that part of the equation involving v and x conſult Waring's Meditat. Analyt. p. 123. See this problem ſolved by Euler, Hiſtoire de l'Academie Royale des Sciences. Ann. 1784; where $v^2 = \frac{4asb}{7} \times \overline{b + \frac{1}{a} \times 1 - L^{\frac{x}{b}}} - x$; from which value of the velocity, the greateſt height to which the globe can aſcend, and time are eaſily aſcertained.

† Upon the reſiſtances of moving bodies ſee Newton's Principia, Vol. II. Euler's Mechan. Comment. on Robins's gunnery. Dr. Horſeley's Newton, Vol. II. p. 320. Maclaurin's fluxions, Sect. 606. Atwood, Sect. 5. of rectilineal motion. Saunderſon's, Simpſon's, Lyons's, Emerſon's, fluxions.

Plate X.

FIG. LXXV.

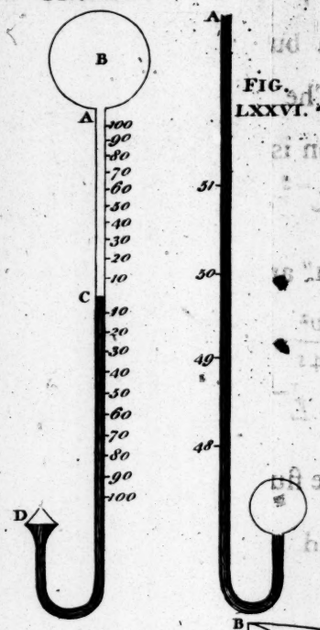


FIG. LXXVI.

FIG. LXXVII.



FIG. LXXVIII.

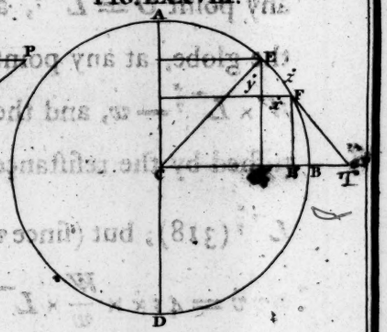


FIG. LXXIX.

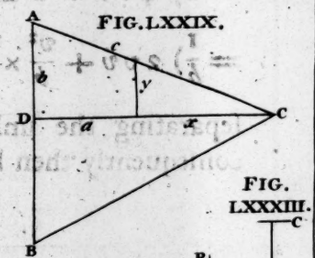


FIG. LXXX.

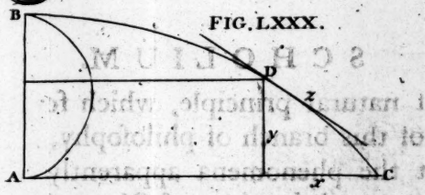


FIG. LXXXI.

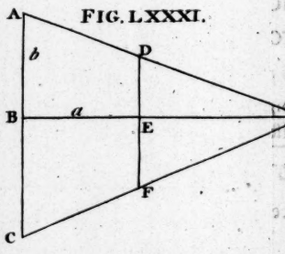


FIG. LXXXII.

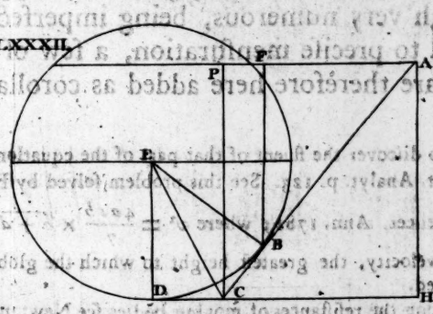


FIG. LXXXIII.

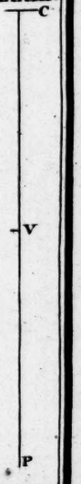
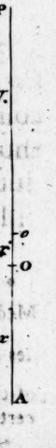


FIG. LXXXIV.




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Preſuming that evaporation is a gradual ſolution of water in air, which ſeems to be ſatisfactorily eſtabliſhed (52), and, like other ſolutions, promoted by heat and checked by cold, the following appearances are deduced from it. 1. In every ſolution to the point of ſaturation, the fluid will retain the diſſolved body always ſuſpended, provided the temperature be not altered; and if the temperature and denſity of the atmoſphere were to be invariably the ſame, and ſaturated with watery particles, theſe would for ever remain ſuſpended in it; conſequently a perpetual circulation of vapours is only preſerved by frequent changes of heat and cold, condenſation and rarefaction, in the atmoſphere. 2. When a hot fluid is ſaturated with any body, diſſolved in it, and ſuffered to cool, it becomes opake, and particles of the body deſcend, by their greater ſpecific gravity, to the bottom; and in the ſame manner, the air, towards the evening of a hot day, being cooled by the ſun's abſence, becomes hazy and opake, and particles of water, returning from ſolution, are precipitated in dew. This hazineſs continues to near the morning, and about ſun riſe appears again, when the dew is mixed with the air, previous to its ſolution. 3. As air when partially heated aſcends and the higher parts of the atmoſphere are colder and rarer than the lower, air, which has been much heated near the earth's ſurface, will aſcend together with the water diſſolved in it, till it be ſtopped by air too thin and cold to preſerve the ſolution, and then particles of water become viſible and form clouds; or riſing up the ſides of mountains by the force of winds, or their attraction of gravitation, and cooled in their aſcent, watery particles are precipitated upon their ſurfaces, or viſibly adhere to their ſummits. The height at which theſe viſible vapours are formed, depends upon the quantity of vapour raiſed, and the denſity and heat of the upper parts of the atmoſphere: they are therefore generally higher in ſummer than winter. But they are often ſeen upon the ſurface of low marſhy land, ariſing from a ſudden change of temperature, as from the heat of day, to the coolneſs of the ſucceeding night. When clouds are encreaſed by additional vapours, or the par-

particles of water contained in them are driven together by winds or any violent agitations of the air, they fall down in drops of rain, the magnitude of which depends upon theſe agitations and upon the height deſcended, and are therefore greater in ſummer than winter, and ſtill greater in thunder ſhowers. When the upper parts of the atmoſphere are very cold, clouds are frequently frozen and deſcend in ſnow; or in hail, if the particles be firſt collected into drops and then frozen. (Hamilton on the aſcent of vapours.)

Whether the ſuſpenſion of water in air, and of metals in the fluids diſſolving them, be an effect of the ſame power, by which water aſcends into dry porous ſubſtances, and up the capillary tubes of glaſs, plants, &c. or of ſome other undiſcovered cauſe, hath been doubted; but ſatisfactory reaſons ought to be aſſigned againſt the actual exerciſe of this attraction, or its incompetency, to produce the effect, proved, before any new principle be introduced.



